AQA

Please write clearly in block capitals.

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AS MATHEMATICS

Paper 1

Wednesday 16 May 2018 Morning Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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Section A

Answer **all** questions in the spaces provided.

1 Three of the following points lie on the same straight line.

Which point does **not** lie on this line?

Tick **one** box.

[1 mark]

(−2, 14) ☐

(−1, 8) ☑

(1, −1) ☐

(2, −6) ☐

2 A circle has equation $(x-2)^2+(y+3)^2=13$

Find the gradient of the tangent to this circle at the origin.

Circle your answer.

[1 mark]

$-\frac{3}{2}$ $-\frac{2}{3}$ $\frac{2}{3}$ $\frac{3}{2}$

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3 State the interval for which $\sin x$ is a decreasing function for $0° \leq x \leq 360°$

[2 marks]

sin x has a maximum at x = 90° and a minimum at x = 270°

so sin x is decreasing between these two points:

90° < x < 270°

Turn over for the next question

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4 (a) Find the first three terms in the expansion of $(1 - 3x)^4$ in ascending powers of x.

[3 marks]

$(1 - 3x)^4 = 1 + {}^4C_1(-3x) + {}^4C_2(-3x)^2 + \ldots$

$= 1 + 4(-3x) + 6(9x^2) + \ldots$

$= 1 - 12x + 54x^2$

4 (b) Using your expansion, approximate $(0.994)^4$ to six decimal places.

[2 marks]

$0.994^4 = (1 - 3x)^4$ where $1 - 3x = 0.994 \Rightarrow x = 0.002$

$0.994^4 \approx 1 - 12(0.002) + 54(0.002)^2$

$= 1 - 0.024 + 0.000216$

$= 0.976216$

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5 Point C has coordinates $(c, 2)$ and point D has coordinates $(6, d)$.

The line $y + 4x = 11$ is the perpendicular bisector of CD.

Find c and d.

[5 marks]

$y + 4x = 11 \Rightarrow y = 11 - 4x$, gradient $= -4$

CD is perpendicular to this line, so gradient of $CD = \frac{1}{4}$

Gradient of $CD = \frac{d-2}{6-c} = \frac{1}{4}$

$4(d-2) = 6 - c \Rightarrow 4d - 8 = 6 - c \Rightarrow c + 4d = 14 \quad (1)$

Midpoint of $CD = \left(\frac{c+6}{2}, \frac{2+d}{2}\right)$ lies on $y + 4x = 11$:

$\frac{2+d}{2} + 4\frac{c+6}{2} = 11$

$2 + d + 4c + 24 = 22 \Rightarrow 4c + d = -4 \quad (2)$

From (1): $c = 14 - 4d$

Sub into (2): $4(14 - 4d) + d = -4$

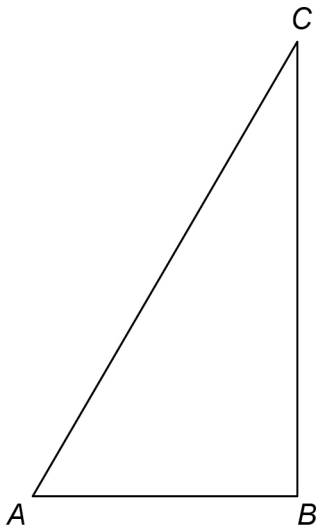
$56 - 15d = -4 \Rightarrow 15d = 60 \Rightarrow d = 4$

$c = 14 - 4(4) = -2$

$c = -2, \ d = 4$

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6 ABC is a right-angled triangle.



D is the point on hypotenuse AC such that $AD = AB$.

The area of ΔABD is equal to half that of ΔABC.

6 (a) Show that $\tan A = 2 \sin A$

[4 marks]

Let $AB = AD = c$.

Area of $ABD = \frac{1}{2} \times AB \times AD \times \sin A = \frac{1}{2} c^2 \sin A$

In triangle ABC: $\tan A = \frac{BC}{AB} \Rightarrow BC = c \tan A$

Area of $ABC = \frac{1}{2} \times AB \times BC = \frac{1}{2} c^2 \tan A$

Area of ABC = 2 × area of ABD:

$\frac{1}{2} c^2 \tan A = 2 \times \frac{1}{2} c^2 \sin A$

Dividing by $\frac{1}{2} c^2$ $(c \neq 0)$:

$\therefore \tan A = 2 \sin A$ as required

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6 (b) (i) Show that the equation given in part **(a)** has two solutions for $0° \leq A \leq 90°$

[2 marks]

$\frac{\sin A}{\cos A} = 2 \sin A \Rightarrow \sin A = 2 \sin A \cos A$

$\sin A - 2 \sin A \cos A = 0 \Rightarrow \sin A(1 - 2 \cos A) = 0$

$\sin A = 0 \Rightarrow A = 0°$ or $\cos A = \frac{1}{2} \Rightarrow A = 60°$

For $0° \leq A \leq 90°$ the solutions are $A = 0°$ and $A = 60°$,

so the equation has two solutions.

6 (b) (ii) State the solution which is appropriate in this context.

[1 mark]

$A = 60°$ ($A = 0°$ is impossible in a triangle)

Turn over for the next question

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7 Prove that

n is a prime number greater than $5 \Rightarrow n^4$ has final digit 1

[5 marks]

The final digit of n^4 depends only on the final digit of n.

n is prime and n > 5, so n is not even (else divisible by 2)

and n does not end in 5 (else divisible by 5).

So the final digit of n must be 1, 3, 7 or 9.

Ends in 1: $1^4 = 1$, so n^4 ends in 1

Ends in 3: $3^4 = 81$, so n^4 ends in 1

Ends in 7: $7^4 = 2401$, so n^4 ends in 1

Ends in 9: $9^4 = 6561$, so n^4 ends in 1

All possible cases give final digit 1, so by exhaustion

n is a prime number greater than $5 \Rightarrow n^4$ has final digit 1

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Turn over for the next question

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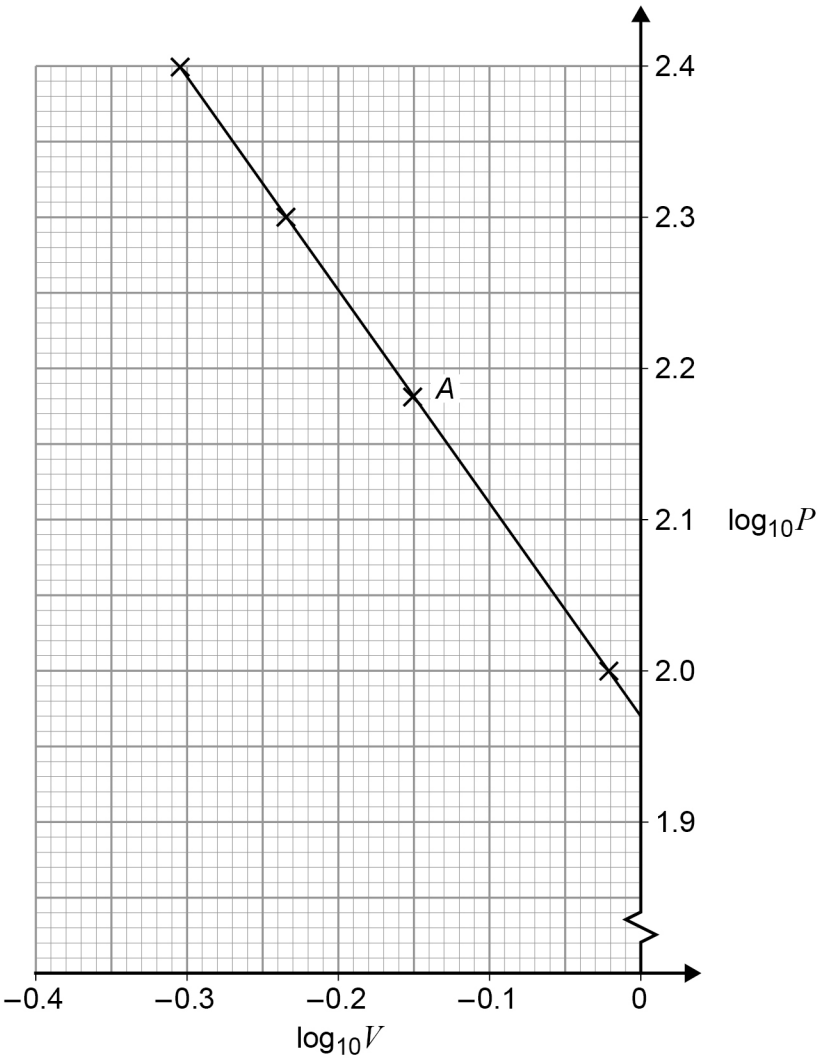
8 Maxine measures the pressure, P kilopascals, and the volume, V litres, in a fixed quantity of gas.

Maxine believes that the pressure and volume are connected by the equation

$$P = cV^d$$

where c and d are constants.

Using four experimental results, Maxine plots $\log_{10}P$ against $\log_{10}V$, as shown in the graph below.



8 (a) Find the value of P and the value of V for the data point labelled *A* on the graph.

[2 marks]

At A: $\log_{10}P = 2.18$, $\log_{10}V = -0.15$

$P = 10^{2.18} = 151.35... \approx 151$

$V = 10^{-0.15} = 0.70794... \approx 0.708$

$P = 151$ kPa, $V = 0.708$ litres

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8 (b) Calculate the value of each of the constants c and d.

[4 marks]

$P = cV^d \Rightarrow \log_{10}P = \log_{10}c + d\log_{10}V$

so the gradient of the line is d and the intercept is $\log_{10}c$

Gradient using (−0.3, 2.4) and (−0.02, 2.0):

$d = \frac{2.0 - 2.4}{-0.02 - (-0.3)} = \frac{-0.4}{0.28} = -1.428... \Rightarrow d \approx -1.4$

Intercept on $\log_{10}P$ axis ($\log_{10}V = 0$): $\log_{10}c = 1.97$

$c = 10^{1.97} = 93.32... \Rightarrow c \approx 93.3$

$c = 93.3$, $d = -1.4$

8 (c) Estimate the pressure of the gas when the volume is 2 litres.

[2 marks]

$P = cV^d = 93.3 \times 2^{-1.4}$

$= 93.3 \times 0.37892... = 35.35...$

$P \approx 35.4$ kilopascals

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9 Craig is investigating the gradient of chords of the curve with equation $f(x) = x - x^2$

Each chord joins the point $(3, -6)$ to the point $(3 + h, f(3 + h))$

The table shows some of Craig's results.

x	$f(x)$	h	$x + h$	$f(x + h)$	Gradient
3	−6	1	4	−12	−6
3	−6	0.1	3.1	−6.51	−5.1
3	−6	0.01	3.01	−6.0501	−5.01
3	−6	0.001			
3	−6	0.0001			

9 (a) Show how the value -5.1 has been calculated.

[1 mark]

$f(3.1) = 3.1 - 3.1^2 = -6.51$

$\text{Gradient} = \frac{f(3.1) - f(3)}{3.1 - 3} = \frac{-6.51 - (-6)}{0.1} = \frac{-0.51}{0.1} = -5.1$

9 (b) Complete the third row of the table above.

[2 marks]

$x + h = 3 + 0.01 = 3.01$

$f(3.01) = 3.01 - 3.01^2 = 3.01 - 9.0601 = -6.0501$

$\text{Gradient} = \frac{-6.0501 - (-6)}{0.01} = \frac{-0.0501}{0.01} = -5.01$

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9 (c) State the limit suggested by Craig's investigation for the gradient of these chords as h tends to 0

[1 mark]

The gradient tends to -5

9 (d) Using differentiation from first principles, verify that your result in part **(c)** is correct.

[4 marks]

Gradient of chord $= \dfrac{f(3+h)-f(3)}{(3+h)-3}$

$f(3+h) = (3+h)-(3+h)^2 = 3+h-9-6h-h^2 = -6-5h-h^2$

$f(3) = 3-3^2 = -6$

Gradient $= \dfrac{(-6-5h-h^2)-(-6)}{h} = \dfrac{-5h-h^2}{h}$

$= -5-h \quad (h \neq 0)$

As $h \to 0$, $-5-h \to -5$

So the gradient of the curve at $x = 3$ is -5,

which verifies the result in part (c).

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10 A curve has equation $y = 2x^2 - 8x\sqrt{x} + 8x + 1$ for $x \geq 0$

10 (a) Prove that the curve has a maximum point at (1, 3)

Fully justify your answer.

[9 marks]

When $x = 1$: $y = 2(1)^2 - 8(1)\sqrt{1} + 8(1) + 1 = 2 - 8 + 8 + 1 = 3$ ✓

$x\sqrt{x} = x^{3/2}$ so $y = 2x^2 - 8x^{3/2} + 8x + 1$

$\frac{dy}{dx} = 4x - 8 \times \frac{3}{2}x^{1/2} + 8 = 4x - 12\sqrt{x} + 8$

Stationary points occur where $\frac{dy}{dx} = 0$:

$4x - 12\sqrt{x} + 8 = 0 \Rightarrow x - 3\sqrt{x} + 2 = 0$

$(\sqrt{x} - 1)(\sqrt{x} - 2) = 0$

$\sqrt{x} = 1$ or $\sqrt{x} = 2 \Rightarrow x = 1$ or $x = 4$

So $x = 1$ gives a stationary point, at (1, 3).

$\frac{d^2y}{dx^2} = 4 - 12 \times \frac{1}{2}x^{-1/2} = 4 - \frac{6}{\sqrt{x}}$

When $x = 1$: $\frac{d^2y}{dx^2} = 4 - \frac{6}{1} = -2$

$\frac{d^2y}{dx^2} < 0$ so the stationary point is a maximum.

∴ the curve has a maximum point at (1, 3)

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10 (b) Find the coordinates of the other stationary point of the curve and state its nature.

[2 marks]

From (a), the other stationary point is at $x = 4$:

$y = 2(16) - 8(4)(2) + 8(4) + 1 = 32 - 64 + 32 + 1 = 1$

When $x = 4$: $\frac{d^2y}{dx^2} = 4 - \frac{6}{2} = 1 > 0 \Rightarrow$ minimum

(4, 1) is a minimum point

Turn over for Section B

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Section B

Answer **all** questions in the spaces provided.

11 **In this question use $g = 9.8\,\text{m s}^{-2}$**

A ball, initially at rest, is dropped from a height of 40 m above the ground.

Calculate the speed of the ball when it reaches the ground.

Circle your answer.

[1 mark]

$-28\,\text{m s}^{-1}$ $\quad$ (circled) $28\,\text{m s}^{-1}$ $\quad$ $-780\,\text{m s}^{-1}$ $\quad$ $780\,\text{m s}^{-1}$

12 An object of mass 5 kg is moving in a straight line.

As a result of experiencing a forward force of F newtons and a resistant force of R newtons it accelerates at $0.6\,\text{m s}^{-2}$

Which one of the following equations is correct?

Circle your answer.

[1 mark]

$F - R = 0$ $\quad$ $F - R = 5$ $\quad$ (circled) $F - R = 3$ $\quad$ $F - R = 0.6$

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13 A vehicle, which begins at rest at point *P*, is travelling in a straight line.

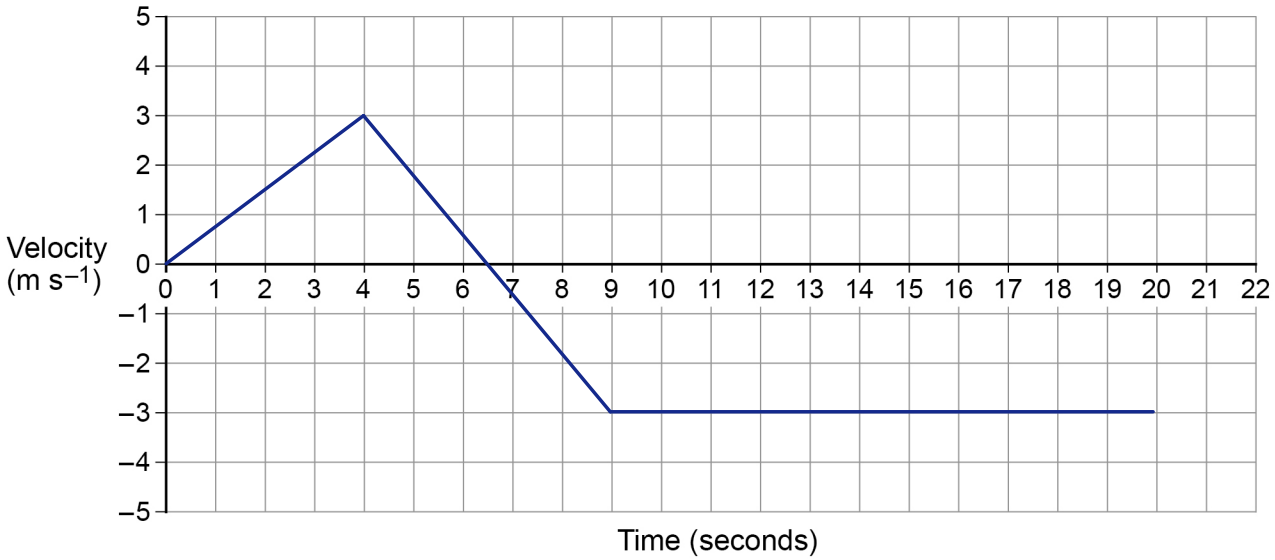
For the first 4 seconds the vehicle moves with a constant acceleration of $0.75\,\text{m s}^{-2}$

For the next 5 seconds the vehicle moves with a constant acceleration of $-1.2\,\text{m s}^{-2}$

The vehicle then immediately stops accelerating, and travels a further 33 m at constant speed.

13 (a) Draw a velocity–time graph for this journey on the grid below.

[3 marks]



13 (b) Find the distance of the car from *P* after 20 seconds.

[3 marks]

0 to 6.5 s (above axis): $\frac{1}{2} \times 6.5 \times 3 = 9.75$ m

6.5 to 9 s: $\frac{1}{2} \times 2.5 \times 3 = 3.75$ m backwards

9 to 20 s: $11 \times 3 = 33$ m backwards

Displacement = $9.75 - 3.75 - 33 = -27$ m

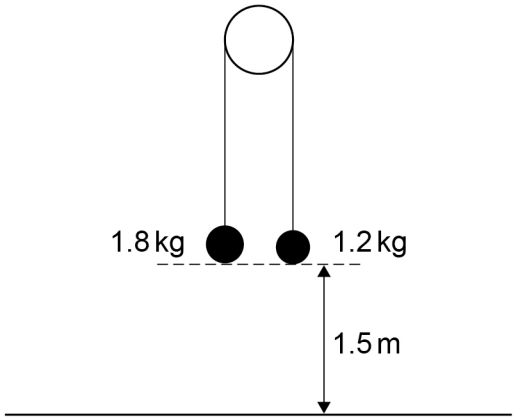
Distance from P = 27 m

Turn over ▶

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14 **In this question use $g = 9.81\,\text{m s}^{-2}$**

Two particles, of mass 1.8 kg and 1.2 kg, are connected by a light, inextensible string over a smooth peg.



14 (a) Initially the particles are held at rest 1.5 m above horizontal ground and the string between them is taut.

The particles are released from rest.

Find the time taken for the 1.8 kg particle to reach the ground.

[5 marks]

For 1.8 kg particle (moving down): $1.8g - T = 1.8a$

For 1.2 kg particle (moving up): $T - 1.2g = 1.2a$

Adding: $0.6g = 3a \Rightarrow a = 0.2g = 0.2 \times 9.81 = 1.962\ \text{m s}^{-2}$

For the 1.8 kg particle: $s = 1.5$, $u = 0$, $a = 1.962$, $t = ?$

$s = ut + \frac{1}{2}at^2 \Rightarrow 1.5 = 0 + \frac{1}{2} \times 1.962 \times t^2$

$t^2 = \frac{1.5}{0.981} = 1.5290...$

$t = 1.2365... \Rightarrow t = 1.24\ \text{s}$

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14 (b) State one assumption you have made in answering part **(a)**.

[1 mark]

The string is long enough that the 1.2 kg particle does not reach the peg before the 1.8 kg particle hits the ground.

Turn over for the next question

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15 A cyclist, Laura, is travelling in a straight line on a horizontal road at a constant speed of 25 km h^{-1}

A second cyclist, Jason, is riding closely and directly behind Laura. He is also moving with a constant speed of 25 km h^{-1}

15 (a) The driving force applied by Jason is likely to be less than the driving force applied by Laura.

Explain why.

[1 mark]

Jason experiences less air resistance than Laura because

he is in her slipstream, so needs less force to keep a constant speed.

15 (b) Jason has a problem and stops, but Laura continues at the same constant speed.

Laura sees an accident 40 m ahead, so she stops pedalling and applies the brakes.

She experiences a total resistance force of 40 N

Laura and her cycle have a combined mass of 64 kg

15 (b) (i) Determine whether Laura stops before reaching the accident.

Fully justify your answer.

[4 marks]

$F = ma$: $-40 = 64a \Rightarrow a = -0.625 \text{ m s}^{-2}$

$u = 25 \text{ km h}^{-1} = \frac{25 \times 1000}{3600} = 6.944... \text{ m s}^{-1}$, $v = 0$

$v^2 = u^2 + 2as \Rightarrow 0 = 6.944...^2 + 2(-0.625)s$

$s = \frac{48.225...}{1.25} = 38.58... \approx 38.6 \text{ m}$

$38.6 \text{ m} < 40 \text{ m}$, so Laura stops before reaching the accident.

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15 (b) (ii) State one assumption you have made that could affect your answer to part **(b)(i)**.

[1 mark]

It assumes she starts braking immediately (no reaction time).

With a reaction time she would travel further before braking,

so she might not stop before the accident.

Turn over for the next question

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16 A remote-controlled toy car is moving over a horizontal surface. It moves in a straight line through a point A.

The toy is initially at the point with displacement 3 metres from A. Its velocity, v m s^{-1}, at time t seconds is defined by

$$v = 0.06(2 + t - t^2)$$

16 (a) Find an expression for the displacement, r metres, of the toy from A at time t seconds.

[4 marks]

$r = \int v\,dt = \int 0.06(2 + t - t^2)\,dt$

$= 0.06\left(2t + \frac{t^2}{2} - \frac{t^3}{3}\right) + c$

$= 0.12t + 0.03t^2 - 0.02t^3 + c$

When $t = 0$, $r = 3 \Rightarrow c = 3$

$r = 0.12t + 0.03t^2 - 0.02t^3 + 3$

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16 (b) **In this question use $g = 9.8\,\text{m s}^{-2}$**

At time $t = 2$ seconds, the toy launches a ball which travels directly upwards with initial speed $3.43\,\text{m s}^{-1}$

Find the time taken for the ball to reach its highest point.

[3 marks]

Taking upwards as positive: $u = 3.43$, $v = 0$ (at highest point),

$a = -9.8$

$v = u + at \Rightarrow 0 = 3.43 - 9.8t$

$t = \frac{3.43}{9.8} = 0.35$ s

Time to reach highest point = 0.35 s

END OF QUESTIONS

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