AS
FURTHER MATHEMATICS
7366/2D

Paper 2 Discrete

Mark scheme

June 2024

Version: 1.0 Final

246A7366/2D/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

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Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

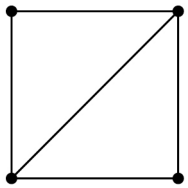
When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles 2nd answer	1.2	B1	$v-e+f=2$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles 4th answer	2.2a	B1	mn
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Ticks 3rd box	1.1b	B1	
	Question total		**1**	

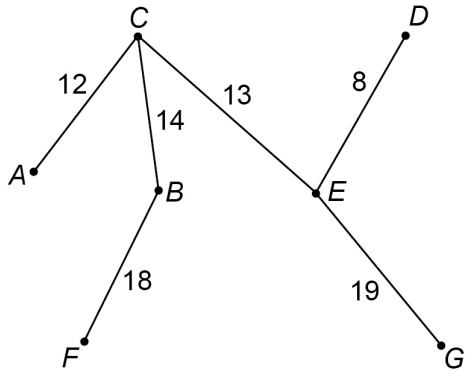
Q	Marking instructions	AO	Marks	Typical solution
4(a)	Completes table with at least two correct rows or two correct columns	1.1a	M1	
	Completes the table fully correctly	1.1b	A1	
	Subtotal		**2**	

$\times_5$	**1**	**2**	**3**	**4**
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

Q	Marking instructions	AO	Marks	Typical solution
4(b)	States 1	1.1b	B1	1
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(c)	States 1 and 4, and no others	1.1b	B1	1 and 4
	Subtotal		**1**	

	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Draws a tree with all vertices labelled and at least four edges correct	3.4	M1	
	Draws a spanning tree with all vertices labelled and exactly 6 edges	1.1a	M1	
	Draws the fully correct spanning tree of minimum total length with all vertices labelled	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Obtains the correct total length for their spanning tree from **part (a)** Condone missing units	1.1b	B1F	84 miles
	Subtotal		**1**	

	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6	Defines two variables to represent the **number of** blueberry cakes and the **number of** chocolate cakes	3.1b	B1	Let x = number of blueberry cakes made Let y = number of chocolate cakes made
	Obtains one of $x + y \leq 200$, $x \geq 2y$ or $20x + 15y$ OE Condone strict inequality	1.1a	M1	Maximise $20x + 15y$ subject to $x + y \leq 200$ $x \geq 2y$ $x \geq 0, y \geq 0$ x and y are integers
	Obtains $x + y \leq 200$ and $x \geq 2y$	1.1b	A1	
	Formulates the linear programming problem correctly with a statement of maximising a correct objective function and all constraints fully correct	2.5	A1	
	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Obtains $b \nabla a = b + a + ba$	3.1a	M1	$b \nabla a = b + a + ba$ $= a + b + ab$ $= a \nabla b$ Hence ∇ is commutative on P
	Completes reasoned argument to conclude that ∇ is commutative	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Writes down both $(a \nabla b) \nabla c$ and $a \nabla (b \nabla c)$	3.1a	M1	$(a \nabla b) \nabla c = (a + b + ab) \nabla c$ $= a + b + ab + c + ac + bc + abc$ $a \nabla (b \nabla c) = a \nabla (b + c + bc)$ $= a + b + c + bc + ab + ac + abc$ As $(a \nabla b) \nabla c = a \nabla (b \nabla c)$ then ∇ is associative on P
	Fully expands at least one of $(a \nabla b) \nabla c$ or $a \nabla (b \nabla c)$ correctly	1.1b	M1	
	Completes reasoned argument, expanding both expressions correctly to prove that ∇ is associative	2.1	R1	
	Subtotal		**3**	

	Question total		**5**	

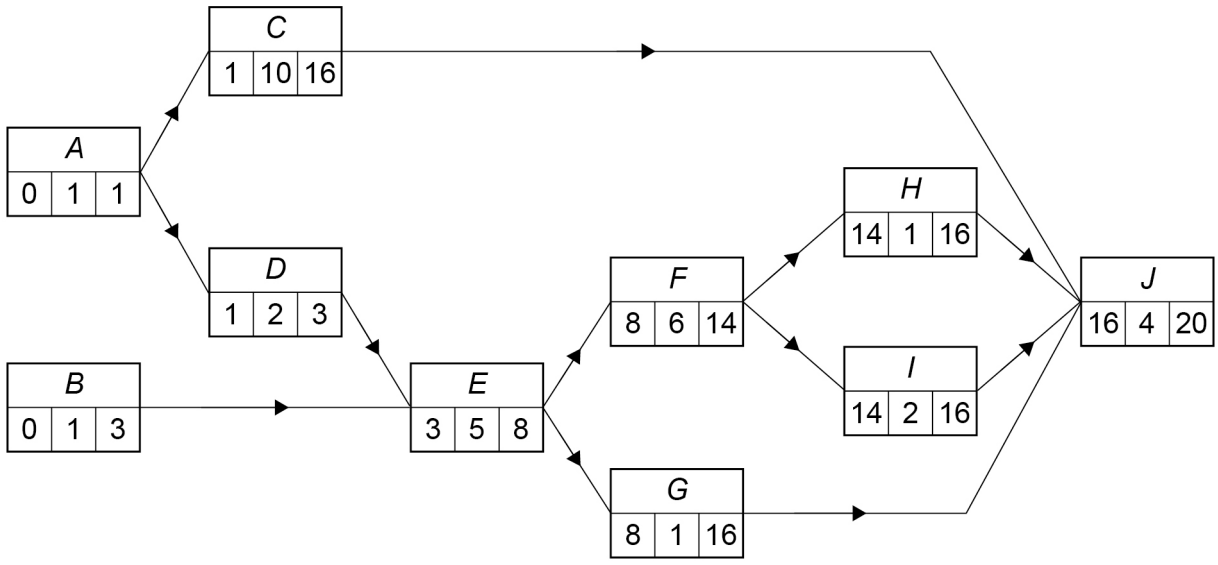
Q	Marking instructions	AO	Marks	Typical solution
8(a)	Adds together current flows on *BT*, *CT* and *ET* or flows on *SA*, *SC* and *SD* or states the current flow through the network is 25	3.1b	M1	Total flow into sink $= BT + CT + ET$ $= 12 + 3 + 10$ $= 25$
	Obtains flows of 25 and $18 + x$	1.1b	A1	Total flow from the source $= SA + SC + SD$ $= 10 + 8 + x$ $= 18 + x$
	Completes reasoned argument referring to the total flow from the source equalling the total flow into the sink to show that $x = 7$	2.1	R1	The total flow from source must be equal to the total flow into sink, so $25 = 18 + x$, hence $x = 7$
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)(i)	States 5	1.1b	B1	$y = 5$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)(ii)	States 4	1.1b	B1	$z = 4$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)	Identifies the cut through arcs *BT*, *CT* and *ET*	3.1b	M1	A cut through the arcs *BT*, *CT* and *ET* has a value of 27 $m^3\ s^{-1}$
	Completes reasoned argument using the maximum flow-minimum cut theorem to establish that the maximum flow through the network is at most 27 $m^3\ s^{-1}$ Condone missing units	2.1	R1	As the maximum flow through a network is less than or equal to the value of any cut in the network, by the maximum flow-minimum cut theorem, the maximum flow is at most 27 $m^3\ s^{-1}$
	Subtotal		**2**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Constructs an activity network with at least 9 activities and at least 4 correct connections	3.1b	M1	See below
	Activity network fully correct with all activities and correct connections Condone omission of arrows	1.1b	A1	
	Finds the correct earliest start time for each activity on the correct network	1.1b	A1	
	Finds the correct latest finish time for each activity on the network	1.1b	A1	
				
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Identifies the minimum completion time for their activity network and compares it to 20 days to make a comment about Robert's claim	3.2a	E1F	As the minimum completion time of the project is 20 days, Robert's claim appears to be valid. However, it may take longer than 20 days if Robert's team is not large enough to begin all activities at the earliest start times.
	Identifies a possible reason for Robert's claim not being valid, for example: • Depends if activities can be worked on simultaneously • Depends if there are enough builders to allow each activity to begin at its earliest start time • Depends on if there are any delays to the earliest start time for any of the critical activities *A*, *D*, *E*, *F*, *I* or *J* OR Identifies a possible condition that needs to be met for Robert's claim to be valid, for example: • Activities will need to be worked on simultaneously • There must be enough builders to allow each activity to begin at its earliest start time • There must be no delay to beginning any of the critical activities	2.2b	E1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Finds at least 4 of the row minima and column maxima	3.1a	M1	Row minima $= -2$, -3, -1 or x Column maxima $= 4$, -1 or x, 1
	Obtains row minima of -2, -3, -1 or x or Obtains column maxima of 4, x or -1, 1	1.1b	A1	max(row minima) $= -1$ or x min(column maxima) $= -1$ or x For a stable solution, max(row minima) = min(column maxima)
	Obtains max(row minima) $= -1$ or x	2.2a	A1	x must equal -1 to ensure that max(row minima) $= -1$ min(column maxima) $= -1$
	Obtains min(column maxima) $= -1$ or x	1.1b	A1	Therefore a stable solution occurs only when $x = -1$
	States or uses max(row minima) = min(column maxima)	1.1b	B1	
	Completes fully reasoned argument to show that $x = -1$ is the **only** value that gives a stable solution	2.1	R1	
	Subtotal		**6**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	States -1	1.1b	B1	The value of the game for Bilal is -1
	Subtotal		**1**	

	Question total		**7**	

	Question Paper total		**40**	