AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Statistics

Thursday 14 May 2020 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The discrete random variable X has the following probability distribution function.

$$\mathrm{P}(X = x) = \begin{cases} 0.2 & x = 1 \\ 0.3 & x = 2 \\ 0.1 & x = 3,\ 4 \\ 0.25 & x = 5 \\ 0.05 & x = 6 \\ 0 & \text{otherwise} \end{cases}$$

Find the mode of X.

Circle your answer.

[1 mark]

0.1 0.25 (2) 3

Largest probability is 0.3 when x = 2, so mode = 2

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2 A χ^2 test is carried out in a school to test for association between the class a student belongs to and the number of times they are late to school in a week.

The contingency table below gives the expected values for the test.

		Number of times late				
		0	**1**	**2**	**3**	**4**
Class	**A**	8.12	14	15.12	14	4.76
	B	8.99	15.5	16.74	15.5	5.27
	C	11.89	20.5	22.14	20.5	6.97

Find a possible value for the degrees of freedom for the test.

Circle your answer.

[1 mark]

(6) 8 12 15

Expected values for 4 times late are all < 5, so combine the 3 and 4 columns:
Degrees of freedom = (3 − 1)(4 − 1) = 6

Turn over for the next question

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3 The random variable X represents the value on the upper face of an eight-sided dice after it has been rolled. The faces are numbered 1 to 8

The random variable X is modelled by a discrete uniform distribution with $n = 8$

3 (a) Find $\mathrm{E}(X)$

[1 mark]

$E(X) = \frac{n+1}{2} = \frac{8+1}{2}$

$E(X) = 4.5$

3 (b) Find $\mathrm{Var}(X)$

[1 mark]

$Var(X) = \frac{n^2-1}{12} = \frac{8^2-1}{12} = \frac{63}{12}$

$Var(X) = 5.25$

3 (c) Find $\mathrm{P}(X \geq 6)$

[1 mark]

$P(X \geq 6) = P(6) + P(7) + P(8) = 3 \times \frac{1}{8}$

$P(X \geq 6) = 0.375$

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3 (d) The dice was rolled 800 times and the results below were obtained.

x	1	2	3	4	5	6	7	8
Frequency	103	63	84	110	74	41	85	240

State, with a reason, how you would refine the model for the random variable X.

[2 marks]

Under the uniform model each expected frequency is 800 ÷ 8 = 100,

but the frequencies are not approximately equal (e.g. 240 for 8

and 41 for 6), so the dice appears to be biased and the discrete

uniform model is not suitable.

Refinement: model X with a (non-uniform) discrete distribution where

each probability is estimated by the relative frequency,

e.g. $P(X = 1) = \frac{103}{800}$, $P(X = 8) = \frac{240}{800} = 0.3$, *etc.*

Turn over for the next question

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4 Murni is investigating the annual salary of people from a particular town.

She takes a random sample of 200 people from the town and records their annual salary.

The mean annual salary is £28 500 and the standard deviation is £5100

Calculate a 97% confidence interval for the population mean of annual salaries for the people who live in the town, giving your values to the nearest pound.

[3 marks]

$\bar{x} = 28500$, $s = 5100$, $n = 200$ (large, so $\bar{x}$ approx. normal by CLT)

97% confidence: $P(Z < z) = 0.985 \Rightarrow z = 2.1700...$

$CI = \bar{x} \pm z \frac{s}{\sqrt{n}} = 28500 \pm 2.1700 \times \frac{5100}{\sqrt{200}}$

$= 28500 \pm 782.587...$

$= (27717.41..., 29282.58...)$

97% confidence interval = (£27 717, £29 283)

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5 The discrete random variable X has the following probability distribution.

x	2	4	6	9
$\mathbf{P}(X = x)$	0.2	0.6	0.1	0.1

5 (a) Find $P(X \leq 6)$

[1 mark]

$P(X \leq 6) = 0.2 + 0.6 + 0.1 = 0.9$

5 (b) Let $Y = 3X + 2$

Show that $\text{Var}(Y) = 32.49$

[5 marks]

$E(X) = 2 \times 0.2 + 4 \times 0.6 + 6 \times 0.1 + 9 \times 0.1$

$= 0.4 + 2.4 + 0.6 + 0.9 = 4.3$

$E(X^2) = 2^2 \times 0.2 + 4^2 \times 0.6 + 6^2 \times 0.1 + 9^2 \times 0.1$

$= 0.8 + 9.6 + 3.6 + 8.1 = 22.1$

$\text{Var}(X) = E(X^2) - (E(X))^2 = 22.1 - 4.3^2$

$= 22.1 - 18.49 = 3.61$

$\text{Var}(Y) = \text{Var}(3X + 2) = 3^2 \text{Var}(X)$

$= 9 \times 3.61$

$\therefore \text{Var}(Y) = 32.49$ as required

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5 (c) The continuous random variable T is independent of Y.

Given that $\text{Var}(T) = 5$, find $\text{Var}(T + Y)$

[1 mark]

T and Y independent, so Var(T + Y) = Var(T) + Var(Y)

= 5 + 32.49

Var(T + Y) = 37.49

Turn over for the next question

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6 The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{4}{45}(x^3 - 10x^2 + 29x - 20) & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

6 (a) Find $P(X < 2)$

[2 marks]

$P(X < 2) = \frac{4}{45}\int_1^2 (x^3 - 10x^2 + 29x - 20)\,dx$

$= \frac{4}{45}\left[\frac{x^4}{4} - \frac{10x^3}{3} + \frac{29x^2}{2} - 20x\right]_1^2$

$= \frac{4}{45}\left[\left(4 - \frac{80}{3} + 58 - 40\right) - \left(\frac{1}{4} - \frac{10}{3} + \frac{29}{2} - 20\right)\right]$

$= \frac{4}{45}\left[-\frac{14}{3} + \frac{103}{12}\right] = \frac{4}{45} \times \frac{47}{12}$

$P(X < 2) = \frac{47}{135}$ $(= 0.348)$

6 (b) Verify that the median of X is 2.3, correct to two significant figures.

[4 marks]

Median m satisfies $\frac{4}{45}\int_1^m (x^3 - 10x^2 + 29x - 20)\,dx = 0.5$

$\frac{4}{45}\left[\frac{x^4}{4} - \frac{10x^3}{3} + \frac{29x^2}{2} - 20x\right]_1^m = 0.5$

$F(m) = \frac{1}{45}m^4 - \frac{8}{27}m^3 + \frac{58}{45}m^2 - \frac{16}{9}m + \frac{103}{135} = 0.5$

Let $g(m) = \frac{1}{45}m^4 - \frac{8}{27}m^3 + \frac{58}{45}m^2 - \frac{16}{9}m + \frac{71}{270} = 0$

$g(2.25) = -0.0175\ldots < 0$

$g(2.35) = 0.0355\ldots > 0$

Sign change and g continuous, so $2.25 < m < 2.35$

(solving gives $m = 2.2828\ldots$)

$\therefore$ median of $X = 2.3$ to 2 s.f. as required

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6 (c) Find the mean of X.

[2 marks]

$E(X) = \frac{4}{45}\int_1^4 x(x^3 - 10x^2 + 29x - 20)\,dx$

$= \frac{4}{45}\left[\frac{x^5}{5} - \frac{10x^4}{4} + \frac{29x^3}{3} - 10x^2\right]_1^4$

$= \frac{4}{45}\left[\frac{352}{15} - \left(-\frac{79}{30}\right)\right] = \frac{4}{45} \times \frac{783}{30}$

$E(X) = 2.32$

Turn over for the next question

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7 A restaurant has asked Sylvia to conduct a χ^2 test for association between meal ordered and age of customer.

7 (a) State the hypotheses that Sylvia should use for her test.

[1 mark]

H_0: there is no association between meal ordered and age of customer

H_1: there is an association between meal ordered and age of customer

7 (b) Sylvia correctly calculates her value of the test statistic to be 44.1

She uses a 5% level of significance and the degrees of freedom for the test is 30

Sylvia accepts the null hypothesis.

Explain whether or not Sylvia was correct to accept the null hypothesis.

[4 marks]

Critical value: χ^2 with 30 degrees of freedom at 5% level

$\chi^2_{30}(0.05) = 43.773$

Test statistic 44.1 > 43.773

so 44.1 lies in the critical region and H_0 should be rejected.

Sylvia was not correct to accept the null hypothesis;

she should have rejected H_0.

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7 (c) State in context the correct conclusion to Sylvia's test.

[1 mark]

There is sufficient evidence at the 5% level to suggest that there is an association between meal ordered and age of customer.

Turn over for the next question

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8 There are two hospitals in a city.

Over a period of time, the first hospital recorded an average of 20 births a day.

Over the same period of time, the second hospital recorded an average of 5 births a day.

Stuart claims that birth rates in the hospitals have changed over time.

On a randomly chosen day, he records a total of 16 births from the two hospitals.

8 (a) Investigate Stuart's claim, using a suitable test at the 5% level of significance.

[6 marks]

Let X = total number of births in a day at the two hospitals

Under the original rates, $X \sim Po(20 + 5) = Po(25)$

H_0: $\lambda = 25$

H_1: $\lambda \neq 25$ (two-tailed, 2.5% in each tail)

$P(X \leq 16) = 0.03775... = 0.0377$

$0.0377 > 0.025$

so not significant: accept (do not reject) H_0

There is insufficient evidence at the 5% level to suggest that the birth rates at the hospitals have changed over time, so Stuart's claim is not supported.

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8 (b) For a test of the type carried out in part **(a)**, find the probability of making a Type I error, giving your answer to two significant figures.

[3 marks]

$X \sim Po(25)$, two-tailed test at 5% level:

Lower tail: $P(X \leq 15) = 0.02229 < 0.025$, $P(X \leq 16) = 0.03775 > 0.025$

Upper tail: $P(X \geq 36) = 0.02246 < 0.025$, $P(X \geq 35) = 0.03384 > 0.025$

Critical region: $X \leq 15$ or $X \geq 36$

$P(\text{Type I error}) = P(X \leq 15) + P(X \geq 36)$

$= 0.02229 + 0.02246 = 0.04475...$

$P(\text{Type I error}) = 0.045$ (2 s.f.)

END OF QUESTIONS

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