AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Statistics

Friday 17 May 2024 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The discrete random variable X has probability distribution function

$$P(X = x) = \begin{cases} 0.45 & x = 1 \\ 0.25 & x = 2 \\ 0.25 & x = 3 \\ 0.05 & x = 4 \\ 0 & \text{otherwise} \end{cases}$$

State the mode of X

Circle your answer.

[1 mark]

0.25 | 0.45 | (1) | 2.5

2 A test for association is to be carried out.

The tables below show the observed frequencies and the expected frequencies that are to be used for the test.

Observed	X	Y	Z
A	28	6	66
B	8	8	4
C	54	16	10

Expected	X	Y	Z
A	45	15	40
B	9	3	8
C	36	12	32

It is necessary to merge some rows or columns before the test can be carried out.

Find the entry in the tables that provides evidence for this.

Circle your answer.

[1 mark]

Observed A-Z | Observed B-Z | Expected A-X | (Expected B-Y)

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3 The random variable X has a normal distribution with known variance 15.7

A random sample of size 120 is taken from X

The sample mean is 68.2

Find a 94% confidence interval for the population mean of X

Give your limits to three significant figures.

[3 marks]

94% confidence: $z = 1.8808$

$\bar{x} \pm z \times \frac{\sigma}{\sqrt{n}} = 68.2 \pm 1.8808 \times \frac{\sqrt{15.7}}{\sqrt{120}}$

$= 68.2 \pm 1.8808 \times 0.36171...$

$= 68.2 \pm 0.68030...$

Lower limit = 67.5197..., upper limit = 68.8803...

94% confidence interval: (67.5, 68.9)

Turn over for the next question

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4 The discrete random variable Y has probability distribution

y	15	21	36	43
$P(Y = y)$	0.16	0.32	0.29	0.23

The standard deviation of Y is s

4 (a) Show that $s = 10.53$ correct to two decimal places.

[4 marks]

$E(Y) = 15 \times 0.16 + 21 \times 0.32 + 36 \times 0.29 + 43 \times 0.23$

$= 2.4 + 6.72 + 10.44 + 9.89 = 29.45$

$E(Y^2) = 15^2 \times 0.16 + 21^2 \times 0.32 + 36^2 \times 0.29 + 43^2 \times 0.23$

$= 36 + 141.12 + 375.84 + 425.27 = 978.23$

$Var(Y) = E(Y^2) - [E(Y)]^2$

$= 978.23 - 29.45^2$

$= 978.23 - 867.3025 = 110.9275$

$s = \sqrt{110.9275} = 10.5322...$

$s = 10.53$ correct to two decimal places, as required

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4 (b) The median of Y is m

Find $P(Y > m - 1.5s)$

[3 marks]

$P(Y \leq 21) = 0.16 + 0.32 = 0.48 < 0.5$

$P(Y \leq 36) = 0.48 + 0.29 = 0.77 \geq 0.5$, so $m = 36$

$m - 1.5s = 36 - 1.5 \times 10.53 = 36 - 15.795 = 20.205$

$P(Y > 20.205) = P(Y \geq 21)$

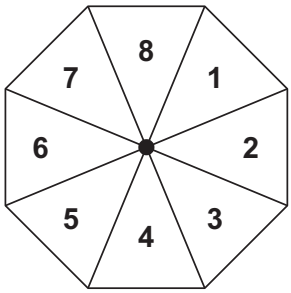
$= 0.32 + 0.29 + 0.23$

$= 0.84$

Turn over for the next question

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5 A spinner has 8 equal areas numbered 1 to 8, as shown in the diagram below.



The spinner is spun and lands with one of its edges on the ground.

5 (a) Assume that the spinner lands on each number with equal probability.

5 (a) (i) State a distribution that could be used to model the number that the spinner lands on.

[1 mark]

Discrete uniform distribution (on the integers 1 to 8)

5 (a) (ii) Use your distribution from part **5 (a) (i)** to find the probability that the spinner lands on a number greater than 5

[1 mark]

$P(X > 5) = P(6) + P(7) + P(8) = 3 \times \frac{1}{8} = \frac{3}{8}$

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5 (b) Clare spins the spinner 1000 times and records the results in the following table.

Number landed on	1	2	3	4	5	6	7	8
Frequency	37	64	112	161	308	156	109	53

5 (b) (i) Explain how the data shows that the model used in part **(a)** may not be valid.

[2 marks]

Using the model, we would expect each frequency to be close to 1000 ÷ 8 = 125 (relative frequency 0.125). However, the observed frequencies are not close to 125: for example, 5 was landed on 308 times but 1 only 37 times, so the spinner appears biased towards 5 and the outcomes are not equally likely.

5 (b) (ii) Describe how Clare's results could be used to adjust the model.

[2 marks]

Calculate the relative frequency of each number from Clare's results, e.g. P(1) = 37/1000 = 0.037, P(5) = 308/1000 = 0.308, etc. Replace the discrete uniform distribution with a discrete random variable whose probabilities are these relative frequencies.

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6 The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{3x}{44} + \frac{1}{22} & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

6 (a) Find $P(X > 2)$

[2 marks]

$P(X > 2) = \int_2^5 \left(\frac{3x}{44} + \frac{1}{22} \right) dx$

$= \left[\frac{3x^2}{88} + \frac{x}{22} \right]_2^5$

$= \left(\frac{75}{88} + \frac{5}{22} \right) - \left(\frac{12}{88} + \frac{2}{22} \right) = \frac{95}{88} - \frac{20}{88}$

$= \frac{75}{88}$ $(= 0.852...)$

6 (b) Find the upper quartile of X

Give your answer to two decimal places.

[4 marks]

Let q be the upper quartile: $\int_1^q \left(\frac{3x}{44} + \frac{1}{22} \right) dx = \frac{3}{4}$

$\left[\frac{3x^2}{88} + \frac{x}{22} \right]_1^q = \frac{3}{4}$

$\frac{3q^2}{88} + \frac{q}{22} - \frac{3}{88} - \frac{1}{22} = \frac{3}{4}$

$\frac{3q^2}{88} + \frac{q}{22} - \frac{73}{88} = 0 \quad (\times 88)$

$3q^2 + 4q - 73 = 0$

$q = \frac{-4 \pm \sqrt{(16 + 876)}}{6} = \frac{-4 \pm \sqrt{892}}{6}$

$q = 4.3110...$ or $q = -5.6443...$ (reject, as $1 \leq x \leq 5$)

Upper quartile = 4.31 (2 d.p.)

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6 (c) Find $\text{Var}\left(44X^{-3}\right)$

Give your answer to three decimal places.

[5 marks]

$E(44X^{-3}) = \int_1^5 44x^{-3}\left(\frac{3x}{44} + \frac{1}{22}\right) dx = \int_1^5 (3x^{-2} + 2x^{-3})\, dx$

$= \left[-3x^{-1} - x^{-2}\right]_1^5 = \left(-\frac{3}{5} - \frac{1}{25}\right) - (-3 - 1)$

$= \frac{84}{25} \quad (= 3.36) \qquad [\text{so } E(X^{-3}) = \frac{21}{275}]$

$E((44X^{-3})^2) = E(44^2X^{-6}) = \int_1^5 1936x^{-6}\left(\frac{3x}{44} + \frac{1}{22}\right) dx$

$= \int_1^5 (132x^{-5} + 88x^{-6})\, dx$

$= \left[-33x^{-4} - \frac{88}{5}x^{-5}\right]_1^5$

$= \left(-\frac{33}{625} - \frac{88}{15625}\right) - \left(-33 - \frac{88}{5}\right)$

$= 50.541568 \qquad [\text{so } E(X^{-6}) = 0.026106\ldots]$

$\text{Var}(44X^{-3}) = E(44^2X^{-6}) - [E(44X^{-3})]^2$

$= 50.541568 - \left(\frac{84}{25}\right)^2$

$= 50.541568 - 11.2896$

$= 39.251968$

$\text{Var}(44X^{-3}) = 39.252 \quad (3\text{ d.p.})$

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7 Over a period of time, it has been shown that the mean number of customers entering a small store is 6 per hour.

The store runs a promotion, selling many products at lower prices.

7 (a) Luke randomly selects an hour during the promotion and counts 11 customers entering the store.

He claims that the promotion has changed the mean number of customers per hour entering the store.

Investigate Luke's claim, using the 5% level of significance.

[6 marks]

Let X = number of customers entering the store in one hour

H_0: $\lambda = 6$ H_1: $\lambda \neq 6$ (λ = mean number of customers per hour)

Under H_0, $X \sim Po(6)$

Two-tailed test at 5%: compare with 0.025 in each tail

$P(X \geqslant 11) = 1 - P(X \leqslant 10)$

$= 1 - 0.95738$

$= 0.04262... = 0.043$

$0.043 > 0.025$, so the result is not significant

Do not reject H_0

There is not sufficient evidence at the 5% level to suggest that the mean number of customers entering the store per hour has changed, so Luke's claim is not supported.

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7 (b) Luke randomly selects another hour and carries out the same investigation as in part **(a)**.

Find the probability of a Type I error, giving your answer to four decimal places.

Fully justify your answer.

[4 marks]

Critical region: each tail must have probability ≤ 0.025

Lower tail: $P(X \leq 1) = 0.01735 \leq 0.025$, $P(X \leq 2) = 0.06197 > 0.025$

$\Rightarrow$ lower critical region $X \leq 1$

Upper tail: $P(X \geq 11) = 1 - P(X \leq 10) = 1 - 0.95738 = 0.04262 > 0.025$

$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.97991 = 0.02009 \leq 0.025$

$\Rightarrow$ upper critical region $X \geq 12$

Critical region: $X \leq 1$ or $X \geq 12$

$P(\text{Type I error}) = P(X \leq 1) + P(X \geq 12)$

$= 0.01735 + 0.02009$

$= 0.03744\ldots = 0.0374$ (4 d.p.)

7 (c) When observing the store, Luke notices that some customers enter the store together as a group.

Explain why the model used in parts **(a)** and **(b)** might not be valid.

[1 mark]

Customers arriving in groups do not enter the store independently,

so the Poisson condition of independent events does not hold.

END OF QUESTIONS

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