Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Statistics

Friday 16 May 2025 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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ANSWER IN THE SPACES PROVIDED

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Answer **all** questions in the spaces provided.

1 The continuous random variable A has probability density function $\mathrm{f}(a)$ and variance 16

It is given that $\mathrm{f}(a) = 0$ for $a < 0$ and $\int_0^\infty a^2 \mathrm{f}(a)\mathrm{d}a = 52$

Find the mean of A

Circle your answer.

[1 mark]

4 (6) 36 68

$E(A^2) - [E(A)]^2 = 16 \Rightarrow [E(A)]^2 = 52 - 16 = 36 \Rightarrow E(A) = 6$

2 The random variable $X \sim$ Po(6.50)

The random variable $Y \sim$ Po(20.54)

The random variables X and Y are independent.

Find the standard deviation of $X + Y$

Circle your answer.

[1 mark]

3.75 (5.20) 14.04 27.04

$X + Y \sim Po(27.04)$, $Var(X + Y) = 27.04$, $sd = \sqrt{27.04} = 5.20$

Turn over for the next question

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3 The random variable T has a discrete uniform distribution and takes values 1, 2, 3, …, 17

Find the standard deviation of T

Circle your answer.

[1 mark]

$\frac{8\sqrt{3}}{3}$ $\quad$ $2\sqrt{6}$ $\quad$ $21\frac{1}{3}$ $\quad$ 24

$Var(T) = \frac{17^2 - 1}{12} = 24, \quad sd = \sqrt{24} = 2\sqrt{6}$

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4 The discrete random variable X has probability distribution

x	1	2	a
$P(X = x)$	0.2	0.7	0.1

where a is a constant greater than 2

4 (a) Find $P(X \geq 2)$

[1 mark]

$P(X \geq 2) = 0.7 + 0.1 = 0.8$

4 (b) The continuous random variable Y has probability density function $f(y)$ where

$$f(y) = \begin{cases} \frac{1}{36}y(6-y) & 0 \leq y \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

The random variables X and Y are independent.

The mean of $X + Y$ is equal to 5

Find the value of a

[4 marks]

$E(Y) = \int_0^6 \frac{1}{36} y^2(6 - y)\, dy$

$= \frac{1}{36}\left[2y^3 - \frac{y^4}{4}\right]_0^6 = \frac{1}{36}(432 - 324) = 3$

$E(X) = 1 \times 0.2 + 2 \times 0.7 + a \times 0.1 = 1.6 + 0.1a$

$E(X + Y) = E(X) + E(Y) = 5$

$1.6 + 0.1a + 3 = 5$

$0.1a = 0.4 \Rightarrow a = 4$

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5 Journey times from a town to the nearest city have a normal distribution with known standard deviation 8.1 minutes.

A random sample of 50 journey times is taken and has mean 54.7 minutes.

5 (a) Construct an 86% confidence interval for the population mean journey time.

Give your values in minutes to two decimal places.

[3 marks]

86% confidence $\Rightarrow P(Z < z) = 0.93 \Rightarrow z = 1.4758$

$54.7 \pm 1.4758 \times \frac{8.1}{\sqrt{50}}$

$= 54.7 \pm 1.6906...$

Confidence interval = (53.01, 56.39) minutes

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5 (b) The town mayor claims that the population mean journey time is 57.7 minutes.

Dana takes a random sample of n journey times.

Her sample has mean 55.1 minutes.

She uses her sample to construct an 86% confidence interval for the population mean journey time.

Her confidence interval does not support the mayor's claim.

Fox takes a random sample of $n-1$ journey times.

His sample also has a mean of 55.1 minutes.

He uses his sample to construct an 86% confidence interval for the population mean journey time.

His confidence interval supports the mayor's claim.

Find the value of n

[4 marks]

Upper limit of CI = $55.1 + 1.4758 \times \frac{8.1}{\sqrt{n}}$

Dana: CI does not contain 57.7, so

$55.1 + 1.4758 \times \frac{8.1}{\sqrt{n}} < 57.7$

$\sqrt{n} > \frac{1.4758 \times 8.1}{2.6} = 4.5977... \Rightarrow n > 21.14$

Fox: CI contains 57.7, so

$55.1 + 1.4758 \times \frac{8.1}{\sqrt{(n-1)}} > 57.7$

$\sqrt{(n-1)} < 4.5977... \Rightarrow n - 1 < 21.14 \Rightarrow n < 22.14$

n is an integer with $21.14 < n < 22.14$

$\therefore n = 22$

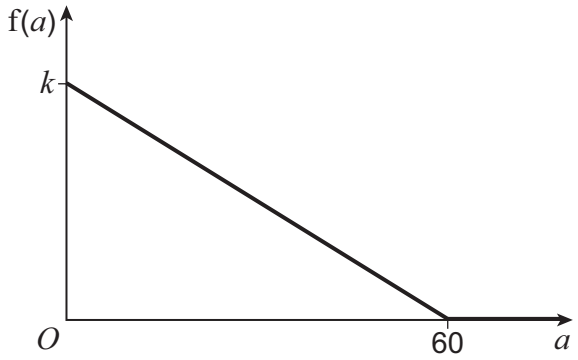
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6 The continuous random variable A represents the score achieved by a player in a particular game.

The probability density function of A is $\mathrm{f}(a)$

The probability density function consists of a line segment between $(0, k)$ and $(60, 0)$, and for all other values of a, $\mathrm{f}(a) = 0$

The probability density function is shown in the graph.



6 (a) Find the value of k

[1 mark]

Area under graph = 1: $\frac{1}{2} \times 60 \times k = 1$

$k = \frac{1}{30}$

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6 (b) Find the median score of the game.

Give your answer to two decimal places.

[5 marks]

Gradient $= \frac{0 - 1/30}{60 - 0} = -\frac{1}{1800}$

$f(a) = 1/30 - a/1800, \quad 0 \leq a \leq 60$

Median m: $\int_0^m \left(\frac{1}{30} - \frac{a}{1800} \right) da = \frac{1}{2}$

$\frac{m}{30} - \frac{m^2}{3600} = \frac{1}{2}$

$m^2 - 120m + 1800 = 0$

$m = \frac{120 \pm \sqrt{(120^2 - 4 \times 1800)}}{2} = 60 \pm \sqrt{1800}$

$m = 102.426...$ or $m = 17.573...$

$m = 102.43$ is not in $0 \leq a \leq 60$, so reject it

Median = 17.57

Turn over ▶

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7 Kristi set up a website for her new home business. The number of visitors per hour to the website is modelled using a Poisson distribution.

During the first month, the mean number of visitors per hour was 3

7 (a) An interval of one hour is chosen at random.

Find the probability that there were at least two visitors to the website.

Give your answer to three decimal places.

[2 marks]

X = number of visitors in one hour, $X \sim Po(3)$

$P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.19915$

$= 0.80085... = 0.801$

7 (b) A year later, Kristi claims that the mean number of visitors per hour has changed.

She selects an interval of one hour at random and counts the number of visitors to the website.

She carries out a hypothesis test to investigate her claim, using the 10% level of significance.

7 (b) (i) Describe, in the context of Kristi's test, what is meant by a Type I error.

[1 mark]

Concluding that the mean number of visitors per hour has changed (from 3) when in fact it has not changed.

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7 (b) (ii) Find the probability that Kristi makes a Type I error.

Give your answer to four decimal places.

Fully justify your answer.

[4 marks]

H_0: $\lambda = 3$ H_1: $\lambda \neq 3$ (two-tailed, 5% in each tail)

Under H_0, $X \sim Po(3)$

Lower tail: $P(X \leqslant 0) = 0.04979 < 0.05$

$P(X \leqslant 1) = 0.19915 > 0.05$

Upper tail: $P(X \geqslant 6) = 1 - P(X \leqslant 5) = 1 - 0.91608 = 0.08392 > 0.05$

$P(X \geqslant 7) = 1 - P(X \leqslant 6) = 1 - 0.96649 = 0.03351 < 0.05$

Critical region: $X = 0$ or $X \geqslant 7$

$P(\text{Type I error}) = P(X \leqslant 0) + P(X \geqslant 7)$

$= 0.04979 + 0.03351$

$= 0.08330... = 0.0833$

7 (c) Kristi looks at the number of visitors to her website over a 24-hour period.

She notices that there are fewer visitors at night.

Explain why the model used in parts **(a)** and **(b)** may not be valid.

[1 mark]

The mean number of visitors per hour is not constant over the 24-hour period (it is lower at night), so a Poisson model with a constant mean of 3 may not be valid.

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8 A company specialising in computer repairs has 200 employees.

The company collects data during a randomly selected week on the number of years of experience employees have and the average time they take to repair a computer.

Some of the data they collected is shown in the table.

		Average time to repair a computer			
		Less than 1 day	**Between 1 and 3 days**	**More than 3 days**	**Total**
Years of experience	**Less than 1 year**				70
	More than 1 year			30	
	Total	80	60		200

A manager at the company claims that there is an association between number of years of experience and average time taken to repair a computer.

A hypothesis test is carried out to investigate the claim.

The test statistic $\sum \frac{(O-E)^2}{E}$ is calculated and found to be 8.645 correct to three decimal places.

All the values of E are greater than 5

8 (a) Test the manager's claim using the 2.5% level of significance.

[4 marks]

H_0: There is no association between years of experience and average time taken to repair a computer

H_1: There is an association between years of experience and average time taken to repair a computer

Degrees of freedom = $(2-1)(3-1) = 2$

χ^2 critical value at 2.5% level = 7.378

$8.645 > 7.378$ so reject H_0

There is sufficient evidence at the 2.5% level to suggest that there is an association between years of experience and average time taken to repair a computer.

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8 (b) A total of x employees have less than 1 year's experience and have an average time to repair a computer of less than 1 day.

8 (b) (i) Find the missing entries in the contingency tables below.

[2 marks]

		Average time to repair a computer		
	Expected (E)	**Less than 1 day**	**Between 1 and 3 days**	**More than 3 days**
Years of experience	**Less than 1 year**	28	21	21
	More than 1 year	52	39	39

		Average time to repair a computer		
	Observed (O)	**Less than 1 day**	**Between 1 and 3 days**	**More than 3 days**
Years of experience	**Less than 1 year**	x	$40 - x$	30
	More than 1 year	$80 - x$	$x + 20$	30

$E = \frac{\text{row total} \times \text{column total}}{200}$, eg $\frac{70 \times 80}{200} = 28$

8 (b) (ii) Find the value of x

[4 marks]

$\Sigma \frac{(O-E)^2}{E} = 8.645$

$\frac{(x-28)^2}{28} + \frac{(28-x)^2}{52} + \frac{(19-x)^2}{21} + \frac{(x-19)^2}{39}$

$+ \frac{(30-21)^2}{21} + \frac{(30-39)^2}{39} = 8.645$

$\frac{5}{91}(x-28)^2 + \frac{20}{273}(x-19)^2 + 5.93406\ldots = 8.645$

$0.128205\ldots x^2 - 5.860805\ldots x + 66.81287\ldots = 0$

$x = 24.001\ldots$ or $x = 21.713\ldots$

x is a frequency so must be a whole number

$\therefore\ x = 24$

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Check x = 24: O = 24, 16, 30, 56, 44, 30

$\Sigma(O - E)^2/E = 0.571 + 1.190 + 3.857 + 0.308 + 0.641 + 2.077$

$= 8.645$ ✓

8 (c) The largest value of $\frac{(O-E)^2}{E}$ is for employees who have less than 1 year's experience and have an average time taken to repair a computer of more than 3 days.

The smallest value of $\frac{(O-E)^2}{E}$ is for employees who have more than 1 year's experience and have an average time taken to repair a computer of less than 1 day.

Interpret in context the association between years of experience and average time taken to repair a computer.

[1 mark]

Employees with less than 1 year's experience have an average time taken to repair a computer of more than 3 days more often than expected (observed 30, expected 21) if there were no association.

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