Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Discrete

Friday 16 May 2025 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 A student is solving a network flow problem involving water pipes.

They have correctly found a flow through the network of 36 $m^3\ s^{-1}$

They have correctly found a cut of the network with value 36 $m^3\ s^{-1}$

Which one of the following is correct?

Tick (✓) **one** box.

[1 mark]

The maximum flow through the network is less than 36 $m^3\ s^{-1}$ ☐

The maximum flow through the network is equal to 36 $m^3\ s^{-1}$ ✓

The maximum flow through the network is greater than 36 $m^3\ s^{-1}$ ☐

It is not possible to make a conclusion about the maximum flow through the network. ☐

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2 A simple-connected graph has n vertices, where $n \geq 2$

State the **minimum** number of edges of the graph.

Circle your answer.

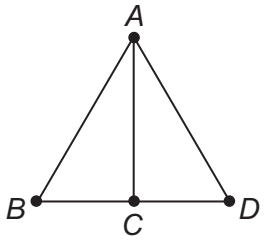
[1 mark]

$n - 1$ $\quad$ n $\quad$ $n + 1$ $\quad$ $\frac{n(n-1)}{2}$

Turn over for the next question

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3 A graph is shown below.



3 (a) Complete the table, to show the degree of each vertex.

[1 mark]

Vertex	Degree
A	3
B	2
C	3
D	2

3 (b) A student makes two statements about the graph.

Statement 1: The graph is Eulerian.

Statement 2: The graph can be drawn in one go without taking the pencil off the paper and without retracing any edges.

3 (b) (i) State, with a reason, whether **Statement 1** is correct or incorrect.

[1 mark]

Incorrect: A and C have odd degree (3), and an Eulerian graph must have every vertex of even degree.

3 (b) (ii) Write down a path to show that **Statement 2** is correct.

[1 mark]

A B C D A C

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4 A company employs five people Alisha, Gary, Jo, Sarif and Ted to complete a project.

This project comprises five tasks *V*, *W*, *X*, *Y* and *Z* which must all be completed.

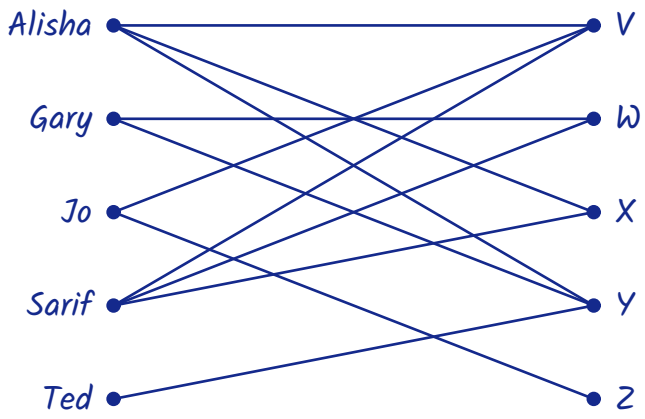
Each person is to be assigned exactly one task.

The table shows the tasks to which each person could possibly be assigned.

Person	Task
Alisha	*V*, *X*, *Y*
Gary	*W*, *Y*
Jo	*V*, *Z*
Sarif	*V*, *W*, *X*
Ted	*Y*

4 (a) Draw a bipartite graph to show this information.

[2 marks]



4 (b) Find all possible ways that each person can be assigned to exactly one task to complete the project.

[2 marks]

Ted can only do Y, so Gary must do W. Only Jo can do Z.

Then Alisha and Sarif share V and X:

Alisha–V, Gary–W, Jo–Z, Sarif–X, Ted–Y

Alisha–X, Gary–W, Jo–Z, Sarif–V, Ted–Y

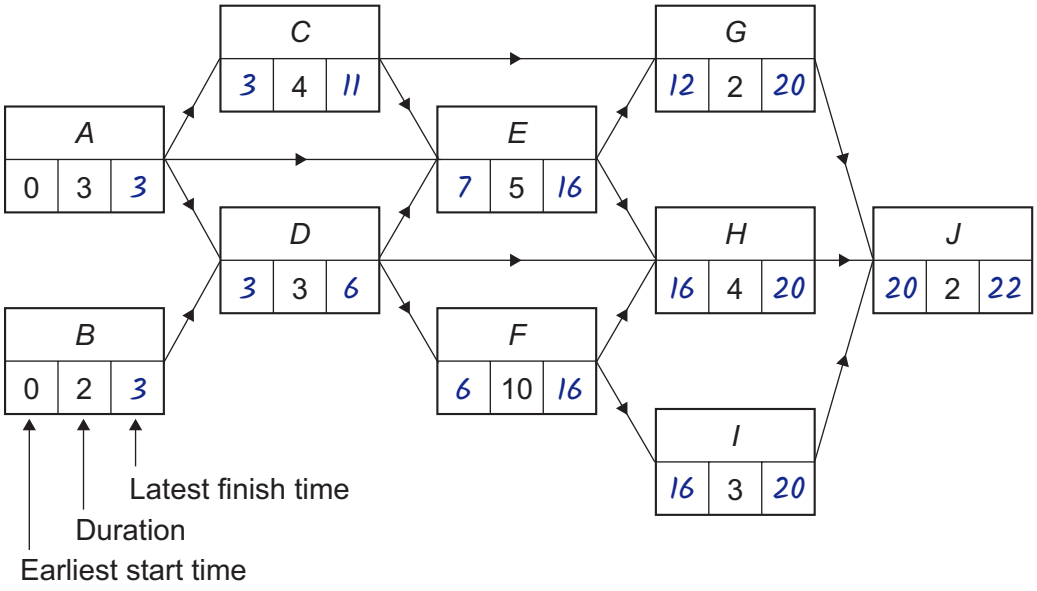
(these are the only two possible ways)

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5 A project consists of 10 activities *A, B, C, D, E, F, G, H, I* and *J*

The activity network for the project along with the duration, in days, of each activity is shown in **Figure 1**

Figure 1



5 (a) Complete the activity network in **Figure 1** by showing the earliest start time and the latest finish time for each activity.

[3 marks]

5 (b) Due to problems with the weather, the project manager predicts that the duration of activity *E* will increase by up to 7 days.

Explain whether the minimum completion time of the project increases.

[2 marks]

E: earliest start 7, latest finish 16, duration 5

Float of E = 16 − 7 − 5 = 4 days

If E increases by 4 days or fewer, the minimum completion time does not increase (it stays at 22 days).

If E increases by 5 or more days (up to 7), the minimum completion time increases, by up to 3 days (to at most 25 days).

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6 The set S is defined as $S = \left\{ x \in \mathbb{R} : x \neq \frac{1}{2} \right\}$

The binary operation $*$ is defined as

$$a * b = a + b - 2ab$$

where a and $b \in S$

It is given that S has exactly one identity element under the binary operation $*$

6 (a) Show that 0 is the identity element of S under the binary operation $*$

[2 marks]

$a * 0 = a + 0 - 2 \times a \times 0 = a$

$0 * a = 0 + a - 2 \times 0 \times a = a$

$(0 \in S$ since $0 \neq \frac{1}{2})$

So $a * 0 = 0 * a = a$ for all $a \in S$

$\therefore$ 0 is the identity element of S under $*$

6 (b) Find an expression for a^{-1}, the inverse of the element a, in S under the binary operation $*$

[2 marks]

Let $b = a^{-1}$: $a * b = 0$

$a + b - 2ab = 0$

$b - 2ab = -a$

$b(1 - 2a) = -a$

$b = \frac{-a}{1 - 2a}$

$a^{-1} = \frac{a}{2a - 1}$

Turn over ▶

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7 Maria is planning a tour of six towns *A, B, C, D, E* and *F*

On the tour, Maria will visit each town at least once, starting and finishing at *A*

Maria's aim is to minimise the total distance travelled on the tour.

The table shows the shortest distances, in miles, between each pair of towns.

	A	*B*	*C*	*D*	*E*	*F*
A	–	16	19	22	24	18
B	16	–	17	14	28	23
C	19	17	–	30	21	27
D	22	14	30	–	15	21
E	24	28	21	15	–	25
F	18	23	27	21	25	–

7 (a) Using the nearest neighbour algorithm starting from *A*, find an upper bound for the total distance travelled by Maria on the tour.

[2 marks]

Nearest neighbour from A:

A → B (16) → D (14) → E (15) → C (21) → F (27) → A (18)

Tour A B D E C F A

16 + 14 + 15 + 21 + 27 + 18 = 111

Upper bound = 111 miles

Do not write outside the box

7 (b) By deleting *A*, find a lower bound for the total distance travelled by Maria on the tour.

[3 marks]

Delete A. Minimum spanning tree of B, C, D, E, F (Prim's from B):

BD = 14, DE = 15, BC = 17, DF = 21

Weight of MST = 14 + 15 + 17 + 21 = 67

Two shortest arcs from A: AB = 16, AF = 18

Lower bound = 67 + 16 + 18

= 101 miles

7 (c) Maria claims that it is possible to travel a total distance of less than 100 miles on the tour.

Comment on the validity of Maria's claim.

[1 mark]

The lower bound is 101 miles and 101 > 100, so every tour is at least 101 miles: Maria's claim is not valid (incorrect).

Turn over for the next question

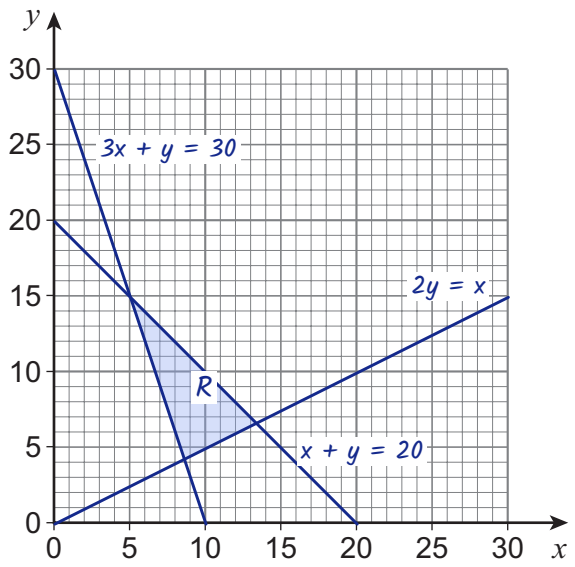
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8 A linear programming problem is represented by the following:

$$\begin{aligned} &\text{Maximise} && P = 3x + 2y \\ &\text{subject to} && x + y \leq 20 \\ & && 3x + y \geq 30 \\ & && 2y \geq x \end{aligned}$$

8 (a) Draw the feasible region R for the linear programming problem on the axes.

[4 marks]



R is the shaded triangular region, labelled R

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8 (b) Solve the linear programming problem, stating the values of x, y and P

[3 marks]

Vertices of R:

$x + y = 20,\ 3x + y = 30 \Rightarrow 2x = 10 \Rightarrow (5, 15)$

$x + y = 20,\ x = 2y \Rightarrow 3y = 20 \Rightarrow \left(\frac{40}{3}, \frac{20}{3}\right)$

$3x + y = 30,\ x = 2y \Rightarrow 7y = 30 \Rightarrow \left(\frac{60}{7}, \frac{30}{7}\right)$

At $(5, 15)$: $P = 15 + 30 = 45$

At $\left(\frac{40}{3}, \frac{20}{3}\right)$: $P = 40 + \frac{40}{3} = \frac{160}{3} = 53.3\ldots$

At $\left(\frac{60}{7}, \frac{30}{7}\right)$: $P = \frac{180}{7} + \frac{60}{7} = \frac{240}{7} = 34.3\ldots$

Maximum $P = \frac{160}{3}$ $(= 53.3\ldots)$ when $x = \frac{40}{3}$, $y = \frac{20}{3}$

Turn over for the next question

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9 Caroline and David play a zero-sum game.

The constant k is such that $0 < k < 3$

The game is represented by the following pay-off matrix for Caroline.

		David		
	Strategy	**D_1**	**D_2**	**D_3**
Caroline	**C_1**	3	-2	-1
	C_2	-2	$k + 1$	$k + 2$
	C_3	2	4	5

9 (a) Show that the game does **not** have a stable solution.

[3 marks]

Row minima: C_1: -2, C_2: -2 (since $k + 1 > 1$), C_3: 2

Column maxima: D_1: 3, D_2: 4 (since $k + 1 < 4$), D_3: 5

max(row minima) = 2

min(column maxima) = 3

max(row minima) = 2 ≠ 3 = min(column maxima)

Hence the game does not have a stable solution.

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9 (b) By first reducing the pay-off matrix to a 2 × 2 matrix, find the optimal mixed strategy for Caroline.

Fully justify your answer.

[6 marks]

Since $0 < k < 3$: $-2 < 2$, $k + 1 < 4$ and $k + 2 < 5$,

so strategy C_3 dominates strategy C_2: remove row C_2

Then $-2 < -1$ and $4 < 5$, so (David wants to minimise Caroline's gain) strategy D_2 dominates strategy D_3: remove column D_3

	D_1	D_2
C_1	3	−2
C_3	2	4

Reduced pay-off matrix

Let Caroline play C_1 with probability p and C_3 with probability $1 - p$

If David plays D_1: expected gain $= 3p + 2(1 - p) = p + 2$

If David plays D_2: expected gain $= -2p + 4(1 - p) = 4 - 6p$

$p + 2$ increases and $4 - 6p$ decreases with p, so the minimum expected gain is maximised where they are equal:

$p + 2 = 4 - 6p$

$7p = 2$

$p = \frac{2}{7}$ $(0 \leqslant p \leqslant 1)$, $1 - p = \frac{5}{7}$ (value of the game $= \frac{16}{7}$)

Caroline should play strategy C_1 with probability $\frac{2}{7}$ and strategy C_3 with probability $\frac{5}{7}$, and never play strategy C_2

END OF QUESTIONS

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