AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Statistics

Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The discrete random variable X has $\text{Var}(X) = 6.5$

Find $\text{Var}(4X - 2)$

Circle your answer.

[1 mark]

24 26 102 104

$\text{Var}(4X - 2) = 4^2 \times 6.5 = 104$

2 The random variable A has a Poisson distribution with mean 2

The random variable B has a Poisson distribution with standard deviation 4

The random variables A and B are independent.

State the distribution of $A + B$

Circle your answer.

[1 mark]

Po(4) Po(6) Po(8) Po(18)

$\text{Var}(B) = 4^2 = 16$, so mean of $B = 16$; $A + B \sim \text{Po}(2 + 16)$

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3 The random variable X has a discrete uniform distribution and takes values 1, 2, 3, …, n

The mean of X is 8

3 (a) Show that $n = 15$

[2 marks]

$E(X) = \frac{n+1}{2} = 8$

$n + 1 = 16$

$n = 15$ as required

3 (b) Find $P(X > 4)$

[1 mark]

$P(X > 4) = P(X = 5, 6, ..., 15) = 11 \times \frac{1}{15}$

$P(X > 4) = \frac{11}{15}$

3 (c) Find the variance of X, giving your answer in exact form.

[2 marks]

$Var(X) = \frac{n^2 - 1}{12} = \frac{15^2 - 1}{12} = \frac{224}{12}$

$Var(X) = \frac{56}{3}$

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4 The distance a particular football player runs in a match is modelled by a normal distribution with standard deviation 0.3 kilometres.

A random sample of n matches is taken.

The distance the player runs in this sample of matches has mean 10.8 kilometres.

The sample is used to construct a 93% confidence interval for the mean, of width 0.0543 kilometres, correct to four decimal places.

4 (a) Find the value of n

[3 marks]

93% confidence $\Rightarrow$ $P(Z < z) = 0.965$ $\Rightarrow$ $z = 1.8119$

Half width $= \frac{0.0543}{2} = 0.02715$

$0.02715 = 1.8119 \times \frac{0.3}{\sqrt{n}}$

$\sqrt{n} = \frac{1.8119 \times 0.3}{0.02715} = 20.0209...$

$n = 20.0209...^2 = 400.8...$

$n = 401$

4 (b) Find the 93% confidence interval for the mean, giving the limits to three decimal places.

[1 mark]

10.8 ± 0.02715

$= (10.77285, 10.82715)$

93% confidence interval $= (10.773, 10.827)$

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4 (c) Alison claims that the population mean distance the player runs is 10.7 kilometres.

She carries out a hypothesis test at the 7% level of significance using the random sample and the hypotheses

H_0: $\mu = 10.7$

H_1: $\mu \neq 10.7$

4 (c) (i) State, with a reason, whether the null hypothesis will be accepted or rejected.

[1 mark]

The null hypothesis will be rejected, because 10.7 lies outside the 93% confidence interval (10.773, 10.827).

4 (c) (ii) Describe, in the context of the hypothesis test in part **(c)(i)**, what is meant by a Type II error.

[1 mark]

A Type II error would be accepting that the mean distance the player runs in a match is 10.7 kilometres, when in fact it is not 10.7 km.

Turn over for the next question

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5 In a game it is known that:

- 25% of players score 0
- 30% of players score 5
- 35% of players score 10
- 10% of players score 20

Players receive prize money, in pounds, equal to 100 times their score.

5 (a) State the modal score.

[1 mark]

Modal score = 10

5 (b) Find the median score.

[2 marks]

x	0	5	10	20
$P(X \leq x)$	0.25	0.55	0.9	1

$P(X \leq 0) = 0.25 < 0.5$, $P(X \leq 5) = 0.55 \geq 0.5$

Median score = 5

5 (c) Find the mean prize money received by a player.

[2 marks]

$E(\text{score}) = 0 \times 0.25 + 5 \times 0.3 + 10 \times 0.35 + 20 \times 0.1$

$= 0 + 1.5 + 3.5 + 2 = 7$

Mean prize money = 100×7

Mean prize money = £700

Turn over ▶

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6 The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{114}(4x+7) & 0 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

6 (a) Show that the median of X is 3.87, correct to three significant figures.

[3 marks]

$\int_0^m \frac{1}{114}(4x + 7)\,dx = 0.5$

$\frac{1}{114}[2x^2 + 7x]_0^m = 0.5$

$\frac{1}{114}(2m^2 + 7m) = 0.5$

$2m^2 + 7m - 57 = 0$

$m = \frac{-7 \pm \sqrt{(49 + 456)}}{4} = \frac{-7 \pm \sqrt{505}}{4}$

$m = 3.86805...$ (or $m = -7.37$ which is rejected as $0 \leq m \leq 6$)

Median = 3.86805... = 3.87 (3 s.f.) as required

6 (b) Find the exact value of $P(X > 2)$

[2 marks]

$P(X > 2) = \int_2^6 \frac{1}{114}(4x + 7)\,dx$

$= \frac{1}{114}[2x^2 + 7x]_2^6$

$= \frac{1}{114}[(72 + 42) - (8 + 14)]$

$= \frac{92}{114}$

$P(X > 2) = \frac{46}{57}$

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6 (c) The continuous random variable Y has probability density function

$$g(y) = \begin{cases} \frac{1}{2}y^2 - \frac{1}{6}y^3 & 1 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

6 (c) (i) Show that $\text{Var}\left(\frac{1}{Y}\right) = \frac{2}{81}$

[4 marks]

$E(\frac{1}{Y}) = \int_1^3 \frac{1}{y}(\frac{1}{2}y^2 - \frac{1}{6}y^3)\,dy = \int_1^3 (\frac{y}{2} - \frac{y^2}{6})\,dy$

$= [\frac{y^2}{4} - \frac{y^3}{18}]_1^3 = (\frac{9}{4} - \frac{3}{2}) - (\frac{1}{4} - \frac{1}{18}) = \frac{5}{9}$

$E(\frac{1}{Y^2}) = \int_1^3 \frac{1}{y^2}(\frac{1}{2}y^2 - \frac{1}{6}y^3)\,dy = \int_1^3 (\frac{1}{2} - \frac{y}{6})\,dy$

$= [\frac{y}{2} - \frac{y^2}{12}]_1^3 = (\frac{3}{2} - \frac{3}{4}) - (\frac{1}{2} - \frac{1}{12}) = \frac{1}{3}$

$\text{Var}(\frac{1}{Y}) = E(\frac{1}{Y^2}) - [E(\frac{1}{Y})]^2 = \frac{1}{3} - (\frac{5}{9})^2$

$= \frac{27}{81} - \frac{25}{81} = \frac{2}{81}$ as required

6 (c) (ii) The random variable X has variance $\frac{939}{361}$ and is independent of Y

Find $\text{Var}\left(2X - \frac{3}{Y}\right)$, giving your answer to three significant figures.

[2 marks]

$\text{Var}(2X - \frac{3}{Y}) = 2^2\,\text{Var}(X) + 3^2\,\text{Var}(\frac{1}{Y})$

$= 4 \times \frac{939}{361} + 9 \times \frac{2}{81}$

$= 10.6266...$

$\text{Var}(2X - 3/Y) = 10.6$ (3 s.f.)

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7 Two employees, *A* and *B*, both produce the same toy for a company.

The company records the total number of errors made per day by each employee during a 40-day period. The results are summarised in the following table.

Number of errors made per day

Employee	0	1	2	3 or more	**Total**
A	8	10	20	2	40
B	18	4	15	3	40
Total	26	14	35	5	80

The company claims that there is an association between employee and number of errors made per day.

7 (a) Test the company's claim, using the 5% level of significance.

[9 marks]

H_0: there is no association between employee and number of errors made per day

H_1: there is an association between employee and number of errors made per day

Expected frequencies E = (row total × column total) ÷ 80:

E	0	1	2	3 or more
A	13	7	17.5	2.5
B	13	7	17.5	2.5

2.5 < 5, so combine the columns "2" and "3 or more" to give "2 or more":

O (E)	0	1	2 or more
A	8 (13)	10 (7)	22 (20)
B	18 (13)	4 (7)	18 (20)

$$\chi^2 = \Sigma\frac{(O-E)^2}{E} = \frac{(8-13)^2}{13} + \frac{(18-13)^2}{13} + \frac{(10-7)^2}{7} + \frac{(4-7)^2}{7}$$

$$+ \frac{(22-20)^2}{20} + \frac{(18-20)^2}{20}$$

$= 1.923 + 1.923 + 1.286 + 1.286 + 0.2 + 0.2 = 6.8175\ldots = 6.82$ (3 s.f.)

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$\nu = (2 - 1)(3 - 1) = 2$

5% critical value of $\chi^2(2) = 5.991$

$6.82 > 5.991$, so the test statistic lies in the critical region

Reject H_0

There is sufficient evidence at the 5% level of significance to support the company's claim that there is an association between employee and number of errors made per day.

7 (b) By considering observed and expected frequencies, interpret in context the association between employee and number of errors made per day.

[2 marks]

The largest contributions to χ^2 come from 0 errors for employee A and employee B ($\frac{(O - E)^2}{E} = 1.92$ each).

Employee A has days with 0 errors less often than expected ($8 < 13$), whereas employee B has days with 0 errors more often than expected ($18 > 13$), so employee B tends to make fewer errors per day than A.

END OF QUESTIONS

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