AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Discrete

Thursday 14 May 2020 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

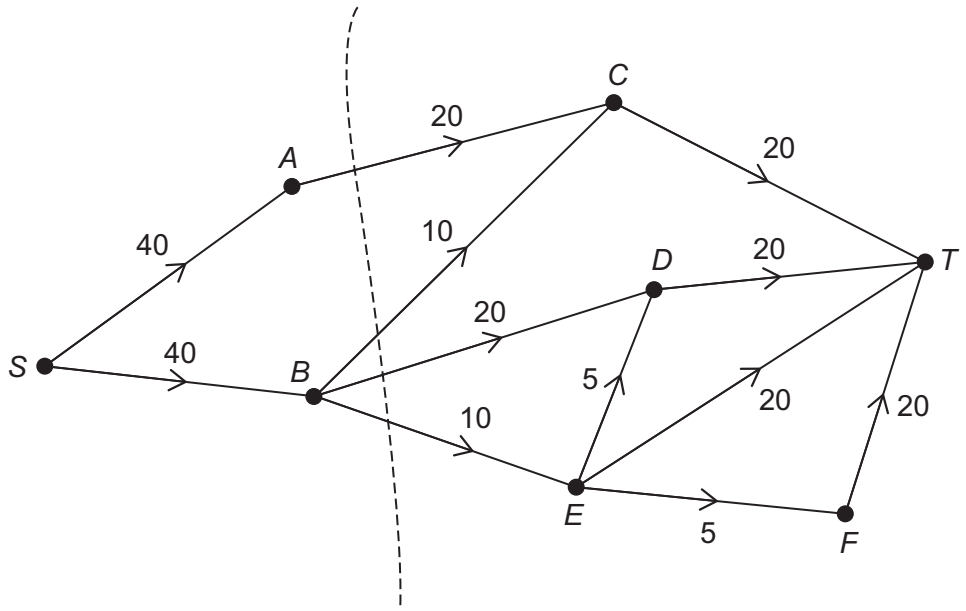
Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The network represents a system of pipes.

The number on each arc represents the upper capacity for each pipe in $cm^3\,s^{-1}$



The value of the cut $\{S, A, B\}$ $\{C, D, E, F, T\}$ is $60\,cm^3\,s^{-1}$

The maximum flow through the system is $M\,cm^3\,s^{-1}$

What does the value of the cut imply about M?

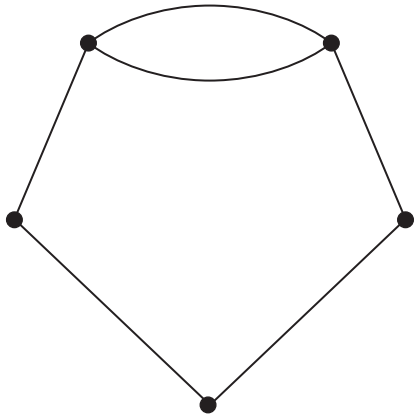
Circle your answer.

[1 mark]

$M < 60$ $\quad$ $M \leq 60$ $\quad$ $M \geq 60$ $\quad$ $M > 60$

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2 The graph G has 5 vertices and 6 edges, as shown below.



Which of the following statements describes the properties of G?

Tick (✓) **one** box.

[1 mark]

G is Eulerian and Hamiltonian. ☐

G is Eulerian but not Hamiltonian. ☐

G is semi-Eulerian and Hamiltonian. ☑

G is semi-Eulerian but not Hamiltonian. ☐

Turn over for the next question

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3 Summer and Haf play a zero-sum game.

The pay-off matrix for the game is shown below.

		Haf		
	Strategy	H_1	H_2	H_3
Summer	S_1	4	−4	0
	S_2	−12	0	10
	S_3	10	4	6

3 (a) Show that the game has a stable solution.

[3 marks]

Row minima: S_1: −4, S_2: −12, S_3: 4

Column maxima: H_1: 10, H_2: 4, H_3: 10

max(row minima) = 4

min(column maxima) = 4

max(row minima) = min(column maxima) = 4

Therefore the game has a stable solution (S_3, H_2).

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3 (b) (i) State the value of the game for Summer.

[1 mark]

Value of the game for Summer = 4

3 (b) (ii) State the play-safe strategy for each player.

[1 mark]

Play-safe strategy for Summer: S_3

Play-safe strategy for Haf: H_2

Turn over for the next question

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4 The connected planar graph P is Eulerian and has at least one vertex of degree x.

Some of the properties of P are shown in the table below.

Number of vertices	Number of edges	Number of faces
$3x + 6$	$x^2 + 8x$	$2x^2 + 2$

Deduce the value of x.

Fully justify your answer.

[4 marks]

P is connected and planar, so by Euler's formula $v - e + f = 2$

$(3x + 6) - (x^2 + 8x) + (2x^2 + 2) = 2$

$x^2 - 5x + 8 = 2$

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

$x = 2$ or $x = 3$

P is Eulerian, so every vertex of P has even degree.

P has a vertex of degree x, so x cannot be 3 (odd).

Therefore $x = 2$

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5 A restoration project is divided into a number of activities.

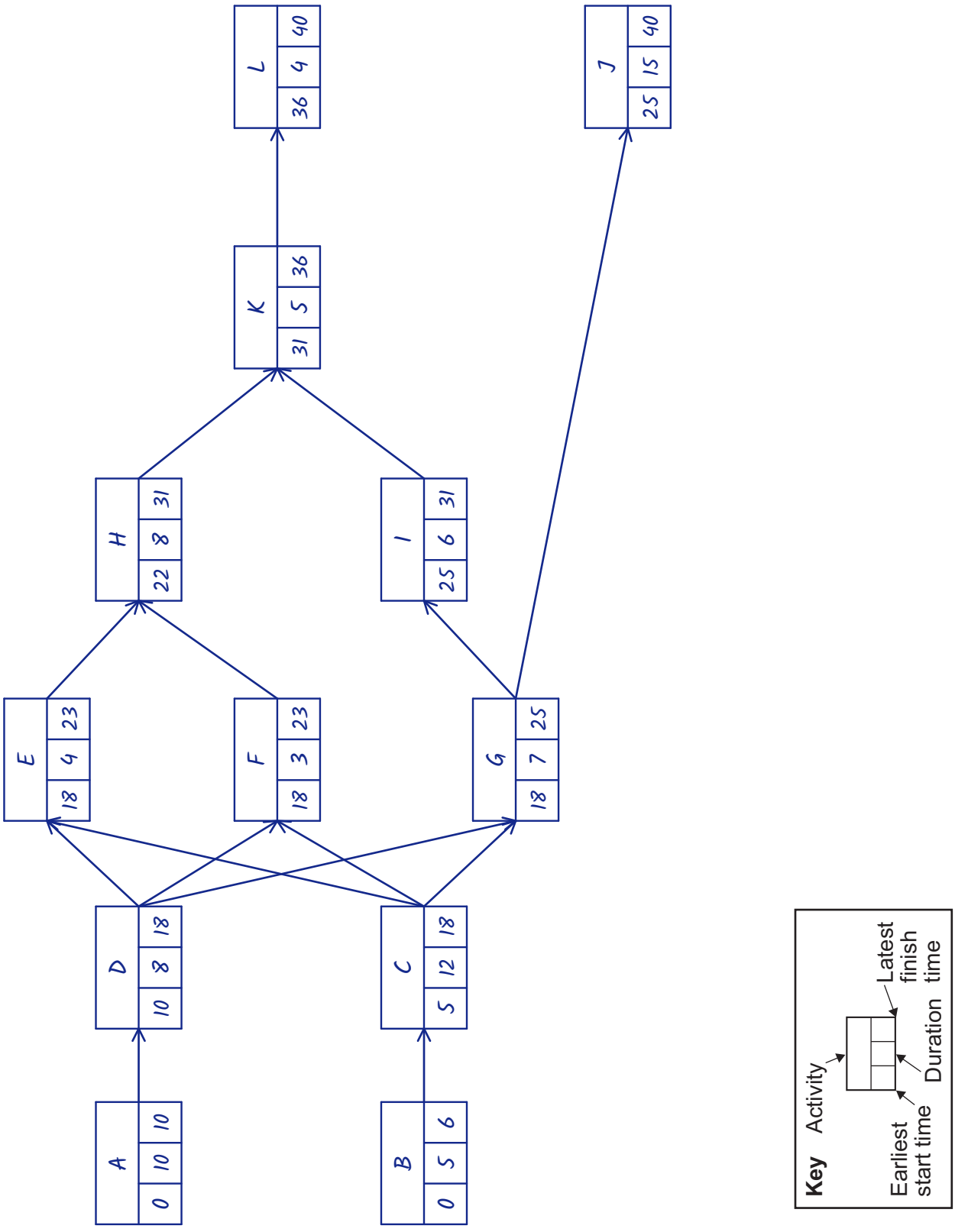
The duration and predecessor(s) of each activity are shown in the table below.

Activity	Immediate predecessor(s)	Duration (weeks)
A	–	10
B	–	5
C	*B*	12
D	*A*	8
E	*C*, *D*	4
F	*C*, *D*	3
G	*C*, *D*	7
H	*E*, *F*	8
I	*G*	6
J	*G*	15
K	*H*, *I*	5
L	*K*	4

5 (a) On the opposite page, construct an activity network for the project and fill in the earliest start time and latest finish time for each activity.

[4 marks]

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5 (b) Due to a change of materials during the project, the duration of activity *C* is extended by 3 weeks.

Determine the new minimum completion time of the project.

[2 marks]

From the network in (a), activity C has float = 18 − 5 − 12 = 1 week

If C is extended by 3 weeks, C finishes at 5 + 15 = 20 weeks,

which is 2 weeks after its latest finish time of 18, so C becomes critical.

Critical path becomes B C G J (or B C G I K L):

5 + 15 + 7 + 15 = 42

The minimum completion time increases by 2 weeks.

New minimum completion time = 42 weeks

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6 A garden has seven statues *A*, *B*, *C*, *D*, *E*, *F* and *G*, with paths connecting each pair of statues, either directly or indirectly.

To provide better access to all the statues, some of the paths are being made wider.

6 (a) State why **six** is the minimum number of paths that need to be made wider.

[2 marks]

The wider paths must connect all 7 statues, so they form a connected graph on 7 vertices. The minimum number of edges is used when this is a spanning tree, which has 7 − 1 = 6 edges. With fewer than 6 paths at least one statue would not be connected, so a minimum of 6 paths need to be made wider.

6 (b) The table below shows the number of trees that need to be removed to make the path between adjacent statues wider.

A dash in the table means that there is no direct path between the two statues.

Statue	*A*	*B*	*C*	*D*	*E*	*F*	*G*
A	–	4	7	–	–	–	–
B	4	–	6	2	3	–	–
C	7	6	–	–	3	–	4
D	–	2	–	–	4	5	–
E	–	3	3	4	–	3	7
F	–	–	–	5	3	–	6
G	–	–	4	–	7	6	–

Find the minimum number of trees that need to be removed.

Fully justify your answer.

[3 marks]

Use Prim's algorithm on the table, starting at A:

Add AB (4), then BD (2), then BE (3),

then EC (3), then EF (3), then CG (4)

Minimum spanning tree: AB, BD, BE, CE, EF, CG

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Total weight = 4 + 2 + 3 + 3 + 3 + 4 = 19

The minimum spanning tree connects all statues with the least total weight, so the minimum number of trees that need to be removed is 19 trees

6 (c) A landscaper identifies that two **new** wide paths could be constructed without removing any trees. However, there are only enough resources to build **one** new wide path.

The new wide path could be between *A* and *D* **or** between *A* and *F*.

Explain clearly how the solution to part **(b)** can be adapted to find the new minimum number of trees that need to be removed.

[2 marks]

Add the new path (AD or AF) to the minimum spanning tree from (b) with weight 0. This creates a cycle: A B D A for AD, or A B E F A for AF. Remove the arc with the largest weight in the cycle. In both cases this is AB (4), which has the largest weight of the arcs connected to A, D, F. So use either AD or AF instead of AB:

new minimum = 19 − 4 = 15 trees

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7 Robyn manages a bakery.

Each day the bakery bakes 900 rolls, 600 teacakes and 450 croissants.

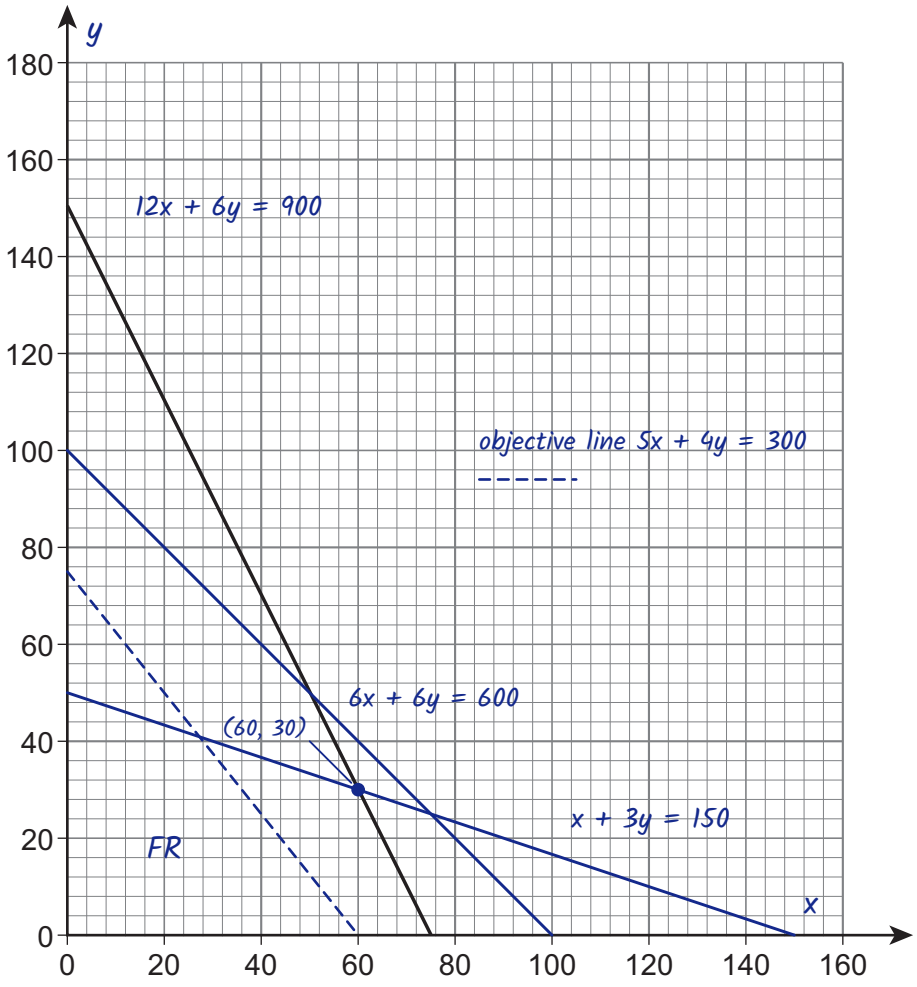
The bakery sells two types of bakery box which contain rolls, teacakes and croissants, as shown in the table below.

Type of bakery box	Number of rolls	Number of teacakes	Number of croissants	Profit per box sold
Standard	12	6	3	£2.50
Luxury	6	6	9	£2.00

Robyn formulates a linear programming problem to find the maximum profit the bakery can make from selling the bakery boxes.

7 (a) Part of a graphical method to solve this linear programming problem is shown on **Figure 1**.

Figure 1



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7 (a) (i) Explain how the line shown on **Figure 1** relates to the linear programming problem.

Clearly define any variables that you introduce.

[3 marks]

Let x = number of standard boxes sold each day

Let y = number of luxury boxes sold each day

The line passes through (0, 150) and (75, 0): it is $12x + 6y = 900$

($y = -2x + 150$). It is the boundary of the constraint $12x + 6y \leq 900$,

the total number of rolls used (900 rolls baked each day), so the

feasible region lies on or below this line.

7 (a) (ii) Use **Figure 1** to find the maximum profit that the bakery can make from selling bakery boxes.

[6 marks]

Teacakes: $6x + 6y \leq 600 \Rightarrow x + y \leq 100$

Croissants: $3x + 9y \leq 450 \Rightarrow x + 3y \leq 150$

Also $x \geq 0$, $y \geq 0$

Lines drawn on Figure 1 and feasible region FR identified.

Maximise $P = 2.5x + 2y$

Objective line $2.5x + 2y = 150$ ($5x + 4y = 300$) drawn; moving it away

from the origin, the last vertex of FR reached is the intersection of

$12x + 6y = 900$ and $3x + 9y = 450$:

$2x + y = 150$ and $x + 3y = 150 \Rightarrow 5y = 150$, $y = 30$, $x = 60$

Optimal vertex (60, 30)

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Check vertices: (75, 0): P = 187.50 (0, 50): P = 100

(60, 30): P = 2.5 × 60 + 2 × 30 = 150 + 60 = 210

Maximum profit at 60 standard boxes and 30 luxury boxes

Maximum profit = £210

7 (b) State an assumption that you have made in part **(a)(ii)**.

[1 mark]

All of the bakery boxes that are made up each day are sold (there is enough demand for every box).

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8 The set S is defined as

$$S = \{a, b, c, d\}$$

Figure 2 shows a Cayley table for S under the commutative binary operation $\odot$

Figure 2

$\odot$	a	b	c	d
a	a	a	a	a
b	a	d	b	c
c	a	b	c	d
d	a	c	d	a

8 (a) (i) Prove that there exists an identity element for S under the binary operation $\odot$

[2 marks]

From the row and column for c in Figure 2:

a ⊙ c = c ⊙ a = a, b ⊙ c = c ⊙ b = b

c ⊙ c = c, d ⊙ c = c ⊙ d = d

Every element of S is unchanged when combined with c,

so x ⊙ c = c ⊙ x = x for all x ∈ S

Therefore c is an identity element for S under ⊙

8 (a) (ii) State the inverse of b under the binary operation $\odot$

[1 mark]

b ⊙ d = d ⊙ b = c, so the inverse of b is d

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8 (b) **Figure 3** shows a Cayley table for multiplication modulo 4

Figure 3

$\times_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Mali says that, by substituting suitable distinct values for a, b, c and d, the Cayley table in **Figure 2** could represent multiplication modulo 4

Use your answers to part **(a)** to show that Mali's statement is **incorrect**.

[3 marks]

From Figure 3, the identity element for $\times_4$ is 1

$1 \times_4 1 = 1$, $3 \times_4 3 = 1$, and 0, 2 have no inverse (no 1 in their rows)

So in Figure 3 the only elements with inverses are self-inverse:

there are no pairs of distinct elements that combine to give 1.

From part (a), the identity in Figure 2 is c and the inverse of b is d,

where $b \neq d$, so Figure 2 has a pair of distinct inverse elements.

Any substitution would have to map c to 1, and then b and d would be

distinct elements combining to give 1, which is impossible.

Therefore the Cayley table in Figure 2 cannot represent

multiplication modulo 4, so Mali's statement is incorrect.

END OF QUESTIONS

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