AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Mechanics

Friday 17 May 2024 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 An elastic string has modulus of elasticity 20 newtons and natural length 2 metres.

The string is stretched so that its extension is 0.5 metres.

Find the elastic potential energy stored in the string.

Circle your answer.

[1 mark]

(1.25 J) 5.5 J 5 J 10 J

2 State the dimensions of impulse.

Circle your answer.

[1 mark]

MLT^{-2} (MLT^{-1}) MLT MLT^{2}

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3 A cyclist travels around a circular track of radius 20 m at a constant speed of 8 m s^{-1}

Find the angular speed of the cyclist in radians per second.

Circle your answer.

[1 mark]

0.2 rad s^{-1} | 0.4 rad s^{-1} | 2.5 rad s^{-1} | 3.2 rad s^{-1}

Turn over for the next question

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4 **In this question use $g = 9.8\ \text{m s}^{-2}$**

A ball of mass 0.5 kg is projected vertically upwards with a speed of 10 m s^{-1}

4 (a) Calculate the initial kinetic energy of the ball.

[1 mark]

$KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.5)(10)^2 = 25\ J$

4 (b) Assuming that the weight is the only force acting on the ball, use an energy method to show that the maximum height reached by the ball is approximately 5.1 m above the point of projection.

[2 marks]

Weight only, so mechanical energy is conserved:

KE lost = PE gained at maximum height

$mgh = 25$

$(0.5)(9.8)h = 25$

$h = 25 \div 4.9 = 5.102...$

$\therefore$ maximum height $\approx$ 5.1 m above the point of projection

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4 (c) (i) A student conducts an experiment to verify the accuracy of the result obtained in part **(b)**.

They observe that the ball rises to a height of 4.4 m above the point of projection and concludes that this height difference is due to a resistance force, R newtons.

Find the total work done against R whilst the ball is moving upwards.

[2 marks]

PE gained at 4.4 m = mgh = (0.5)(9.8)(4.4) = 21.56 J

Work done against R = initial KE − PE gained

= 25 − 21.56

= 3.44 J

4 (c) (ii) Using a model that assumes R is constant, find the magnitude of R

[2 marks]

Work done against R = R × distance

R × 4.4 = 3.44

R = 3.44 ÷ 4.4 = 0.7818...

R = 0.78 N (2 s.f.)

4 (c) (iii) Comment on the validity of the model used in part **(c)(ii)**.

[1 mark]

Not valid: a constant resistance is unrealistic, as air resistance depends on speed, so R would decrease as the ball slows down.

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5 Kang is riding a motorbike along a straight, horizontal road.

The motorbike has a maximum power of 75 000 W

The maximum speed of the motorbike is 50 m s^{-1}

When the speed of the motorbike is v m s^{-1}, the resistance force is kv newtons.

Find the value of k

Fully justify your answer.

[4 marks]

P = Fv, so at maximum power and speed 50 m s^{-1}:

driving force F = 75 000 ÷ 50 = 1500 N

Newton's second law along the road: F − kv = ma

At maximum speed the motorbike cannot accelerate, so a = 0,

i.e. driving force = resistance force

1500 − k(50) = 0

50k = 1500

k = 30

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6 Kepler's Third Law of planetary motion for the period of a circular orbit around the Earth is given by the formula,

$$t = 2\pi\sqrt{\frac{r^3}{Gm}}$$

where,

t is the time taken for one orbit

r is the radius of the circular orbit

m is the mass of the Earth

G is a gravitational constant.

Use dimensional analysis to determine the dimensions of G

[4 marks]

2π is a number, so $[2\pi] = 1$ (dimensionless)

$t^2 = 4\pi^2 r^3/(Gm) \Rightarrow G = 4\pi^2 r^3/(mt^2)$

$[r] = L, \quad [m] = M, \quad [t] = T$

$[G] = [4\pi^2][r]^3 \div ([m][t]^2)$

$[G] = 1 \times L^3 \div (M\,T^2)$

$[G] = L^3 M^{-1} T^{-2}$

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7 A single force, F newtons, acts on a particle moving on a straight, smooth, horizontal line.

The force F acts in the direction of motion of the particle.

At time t seconds, $F = 6e^{t} + 2e^{2t}$ where $0 \leq t \leq \ln 8$

7 (a) Find the impulse of F over the interval $0 \leq t \leq \ln 8$

[2 marks]

$I = \int_0^{\ln 8} (6e^t + 2e^{2t})\, dt = [6e^t + e^{2t}]_0^{\ln 8}$

$= (6(8) + 8^2) - (6 + 1)$

$= 112 - 7 = 105$ N s

7 (b) The particle has a mass of 2 kg and at time $t = 0$ has velocity 5 m s^{-1}

Find the velocity of the particle when $t = \ln 8$

[3 marks]

Impulse = change in momentum: $I = mv - mu$

$2v - 2(5) = 105$

$2v = 115$

$v = 57.5$ m s^{-1}

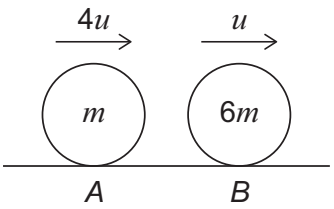
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8 Two spheres, *A* and *B*, of equal size are moving in the same direction along a straight line on a smooth horizontal surface.

Sphere *A* has mass m and is moving with speed $4u$

Sphere *B* has mass $6m$ and is moving with speed u

The diagram shows the spheres and their velocities.



Subsequently *A* collides directly with *B*

The coefficient of restitution between *A* and *B* is e

8 (a) Find, in terms of m and u, the total momentum of the spheres before the collision.

[1 mark]

Total momentum = $m(4u) + 6m(u) = 10mu$

8 (b) Show that the speed of *B* immediately after the collision is $\frac{u(3e+10)}{7}$

[4 marks]

Let the velocities after the collision be v_A and v_B

(both in the original direction of motion).

CoM: $mv_A + 6mv_B = 10mu \Rightarrow v_A + 6v_B = 10u$ (1)

NLR: $v_B - v_A = e(4u - u) = 3ue$ (2)

(1) + (2): $7v_B = 10u + 3ue$

$v_B = u(3e + 10)/7$, which is positive,

so the speed of B is $u(3e + 10)/7$ as required

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8 (c) After the collision sphere A moves in the opposite direction.

Find the range of possible values for e

[5 marks]

From (1): $v_A = 10u - 6v_B = 10u - 6u(3e + 10)/7$

$v_A = (70u - 18ue - 60u)/7$

$v_A = (10u - 18ue)/7$

A moves in the opposite direction, so $v_A < 0$:

$(10u - 18ue)/7 < 0$

$10u < 18ue \quad (u > 0)$

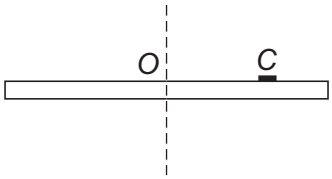
$e > 10/18 = 5/9$

Also $e \leq 1$ (coefficient of restitution)

$\therefore \ 5/9 < e \leq 1$

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9 A small coin is placed at a point *C* on a rough horizontal turntable, with centre *O*, as shown in the diagram below.



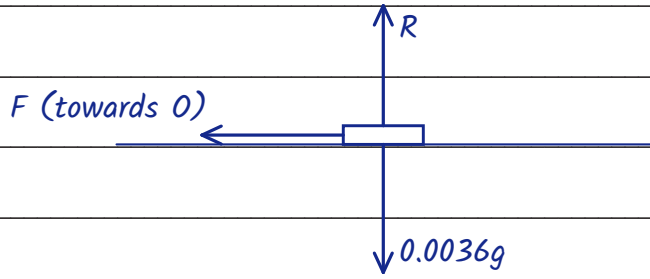
The mass of the coin is 3.6 grams.

The distance *OC* is 20 cm

The turntable rotates about a vertical axis through *O*, with constant angular speed ω radians per second.

9 (a) Draw a diagram to show all the forces acting on the coin.

[1 mark]



9 (b) The maximum value of friction is 0.01 newtons and the coin does not slip during the motion.

Find the maximum value of ω

Give your answer to two significant figures.

[4 marks]

Friction provides the force towards O: $F = mr\omega^2$

$m = 3.6\ g = 0.0036\ kg, \quad r = 20\ cm = 0.2\ m$

Maximum friction: $0.01 = (0.0036)(0.2)\omega^2$

$\omega^2 = 0.01 \div 0.00072 = 13.888...$

$\omega = \sqrt{13.888...} = 3.7267...$

Maximum $\omega = 3.7\ rad\ s^{-1}$ (2 s.f.)

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9 (c) State one modelling assumption you have made to answer part **(b)**.

[1 mark]

The coin is modelled as a particle.

END OF QUESTIONS

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