Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

AS MATHEMATICS

Paper 2

Please note that question 17 uses the original Large Data Set "Family Food". This was replaced by the data set "Transport Stock Vehicle Database" in AS exams from June 2019. If you'd like to see the original data set, please contact maths@aqa.org.uk.

Wednesday 23 May 2018 Morning Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Given that $\frac{dy}{dx} = \frac{1}{6x^2}$ find y.

Circle your answer.

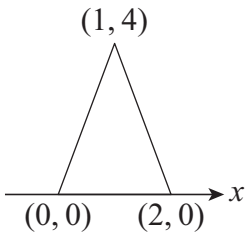
[1 mark]

$\frac{-1}{3x^3} + c$ $\quad$ $\frac{1}{2x^3} + c$ $\quad$ $\frac{-1}{6x} + c$ $\quad$ $\frac{-1}{3x} + c$

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2 **Figure 1** shows $y = f(x)$.

Figure 1



Which figure below shows $y = f(2x)$?

Tick **one** box.

[1 mark]

Figure 2

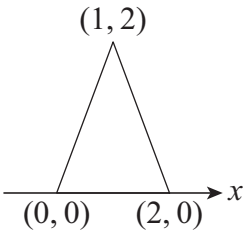


Figure 3

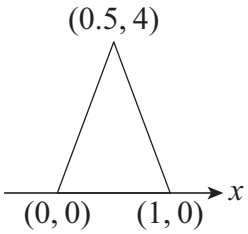


Figure 4

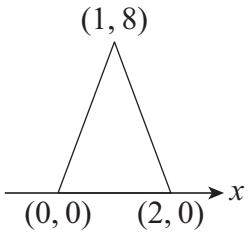


Figure 5

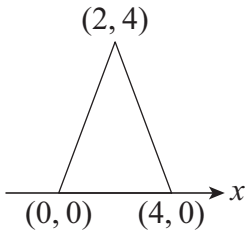


Figure 2 ☐

Figure 3 ☑

Figure 4 ☐

Figure 5 ☐

Turn over ▶

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3 Express as a single logarithm

$$2\log_a 6 - \log_a 3$$

[2 marks]

$2\log_a 6 - \log_a 3 = \log_a 6^2 - \log_a 3$

$= \log_a 36 - \log_a 3 = \log_a \frac{36}{3}$

$= \log_a 12$

4 Solve the equation $\tan^2 2\theta - 3 = 0$ giving all the solutions for $0° \leq \theta \leq 360°$

[4 marks]

$\tan^2 2\theta = 3 \quad \Rightarrow \quad \tan 2\theta = \pm\sqrt{3}$

$0° \leqslant \theta \leqslant 360° \quad \Rightarrow \quad 0° \leqslant 2\theta \leqslant 720°$

$\tan 2\theta = \sqrt{3}: \quad 2\theta = 60°, 240°, 420°, 600°$

$\tan 2\theta = -\sqrt{3}: \quad 2\theta = 120°, 300°, 480°, 660°$

$\Rightarrow \quad \theta = 30°, 120°, 210°, 300°$

$\theta = 60°, 150°, 240°, 330°$

$\theta = 30°, 60°, 120°, 150°, 210°, 240°, 300°, 330°$

Do not write outside the box

5 $f'(x) = \left(2x - \frac{3}{x}\right)^2$ and $f(3) = 2$

Find $f(x)$.

[4 marks]

$f(x) = (2x - \frac{3}{x})^2 = 4x^2 - 12 + \frac{9}{x^2}$

$f(x) = 4x^2 - 12 + 9x^{-2}$

$f(x) = \int (4x^2 - 12 + 9x^{-2})\, dx = \frac{4x^3}{3} - 12x - \frac{9}{x} + c$

$f(3) = 2 : \quad \frac{4 \times 27}{3} - 36 - \frac{9}{3} + c = 2$

$36 - 36 - 3 + c = 2 \quad \Rightarrow \quad c = 5$

$f(x) = \frac{4x^3}{3} - 12x - \frac{9}{x} + 5$

Do not write outside the box

6 Points A $(-7, -7)$, B $(8, -1)$, C $(4, 9)$ and D $(-11, 3)$ are the vertices of a quadrilateral $ABCD$.

6 (a) Prove that $ABCD$ is a rectangle.

[4 marks]

grad $AB = \frac{-1-(-7)}{8-(-7)} = \frac{6}{15} = \frac{2}{5}$ grad $DC = \frac{9-3}{4-(-11)} = \frac{6}{15} = \frac{2}{5}$

grad $BC = \frac{9-(-1)}{4-8} = \frac{10}{-4} = -\frac{5}{2}$ grad $AD = \frac{3-(-7)}{-11-(-7)} = \frac{10}{-4} = -\frac{5}{2}$

grad AB = grad DC, so AB is parallel to DC

grad BC = grad AD, so BC is parallel to AD

Both pairs of opposite sides are parallel, so ABCD is a parallelogram.

grad AB × grad BC = $\frac{2}{5} \times (-\frac{5}{2}) = -1$

so AB is perpendicular to BC, i.e. angle ABC = 90°

A parallelogram with one right angle has all angles 90°

(opposite angles equal, co-interior angles add to 180°).

∴ ABCD has 2 pairs of parallel sides and all angles are 90°,

so ABCD is a rectangle.

6 (b) Find the area of $ABCD$.

[2 marks]

$AB = \sqrt{(15^2 + 6^2)} = \sqrt{261} = 3\sqrt{29}$

$BC = \sqrt{((-4)^2 + 10^2)} = \sqrt{116} = 2\sqrt{29}$

Area = AB × BC = $3\sqrt{29} \times 2\sqrt{29} = 6 \times 29$

Area = 174

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7 (a) Express $2x^2 - 5x + k$ in the form $a(x - b)^2 + c$

[3 marks]

$2x^2 - 5x + k = 2(x^2 - \frac{5}{2}x) + k$

$= 2[(x - \frac{5}{4})^2 - \frac{25}{16}] + k$

$= 2(x - \frac{5}{4})^2 - \frac{25}{8} + k$

$2x^2 - 5x + k = 2(x - \frac{5}{4})^2 + k - \frac{25}{8}$ $\quad (a = 2, b = \frac{5}{4}, c = k - \frac{25}{8})$

7 (b) Find the values of k for which the curve $y = 2x^2 - 5x + k$ does **not** intersect the line $y = 3$

[3 marks]

Minimum point of the curve is at $x = \frac{5}{4}$, where $(x - \frac{5}{4})^2 = 0$

minimum value of $y = k - \frac{25}{8}$

u-shaped curve does not meet $y = 3$ if its minimum is above 3:

$k - \frac{25}{8} > 3$

$k > \frac{49}{8}$

Check: $2x^2 - 5x + k - 3 = 0$ has no real roots

$(-5)^2 - 4 \times 2 \times (k - 3) < 0 \Rightarrow 49 - 8k < 0 \Rightarrow k > \frac{49}{8}$

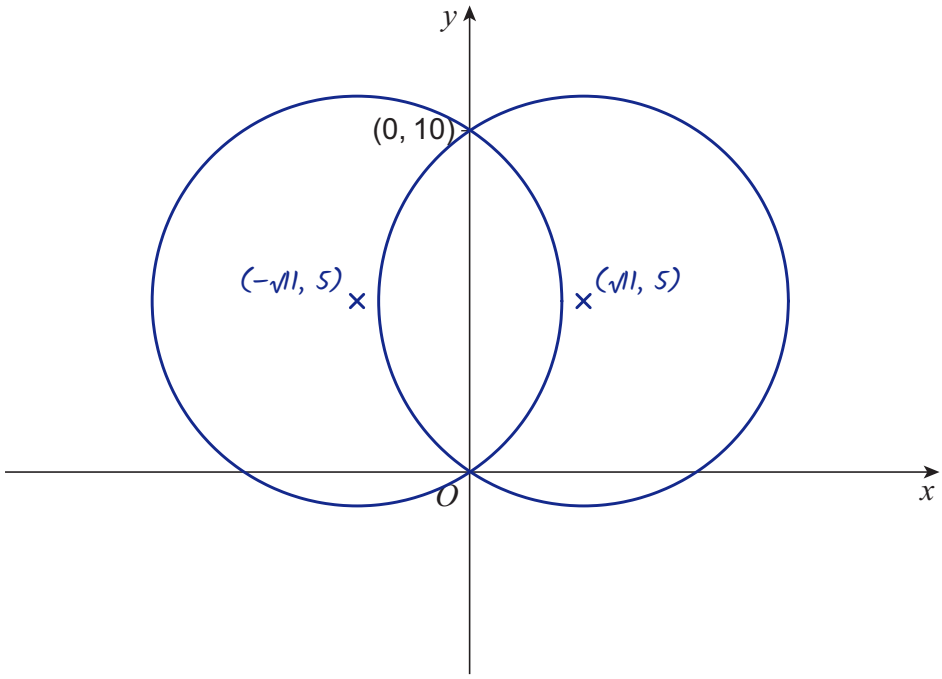
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8 A circle of radius 6 passes through the points (0, 0) and (0, 10).

8 (a) Sketch the two possible positions of the circle.

[1 mark]



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8 (b) Find the equations of the two circles.

[3 marks]

Both points lie on each circle, so the centre is on the perpendicular bisector of (0, 0) and (0, 10): $y = 5$

Centre $(p, 5)$, distance to $(0, 0)$ is the radius 6:

$p^2 + 5^2 = 6^2 \Rightarrow p^2 = 11 \Rightarrow p = \pm\sqrt{11}$

Centres $(\sqrt{11}, 5)$ and $(-\sqrt{11}, 5)$

$(x - \sqrt{11})^2 + (y - 5)^2 = 36$

$(x + \sqrt{11})^2 + (y - 5)^2 = 36$

Turn over for the next question

Do not write outside the box

9 It is given that $\cos 15° = \frac{1}{2}\sqrt{2+\sqrt{3}}$ and $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$

Show that $\tan^2 15°$ can be written in the form $a + b\sqrt{3}$, where a and b are integers.

Fully justify your answer.

[3 marks]

$\tan 15° = \frac{\sin 15°}{\cos 15°}$ so $\tan^2 15° = \frac{\sin^2 15°}{\cos^2 15°}$

$\tan^2 15° = \frac{\frac{1}{4}(2-\sqrt{3})}{\frac{1}{4}(2+\sqrt{3})} = \frac{2-\sqrt{3}}{2+\sqrt{3}}$

$= \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$ (multiply by the conjugate)

$= \frac{(2-\sqrt{3})^2}{4-3} = \frac{4-4\sqrt{3}+3}{1}$

$\tan^2 15° = 7 - 4\sqrt{3}$

which is in the form $a + b\sqrt{3}$ with $a = 7$ and $b = -4$ (integers)

Do not write outside the box

10 In the binomial expansion of $(1+x)^n$, where $n \geq 4$, the coefficient of x^4 is $1\frac{1}{2}$ times the sum of the coefficients of x^2 and x^3

Find the value of n.

[5 marks]

Coefficients: x^2: ${}^nC_2$, x^3: ${}^nC_3$, x^4: ${}^nC_4$

${}^nC_4 = \frac{3}{2}({}^nC_2 + {}^nC_3)$

$\frac{n(n-1)(n-2)(n-3)}{24} = \frac{3}{2}\left(\frac{n(n-1)}{2} + \frac{n(n-1)(n-2)}{6}\right)$

$n \geq 4$ so $n(n-1) \neq 0$; multiply by $\frac{24}{n(n-1)}$:

$(n-2)(n-3) = 18 + 6(n-2)$

$n^2 - 5n + 6 = 6n + 6$

$n^2 - 11n = 0 \Rightarrow n(n-11) = 0$

$n = 0$ or $n = 11$, but $n \geq 4$

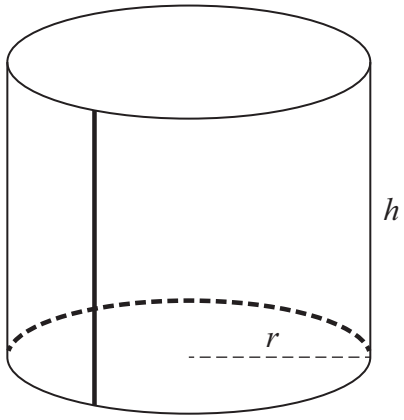
$n = 11$

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11 Rakti makes open-topped cylindrical planters out of thin sheets of galvanised steel.

She bends a rectangle of steel to make an open cylinder and welds the joint. She then welds this cylinder to the circumference of a circular base.



The planter must have a capacity of 8000 cm^3

Welding is time consuming, so Rakti wants the total length of weld to be a minimum.

Calculate the radius, r, and height, h, of a planter which requires the minimum total length of weld.

Fully justify your answers, giving them to an appropriate degree of accuracy.

[9 marks]

Weld: vertical joint (length h) + circumference of base ($2\pi r$)

$w = h + 2\pi r$

Volume: $\pi r^2 h = 8000 \Rightarrow h = \frac{8000}{\pi r^2}$

$w = \frac{8000}{\pi r^2} + 2\pi r = \frac{8000}{\pi} r^{-2} + 2\pi r$

For minimum w, $\frac{dw}{dr} = 0$

$\frac{dw}{dr} = -\frac{16000}{\pi} r^{-3} + 2\pi = -\frac{16000}{\pi r^3} + 2\pi$

$-\frac{16000}{\pi r^3} + 2\pi = 0 \Rightarrow 2\pi^2 r^3 = 16000 \Rightarrow \pi^2 r^3 = 8000$

$r^3 = \frac{8000}{\pi^2} = 810.57... \Rightarrow r = 9.3238...$ cm

$h = \frac{8000}{\pi \times 9.3238...^2} = 29.291...$ cm $(h = \pi r)$

$\frac{d^2w}{dr^2} = \frac{48000}{\pi} r^{-4} = \frac{48000}{\pi r^4}$

at $r = 9.3238...$, $\frac{d^2w}{dr^2} = \frac{48000}{\pi \times 9.3238...^4} = 2.02 > 0$

so this stationary point is a minimum for the length of weld.

Minimum weld when r = 9.3 cm and h = 29.3 cm (3 s.f.)

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Turn over ▶

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12 Trees in a forest may be affected by one of two types of fungal disease, but not by both.

The number of trees affected by disease A, n_A, can be modelled by the formula

$$n_A = ae^{0.1t}$$

where t is the time in years after 1 January 2017.

The number of trees affected by disease B, n_B, can be modelled by the formula

$$n_B = be^{0.2t}$$

On 1 January 2017 a **total** of 290 trees were affected by a fungal disease.

On 1 January 2018 a **total** of 331 trees were affected by a fungal disease.

12 (a) Show that $b = 90$, to the nearest integer, and find the value of a.

[3 marks]

1 Jan 2017, $t = 0$: $a + b = 290 \Rightarrow a = 290 - b$

1 Jan 2018, $t = 1$: $ae^{0.1} + be^{0.2} = 331$

$(290 - b)e^{0.1} + be^{0.2} = 331$

$b(e^{0.2} - e^{0.1}) = 331 - 290e^{0.1}$

$b = \frac{331 - 290e^{0.1}}{e^{0.2} - e^{0.1}} = 90.340...$

$b = 90$ to the nearest integer

$a = 290 - 90 = 200$

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12 (b) Estimate the total number of trees that will be affected by a fungal disease on 1 January 2020.

[1 mark]

1 Jan 2020, $t = 3$: $200e^{0.3} + 90e^{0.6} = 433.96...$

about 434 trees

12 (c) Find the year in which the number of trees affected by disease B will first exceed the number affected by disease A.

[3 marks]

B exceeds A when $90e^{0.2t} > 200e^{0.1t}$

$e^{0.1t} > \frac{200}{90} = \frac{20}{9}$

$0.1t > \ln(\frac{20}{9})$

$t > 10\ln(\frac{20}{9}) = 7.985...$

$t = 7.985...$ years after 1 January 2017 is just before 1 January 2025,

so the number with disease B first exceeds A during 2024.

Year: 2024

12 (d) Comment on the long-term accuracy of the model.

[1 mark]

Both n_A and n_B grow without limit, but there is a finite

number of trees in the forest, so the model will break down

and is not accurate in the long term.

Turn over for Section B

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Section B

Answer **all** questions in the spaces provided.

13 The table below shows the probability distribution for a discrete random variable X.

x	0	1	2	3	4 or more
$\mathbf{P}(X = x)$	0.35	0.25	k	0.14	0.1

Find the value of k.

Circle your answer.

[1 mark]

0.14 (0.16) 0.18 1

14 Given that $\sum x = 364$, $\sum x^2 = 19\,412$, $n = 10$, find σ, the standard deviation of X.

Circle your answer.

[1 mark]

(24.8) 44.1 616.2 1941.2

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15 Nicola, a darts player, is practising hitting the bullseye. She knows from previous experience that she has a probability of 0.3 of hitting the bullseye with each dart.

Nicola throws eight practice darts.

15 (a) Using a binomial distribution, calculate the probability that she will hit the bullseye three or more times.

[2 marks]

X = number of bullseyes, $X \sim B(8, 0.3)$

$P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - 0.55177...$

$P(X \geq 3) = 0.44822... = 0.448$ (3 s.f.)

15 (b) Nicola throws eight practice darts on three different occasions. Calculate the probability that she will hit the bullseye three or more times on all three occasions.

[2 marks]

P(three or more on all three occasions) $= 0.44822...^3$

$= 0.090051...$

$= 0.0901$ (3 s.f.)

15 (c) State two assumptions that are necessary for the distribution you have used in part **(a)** to be valid.

[2 marks]

1. The probability of hitting the bullseye is constant (fixed at 0.3) for every dart.

2. Hitting the bullseye with each dart is independent of hitting the bullseye with any other dart.

Turn over ▶

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16 Kevin is the Principal of a college.

He wishes to investigate types of transport used by students to travel to college.

There are 3200 students in the college and Kevin decides to survey 60 of them.

Describe how he could obtain a simple random sample of size 60 from the 3200 students.

[4 marks]

Obtain a list of all 3200 students (the sampling frame) and give each student a different number from 0001 to 3200.

Use a calculator or computer random number generator to generate random four-digit numbers.

Ignore any number outside the range 0001 to 3200 and ignore any repeats.

Continue until 60 different numbers have been obtained.

Select the 60 students whose numbers match these numbers to form the simple random sample.

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17 The table below is an extract from the Large Data Set, showing the purchased quantities of fats and oils for the South East of England in 2014.

Description	Purchased quantity
Butter	42
Soft margarine	16
Olive oil	17
Other vegetable and salad oils	28

Kim claims that more olive oil was purchased in the South East than soft margarine.

Explain why Kim may be incorrect.

[2 marks]

No units are given for the purchased quantities, so they could be in different units and cannot be compared directly. In the Large Data Set, butter and margarine are measured in grams (g) whereas oils are measured in millilitres (ml), so 17 ml of olive oil is not necessarily more than 16 g of margarine.

Turn over for the next question

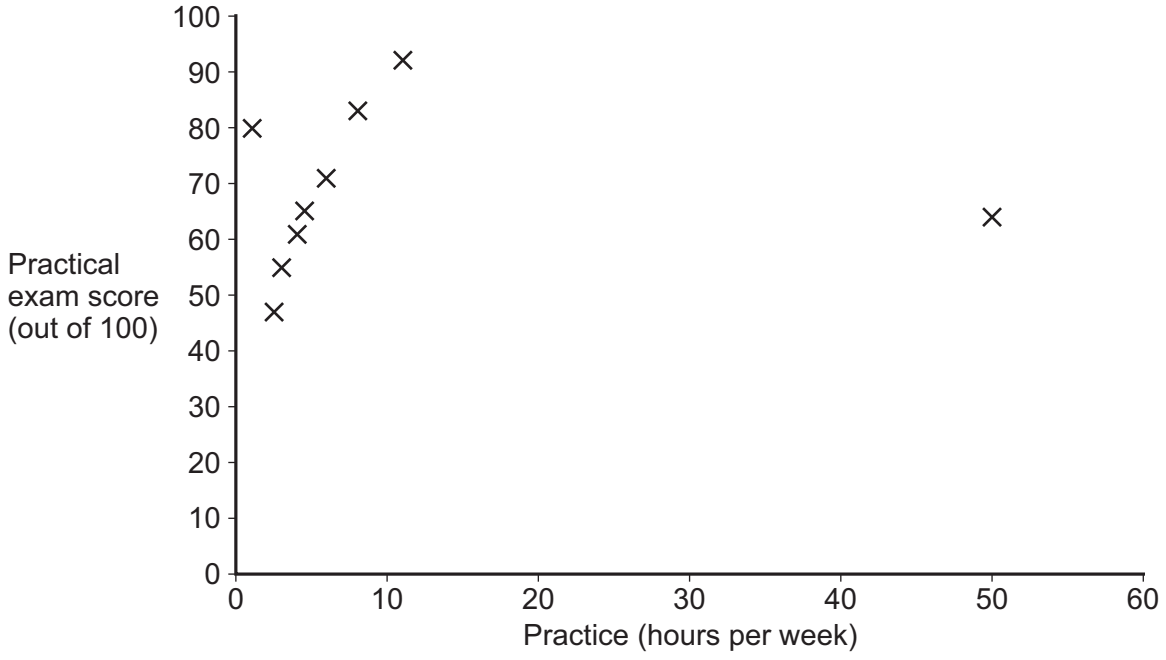
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18 Jennie is a piano teacher who teaches nine pupils.

She records how many hours per week they practice the piano along with their most recent practical exam score.

Student	Practice (hours per week)	Practical exam score (out of 100)
Donovan	50	64
Vazquez	6	71
Higgins	3	55
Begum	2.5	47
Collins	1	80
Coldbridge	4	61
Nedbalek	4.5	65
Carter	8	83
White	11	92

She plots a scatter diagram of this data, as shown below.



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18 (a) Identify two possible outliers by name, giving a possible explanation for the position on the scatter diagram of each outlier.

[4 marks]

First outlier Donovan

Possible reason Data entry error: 50 hours per week is unrealistic, it should probably be 5 hours (fits with the other points).

Second outlier Collins

Possible reason Naturally very able pianist, so scores highly despite only doing 1 hour of practice per week.

18 (b) Jennie discards the two outliers.

18 (b) (i) Describe the correlation shown by the scatter diagram for the remaining points.

[1 mark]

Strong positive correlation

18 (b) (ii) Interpret this correlation in the context of the question.

[1 mark]

Students who practise for more hours per week tend to get higher practical exam scores.

Turn over for the next question

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19 Martin grows cucumbers from seed.

In the past, he has found that 70% of all seeds successfully germinate and grow into cucumber plants.

He decides to try out a new brand of seed.

The producer of this brand claims that these seeds are more likely to successfully germinate than other brands of seeds.

Martin sows 20 of this new brand of seed and 18 successfully germinate.

Carry out a hypothesis test at the 5% level of significance to investigate the producer's claim.

[7 marks]

Let p = probability that a seed of the new brand germinates

H_0: $p = 0.7$

H_1: $p > 0.7$ (one-tailed test, 5% level)

X = number of seeds out of 20 that germinate

Under H_0, $X \sim B(20, 0.7)$

$P(X \geq 18) = 1 - P(X \leq 17)$

$= 1 - 0.96451...$

$= 0.035483... = 0.0355$

$0.0355 < 0.05$, so the result is significant

Reject H_0

There is sufficient evidence at the 5% level to conclude that the new brand of seeds is more likely to germinate, so the evidence supports the producer's claim.

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END OF QUESTIONS

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