Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Discrete

Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

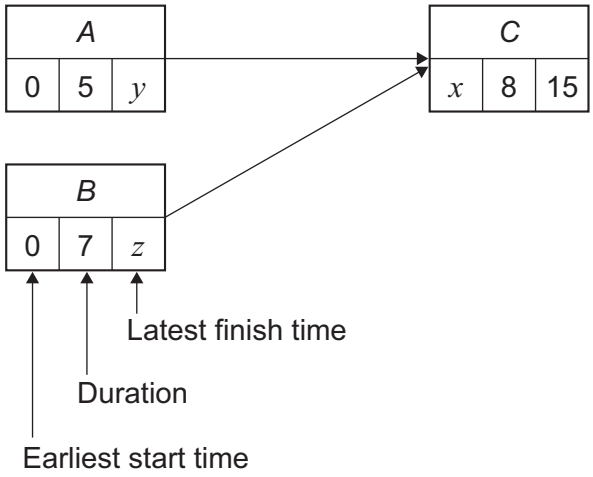
For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Do not write outside the box

Answer **all** questions in the spaces provided.

1 (a) A project consists of three activities *A*, *B* and *C*

An activity network for the project is shown in the diagram below.



Find the value of x

Circle your answer.

[1 mark]

5 7 8 12

1 (b) Find the value of y

Circle your answer.

[1 mark]

5 7 8 15

Do not write outside the box

2 The set S is given by $S = \{0, 2, 4, 6\}$

2 (a) Construct a Cayley table, using the grid below, for S under the binary operation addition modulo 8

[3 marks]

2 (b) State the identity element for S under the binary operation addition modulo 8

[1 mark]

_____

_____

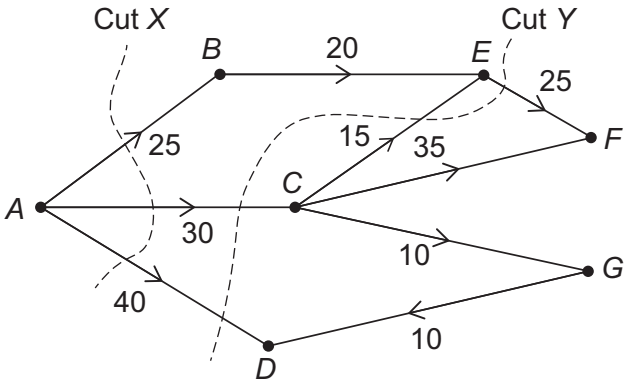
_____

Turn over for the next question

Do not write outside the box

3 The diagram shows a network of pipes.

Each pipe is labelled with its upper capacity in $m^3 s^{-1}$



3 (a) Find the value of Cut *X*

[1 mark]

3 (b) Find the value of Cut *Y*

[1 mark]

3 (c) Add a supersink *T* to the network.

[2 marks]

Do not write outside the box

4 The binary operation $*$ is defined as

$$a * b = ab + 1 \quad \text{where} \quad a, b \in \mathbb{R}$$

4 (a) Prove that $*$ is commutative on $\mathbb{R}$

[2 marks]

4 (b) Prove that $*$ is **not** associative on $\mathbb{R}$

[3 marks]

Turn over for the next question

Do not write outside the box

5 An adjacency matrix for the simple graph *G* is shown below.

	A	*B*	*C*	*D*	*E*
A	0	1	1	0	0
B	1	0	1	0	0
C	1	1	0	0	0
D	0	0	0	0	1
E	0	0	0	1	0

5 (a) Using the adjacency matrix, explain why *G* is **not** a complete graph.

[2 marks]

5 (b) State, with a reason, whether *G* is Eulerian, semi-Eulerian or neither.

[2 marks]

Do not write outside the box

5 (c) Draw a graph that is the complement of G

[3 marks]

Do not write outside the box

6 Vaya and Wynne are playing a zero-sum game.

The game is represented by the pay-off matrix for Vaya.

		Wynne		
	Strategy	$\mathbf{W_1}$	$\mathbf{W_2}$	$\mathbf{W_3}$
Vaya	$\mathbf{V_1}$	−150	300	−250
	$\mathbf{V_2}$	−100	0	75
	$\mathbf{V_3}$	75	−200	50

6 (a) Find the play-safe strategies for Vaya and Wynne.

Fully justify your answer.

[4 marks]

6 (b) Vaya and Wynne decide **not** to play their play-safe strategies.

Deduce the best possible outcome for Wynne.

[2 marks]

Do not write outside the box

Turn over for the next question

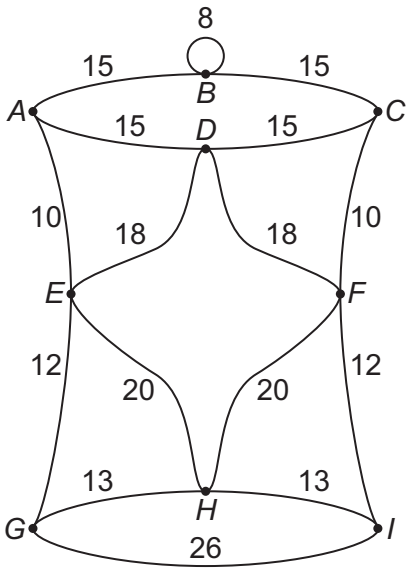
DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

7 A jeweller is making pendants.

Each pendant is made by bending a single, continuous strand of wire.

Each pendant has the same design as shown below.



The lengths on the diagram are in millimetres. The sum of these lengths is 240 mm

As the jeweller does not cut the wire, some sections require a double length of wire.

7 (a) The jeweller makes a pendant by starting and finishing at *B*

Find the minimum length of the strand of wire that the jeweller needs to make the pendant.

Fully justify your answer.

[4 marks]

Do not write outside the box

7 (b) The jeweller makes another pendant of the same design.

Find the minimum possible length for the strand of wire that the jeweller would need.

[2 marks]

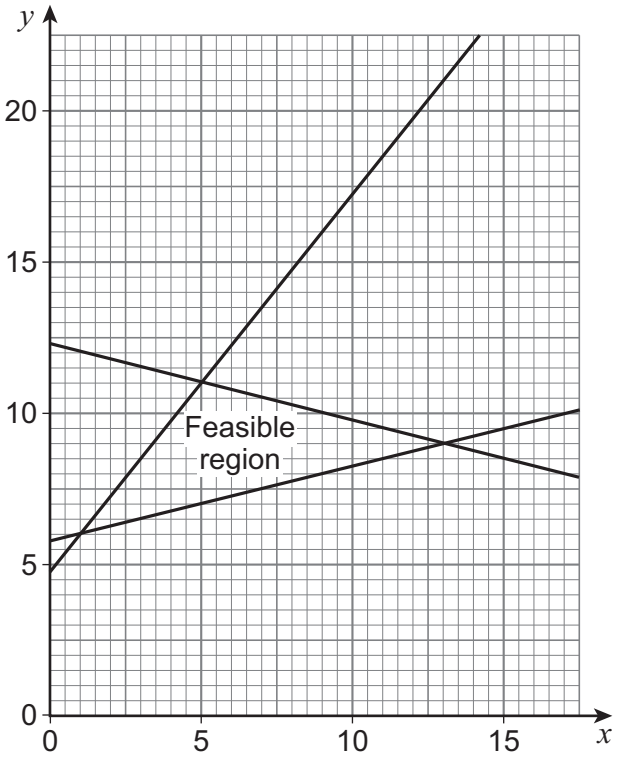
7 (c) By considering the differences between the pendants in part **(a)** and part **(b)**, state one reason why the jeweller may prefer the pendant in part **(a)**.

[1 mark]

Do not write outside the box

8 A linear programming problem is set up to maximise $P = ax + y$ where a is a constant.

P is maximised subject to three linear constraints which form the triangular feasible region shown in the diagram below.



The vertices of the triangle are (1, 6), (5, 11) and (13, 9)

P is maximised at (5, 11)

Find the range of possible values for P

[5 marks]

Do not write outside the box

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Copyright information

For confidentiality purposes, all acknowledgements of third-party copyright material are published in a separate booklet. This booklet is published after each live examination series and is available for free download from www.aqa.org.uk.

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team.

Copyright © 2021 AQA and its licensors. All rights reserved.