AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Discrete

Friday 17 May 2024 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 A connected planar graph has v vertices, e edges and f faces.

Which one of the formulae below is correct?

Circle your answer.

[1 mark]

$v + e + f = 2$ $\quad$ $v - e + f = 2$ $\quad$ $v - e - f = 2$ $\quad$ $v + e - f = 2$

2 Find an expression for the number of edges in the complete bipartite graph, $K_{m,n}$

Circle your answer.

[1 mark]

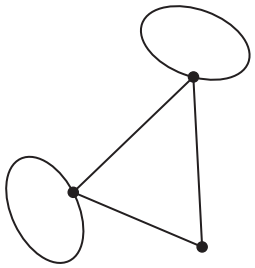
$\frac{m}{n}$ $\quad$ $m - n$ $\quad$ $m + n$ $\quad$ mn

Do not write outside the box

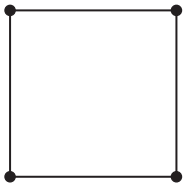
3 Which one of the graphs shown below is semi-Eulerian?

Tick (✓) **one** box.

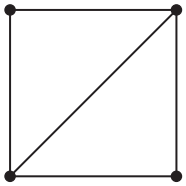
[1 mark]



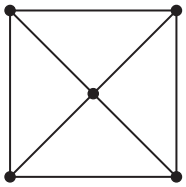
☐



☐



☑



☐

Do not write outside the box

4 The set S is defined as $S = \{1, 2, 3, 4\}$

4 (a) Complete the Cayley Table shown below for S under the binary operation multiplication modulo 5

[2 marks]

$\times_5$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

4 (b) State the identity element for S under multiplication modulo 5

[1 mark]

Identity = 1 (1 × a = a × 1 = a for all a in S)

4 (c) State the self-inverse elements of S under multiplication modulo 5

[1 mark]

1 × 1 = 1 and 4 × 4 = 16 ≡ 1 (mod 5)

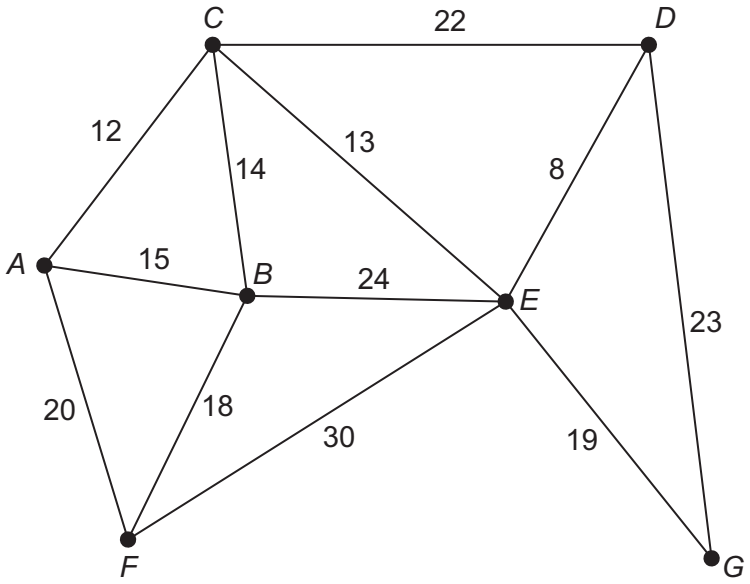
Self-inverse elements: 1 and 4

Do not write outside the box

5 A network of roads connects the villages *A*, *B*, *C*, *D*, *E*, *F* and *G*

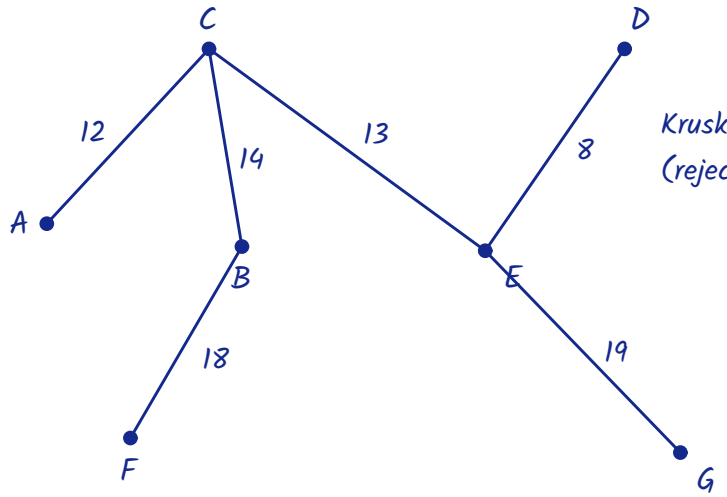
The weight on each arc in the network represents the distance, in miles, between adjacent villages.

The network is shown in the diagram below.



5 (a) Draw, in the space below, the spanning tree of minimum total length for this road network.

[3 marks]



Kruskal: DE 8, AC 12, CE 13, BC 14, (reject AB 15), BF 18, EG 19

5 (b) Find the total length of the spanning tree drawn in part **(a)**.

[1 mark]

8 + 12 + 13 + 14 + 18 + 19 = 84 miles

Turn over ▶

Do not write outside the box

6 A Young Enterprise Company decides to sell two types of cakes at a breakfast club.

The two types of cakes are blueberry and chocolate.

From its initial market research, the company knows that it will:

- sell at most 200 cakes in total
- sell at least twice as many blueberry cakes as they will chocolate cakes
- make 20p profit on each blueberry cake they sell
- make 15p profit on each chocolate cake they sell.

The company's objective is to maximise its profit.

Formulate the Young Enterprise Company's situation as a linear programming problem.

[4 marks]

Let x = number of blueberry cakes sold

Let y = number of chocolate cakes sold

Maximise $P = 20x + 15y$ (profit in pence)

subject to

$x + y \leq 200$

$x \geq 2y$

$x \geq 0, \ y \geq 0$

x and y are integers

Do not write outside the box

7 The binary operation ∇ is defined as

$$a \nabla b = a + b + ab \text{ where } a, b \in \mathbb{R}$$

7 (a) Determine if ∇ is commutative on $\mathbb{R}$

Fully justify your answer.

[2 marks]

$b \nabla a = b + a + ba$

$= a + b + ab$ (addition and multiplication on $\mathbb{R}$ are commutative)

$= a \nabla b$ for all $a, b \in \mathbb{R}$

Hence ∇ is commutative on $\mathbb{R}$

7 (b) Prove that ∇ is associative on $\mathbb{R}$

[3 marks]

$(a \nabla b) \nabla c = (a + b + ab) \nabla c$

$= a + b + ab + c + (a + b + ab)c$

$= a + b + c + ab + ac + bc + abc$

$a \nabla (b \nabla c) = a \nabla (b + c + bc)$

$= a + b + c + bc + a(b + c + bc)$

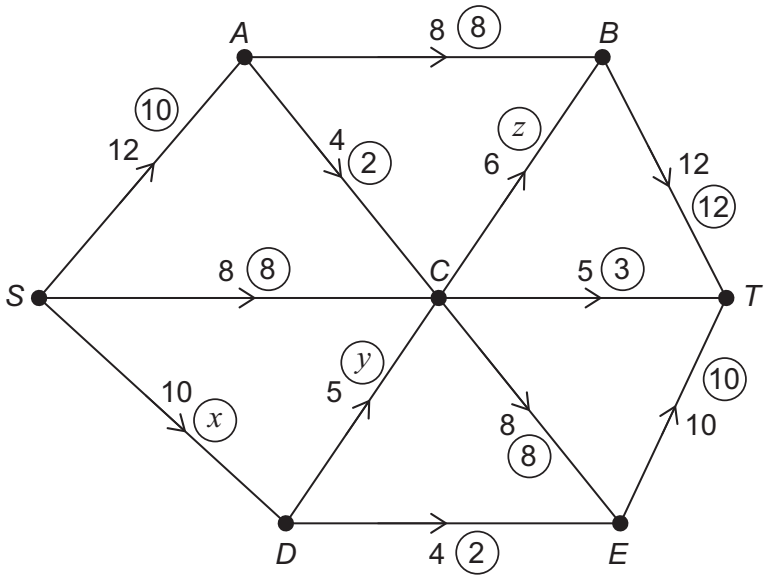
$= a + b + c + ab + ac + bc + abc$

So $(a \nabla b) \nabla c = a \nabla (b \nabla c)$ for all $a, b, c \in \mathbb{R}$

Hence ∇ is associative on $\mathbb{R}$

Do not write outside the box

8 The diagram below shows a network of pipes.



The uncircled numbers on each arc represent the capacity of each pipe in $m^3\ s^{-1}$

The circled numbers on each arc represent an initial feasible flow, in $m^3\ s^{-1}$, through the network.

The initial flow through pipe *SD* is x $m^3\ s^{-1}$

The initial flow through pipe *DC* is y $m^3\ s^{-1}$

The initial flow through pipe *CB* is z $m^3\ s^{-1}$

8 (a) By considering the flows at the source and the sink, explain why $x = 7$

[3 marks]

Total flow into sink = BT + CT + ET = 12 + 3 + 10 = 25

Total flow out of source = SA + SC + SD = 10 + 8 + x = 18 + x

The total flow out of the source must equal the total flow into the sink, so

18 + x = 25

∴ x = 7

Do not write outside the box

8 (b) (i) State the value of y

[1 mark]

At D: flow in = 7, flow out = $y + 2 \Rightarrow y = 7 - 2$

$y = 5$

8 (b) (ii) State the value of z

[1 mark]

At C: flow in = $2 + 8 + 5 = 15$, flow out = $z + 3 + 8 \Rightarrow z = 15 - 11$

$z = 4$

8 (c) Prove that the maximum flow through the network is at most 27 $m^3\ s^{-1}$

[2 marks]

Consider the cut through arcs BT, CT and ET:

value of cut = $12 + 5 + 10 = 27\ m^3\ s^{-1}$

By the maximum flow–minimum cut theorem, the maximum flow is less than or equal to the value of any cut, so the maximum flow through the network is at most $27\ m^3\ s^{-1}$

Do not write outside the box

9 Robert, a project manager, and his team of builders are working on a small building project.

Robert has divided the project into ten activities labelled *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I* and *J* as shown in the precedence table below:

Activity	Immediate Predecessor(s)	Duration (Days)
A	None	1
B	None	1
C	*A*	10
D	*A*	2
E	*B, D*	5
F	*E*	6
G	*E*	1
H	*F*	1
I	*F*	2
J	*C, G, H, I*	4

9 (a) On the opposite page, construct an activity network for the project and fill in the earliest start time and latest finish time for each activity.

[4 marks]

9 (b) Robert claims that the project can be completed in 20 days.

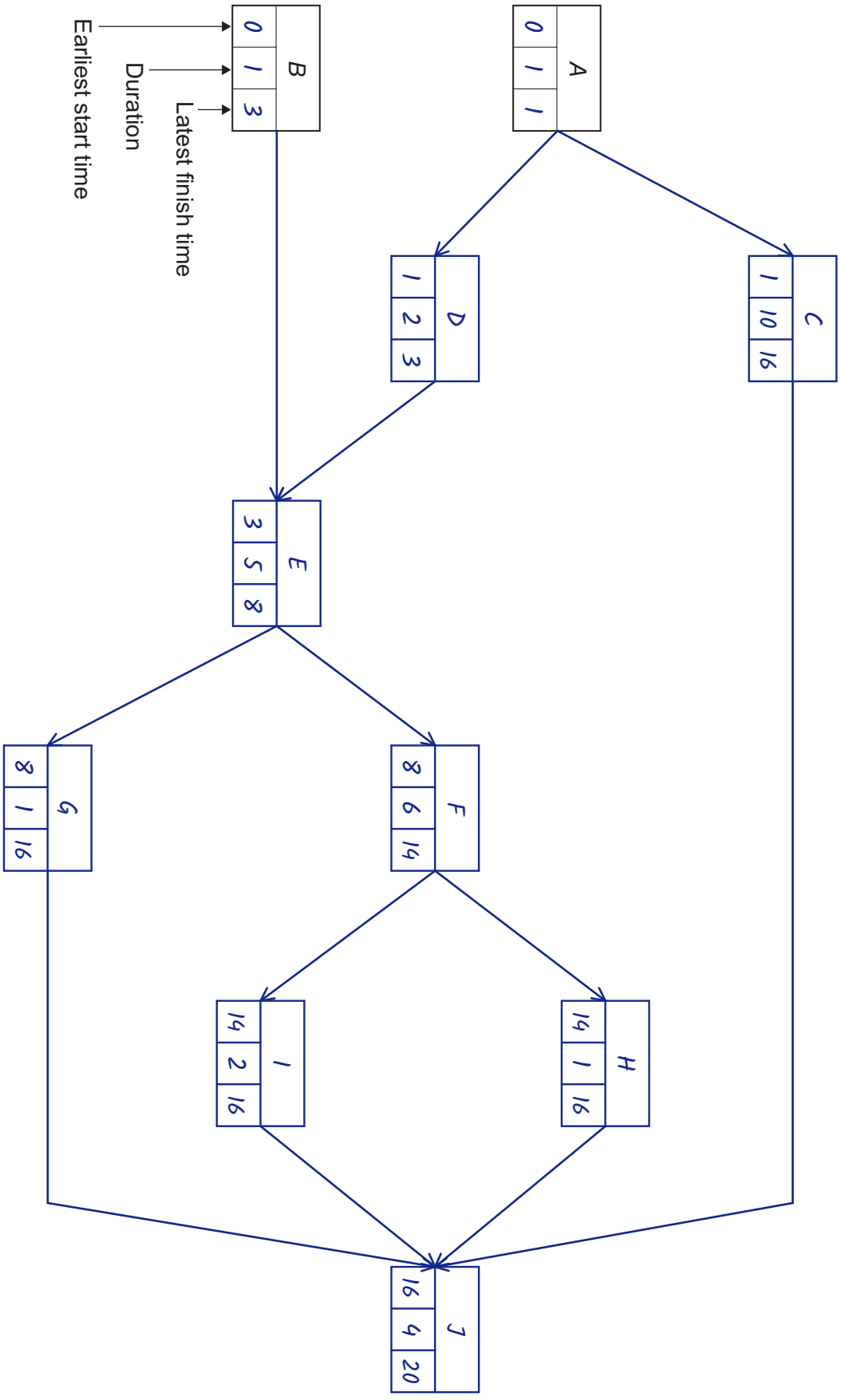
Comment on the validity of Robert's claim.

[2 marks]

The critical path is A D E F I J and the minimum completion time of the project is 20 days (LFT of J = 20), so Robert's claim appears to be valid.

However, this assumes there are enough builders for activities to be worked on simultaneously, so that each activity begins at its earliest start time, and that there are no delays to any of the critical activities. Otherwise the project will take longer than 20 days.

Do not write outside the box



Turn over for the next question

Do not write outside the box

10 Bilal and Mayon play a zero-sum game.

The game is represented by the following pay-off matrix for Bilal, where x is an integer.

		Mayon		
	Strategy	**M_1**	**M_2**	**M_3**
Bilal	**B_1**	-2	-1	1
	B_2	4	-3	1
	B_3	-1	x	0

The game has a stable solution.

10 (a) Show that there is **only** one possible value for x

Fully justify your answer.

[6 marks]

Row minima: B_1: -2, B_2: -3, B_3: -1 if $x \geq -1$, or x if $x < -1$

Column maxima: M_1: 4, M_2: -1 if $x \leq -1$, or x if $x > -1$, M_3: 1

For a stable solution, max(row minima) = min(column maxima)

Case $x \geq 0$:

max(row minima) = max(-2, -3, -1) = -1

min(column maxima) = min(4, x, 1) ≥ 0

$-1 \neq$ min(column maxima), so no stable solution

Case $x = -1$:

max(row minima) = max(-2, -3, -1) = -1

min(column maxima) = min(4, -1, 1) = -1

these are equal, so there is a stable solution

Case $x \leq -2$:

max(row minima) = max(-2, -3, x) = -2

min(column maxima) = min(4, -1, 1) = -1

$-2 \neq -1$, so no stable solution

Do not write outside the box

So x must equal −1 so that max(row minima) = −1 = min(column maxima)

Therefore a stable solution occurs only when x = −1 (B_3 plays M_2),

so there is only one possible value of x: x = −1

10 (b) State the value of the game for Bilal.

[1 mark]

The value of the game for Bilal is −1

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Copyright information

For confidentiality purposes, all acknowledgements of third-party copyright material are published in a separate booklet. This booklet is published after each live examination series and is available for free download from www.aqa.org.uk.

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team.

Copyright © 2024 AQA and its licensors. All rights reserved.