AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Mechanics

Thursday 14 May 2020 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 **In this question use $g = 10\,\text{m}\,\text{s}^{-2}$**

A particle of mass 2 kg is attached to one end of a light elastic string of natural length 0.5 metres and modulus of elasticity 100 N. The other end of the string is attached to the point *O*.

Find the extension of the elastic string when the particle hangs in equilibrium vertically below *O*.

Circle your answer.

[1 mark]

0.01 m | (0.1 m) | 0.2 m | 0.4 m

2 An object moves under the action of a single force F newtons.

It is given that $F = 6x^2$, where x represents the displacement in metres from the initial position of the object.

Find the work done by F in moving the object from $x = 1$ to $x = 2$

Circle your answer.

[1 mark]

12 J | (14 J) | 18 J | 42 J

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3 The time taken for the moon to make one complete orbit around Earth is approximately 27.3 days.

Model this orbit as circular, with a radius of 3.84×10^8 metres.

Find the approximate speed of the moon relative to Earth, in metres per second.

[3 marks]

Time for one orbit: $T = 27.3 \times 24 \times 60 \times 60 = 2\ 358\ 720$ s

$\omega = 2\pi / T = 2\pi / 2\ 358\ 720 = 2.6638... \times 10^{-6}$ rad s^{-1}

$v = r\omega$

$v = 3.84 \times 10^8 \times 2.6638... \times 10^{-6}$

$v = 1022.9...$

Speed of the moon ≈ 1020 m s^{-1} (3 s.f.)

Turn over for the next question

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4 A particle P, of mass m kg, collides with a particle Q, of mass 2 kg

Immediately before the collision the velocity of P is $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ m s^{-1} and the velocity of Q is $\begin{bmatrix} -3 \\ 5 \end{bmatrix}$ m s^{-1}

As a result of the collision the particles coalesce into a single particle which moves with velocity $\begin{bmatrix} k \\ 0 \end{bmatrix}$ m s^{-1}, where k is a constant.

Find the value of k.

[4 marks]

Conservation of momentum:

$m(4i - 2j) + 2(-3i + 5j) = (m + 2)(ki + 0j)$

j components: $-2m + 2(5) = 0$

$-2m + 10 = 0$

$m = 5$

i components: $4m + 2(-3) = k(m + 2)$

$4(5) - 6 = 7k$

$14 = 7k$

$k = 2$

Do not write outside the box

5 A train consisting of an engine and eight carriages moves on a straight horizontal track.

A constant resistive force of 2400 N acts on the engine.

A constant resistive force of 300 N acts on **each** of the eight carriages.

The maximum speed of the train on the track is 120 km h^{-1}

Find the maximum power output of the engine.

Fully justify your answer.

[5 marks]

At maximum speed the acceleration of the train is zero, so the driving force F of the engine equals the total resistance R.

The engine works at its maximum power at this maximum speed.

$R = 2400 + 8 \times 300 = 2400 + 2400 = 4800$ N

$\therefore\ F = R = 4800$ N

$v = 120$ km $h^{-1} = (120 \times 1000) / (60 \times 60) = 100/3$ m s^{-1}

$P = Fv$

$P = 4800 \times 100/3$

$P = 160\,000$ W

Maximum power output = 160 kW

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6 The magnitude of the gravitational force F between two planets of masses m_1 and m_2 with centres at a distance d apart is given by

$$F = \frac{Gm_1m_2}{d^2}$$

where G is a constant.

6 (a) Show that G must have dimensions $L^3M^{-1}T^{-2}$, where L represents length, M represents mass and T represents time.

[2 marks]

$[F] = MLT^{-2}$ $[d] = L$ $[m_1] = [m_2] = M$

$F = Gm_1m_2/d^2 \Rightarrow G = Fd^2/(m_1m_2)$

$[G] = MLT^{-2} \times L^2 / (M \times M)$

$[G] = ML^3T^{-2} / M^2$

$\therefore [G] = L^3M^{-1}T^{-2}$ as required

Do not write outside the box

6 (b) The lifetime t of a planet is thought to depend on its mass m, its radius r, the constant G and a dimensionless constant k such that

$$t = km^a r^b G^c$$

where a, b and c are constants.

Determine the values of a, b and c.

[3 marks]

$[t] = T$, k is dimensionless

$[m^a r^b G^c] = M^a L^b (L^3 M^{-1} T^{-2})^c$

$= M^{a-c} L^{b+3c} T^{-2c}$

Comparing with $M^0 L^0 T^1$:

M: $a - c = 0$

L: $b + 3c = 0$

T: $-2c = 1 \Rightarrow c = -\frac{1}{2}$

$b = -3c = 3/2$

$a = c = -\frac{1}{2}$

$a = -\frac{1}{2}$, $b = 3/2$, $c = -\frac{1}{2}$

Do not write outside the box

7 **In this question use $g = 9.8\,\text{m s}^{-2}$**

As part of a competition, Jo-Jo makes a small pop-up rocket.

It is operated by pressing the rocket vertically downwards to compress a light spring, which is positioned underneath the rocket.

The rocket is released from rest and moves vertically upwards.

The mass of the rocket is 18 grams and the stiffness constant of the spring is $60\,\text{N m}^{-1}$

Initially the spring is compressed by 3 cm

7 (a) Find the speed of the rocket when the spring first reaches its natural length.

[4 marks]

$m = 18\ g = 0.018\ kg, \quad x = 3\ cm = 0.03\ m$

Initial EPE $= \frac{1}{2}kx^2 = \frac{1}{2} \times 60 \times 0.03^2 = 0.027\ J$

Conservation of energy (no air resistance):

EPE lost = KE gained + PE gained

$0.027 = \frac{1}{2}(0.018)v^2 + 0.018 \times 9.8 \times 0.03$

$0.027 = 0.009v^2 + 0.005292$

$0.009v^2 = 0.021708$

$v^2 = 2.412$

$v = 1.5530...$

Speed $= 1.6\ m\ s^{-1}$ (2 s.f.)

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7 (b) By considering energy find the distance that the rocket rises.

[2 marks]

At the highest point KE = 0, so EPE lost = PE gained

$0.027 = 0.018 \times 9.8 \times h$

$h = 0.027 / 0.1764 = 0.15306...$

Distance the rocket rises = 0.15 m (2 s.f.)

7 (c) In order to win a prize in the competition, the rocket must reach a point which is 15 cm vertically above its starting position.

With reference to the assumptions you have made, determine if Jo-Jo wins a prize or not.

Fully justify your answer.

[3 marks]

Assumptions: the rocket is modelled as a particle, and there is no air resistance.

The model predicts a rise of 0.153 m = 15.3 cm, only just above 15 cm.

In reality there would be air resistance, which would do work against the rocket and reduce the height it reaches.

The rocket also has size, so this needs to be taken into account when judging whether the required point is reached.

As the predicted height is only just enough, the effects of air resistance (and the size of the rocket) mean it is unlikely that the rocket reaches 15 cm, so Jo-Jo is unlikely to win a prize.

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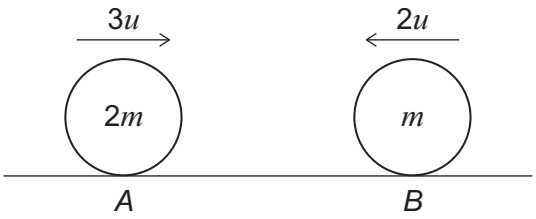
8 Two smooth spheres A and B have the same radius and are free to move on a smooth horizontal surface.

The masses of A and B are $2m$ and m respectively.

Both A and B are initially at rest.

The sphere A is set in motion directly towards B with speed $3u$ and at the same time B is set in motion directly towards A with speed $2u$.

Subsequently A and B collide directly.



The coefficient of restitution between the spheres is e.

8 (a) Show that the speed of B after the collision is given by

$$\frac{2u(2+5e)}{3}$$

[4 marks]

Let v and w be the velocities of A and B after the collision,

taking the direction of A's initial motion as positive.

Conservation of momentum:

$2m(3u) + m(-2u) = 2mv + mw$

$4u = 2v + w$ (1)

Newton's law of restitution:

$w - v = e(3u - (-2u)) = 5ue \Rightarrow v = w - 5ue$

Substitute into (1): $4u = 2(w - 5ue) + w$

$4u = 3w - 10ue$

$3w = 4u + 10ue$

$w = (4u + 10ue)/3$

$\therefore$ speed of B $= 2u(2 + 5e)/3$ as required

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8 (b) Given that the direction of the velocity of A is reversed during the collision, find the range of possible values of e.

Fully justify your answer.

[4 marks]

From (1): $2v = 4u - w = 4u - 2u(2 + 5e)/3$

$6v = 12u - 4u - 10ue = 8u - 10ue$

$v = u(4 - 5e)/3$

A was moving in the positive direction, so if its direction is reversed then $v < 0$:

$u(4 - 5e)/3 < 0$

$4 - 5e < 0$

$e > 4/5$

Also the coefficient of restitution satisfies $e \leq 1$

$\therefore \ 4/5 < e \leq 1$

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8 (c) Given that the magnitude of the impulse that A exerts on B is $\frac{19mu}{3}$, find the value of e.

[4 marks]

Impulse that A exerts on B = change in momentum of B

$I = mw - m(-2u)$

$m \times 2u(2 + 5e)/3 + 2mu = 19mu/3$

$2u(2 + 5e)/3 + 2u = 19u/3$

$4u + 10eu + 6u = 19u$

$10eu = 9u$

$e = 9/10$

END OF QUESTIONS

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