AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Statistics

Friday 19 May 2023 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The continuous random variable X has variance 9

The discrete random variable Y has standard deviation 2 and is independent of X

Find Var$(X + Y)$

Circle your answer.

[1 mark]

5 11 (13) 85

$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) = 9 + 2^2 = 13$

2 The random variable T has a discrete uniform distribution and takes the values 1, 2, 3, 4 and 5

Find the variance of T

Circle your answer.

[1 mark]

$\frac{1}{5}$ $\frac{4}{3}$ (2) $\frac{13}{6}$

$\text{Var}(T) = \frac{n^2 - 1}{12} = \frac{5^2 - 1}{12} = 2$

Turn over for the next question

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3 The discrete random variable X has probability distribution

x	-4	3	8
$P(X = x)$	0.2	0.7	0.1

Show that $E(5X - 7) = 3.5$

[3 marks]

$E(X) = \Sigma\, x\, P(X = x) = -4 \times 0.2 + 3 \times 0.7 + 8 \times 0.1$

$= -0.8 + 2.1 + 0.8 = 2.1$

$E(5X - 7) = 5E(X) - 7$

$= 5 \times 2.1 - 7$

$= 10.5 - 7 = 3.5$ as required

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4 The proportion, p, of people in a particular town who use the local supermarket is unknown.

A random sample of 30 people in the town is taken and each person is asked if they use the local supermarket.

The manager of the supermarket claims that 35% of the people in the town use the local supermarket.

The random sample is used to conduct a hypothesis test at the 5% level of significance with the hypotheses

$$H_0: p = 0.35$$

$$H_1: p \neq 0.35$$

Show that the probability that a Type I error is made is 0.0356, correct to four decimal places.

[4 marks]

X = number who use the supermarket, $X \sim B(30, 0.35)$

Two-tailed test at 5%, so 0.025 in each tail

$P(X \leqslant 5) = 0.02326 < 0.025$, $P(X \leqslant 6) = 0.05857 > 0.025$

$P(X \geqslant 16) = 1 - P(X \leqslant 15) = 1 - 0.96992 = 0.03008 > 0.025$

$P(X \geqslant 17) = 1 - P(X \leqslant 16) = 1 - 0.98764 = 0.01236 < 0.025$

Critical region: $X \leqslant 5$ or $X \geqslant 17$

P(Type I error) $= P(X \leqslant 5) + P(X \geqslant 17)$

$= 0.02326 + 0.01236 = 0.03562$

$= 0.0356$ (4 d.p.) as required

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5 Rebekah is investigating the distances, X light years, between the Earth and visible stars in the night sky.

She determines the distance between the Earth and a star for a random sample of 100 visible stars.

The summarised results are as follows:

$$\sum x = 35\,522 \quad \text{and} \quad \sum x^2 = 32\,902\,257$$

5 (a) Calculate a 97% confidence interval for the population mean of X, giving your values to the nearest light year.

[5 marks]

$\bar{x} = \frac{35522}{100} = 355.22$

$s^2 = \frac{1}{99}\left(32\,902\,257 - \frac{35522^2}{100}\right)$

$= 204\,890.22\ldots \quad (s = 452.648\ldots)$

97% confidence: $P(Z < z) = 0.985 \Rightarrow z = 2.1701$

$CI = \bar{x} \pm z\frac{s}{\sqrt{n}} = 355.22 \pm 2.1701 \times \frac{\sqrt{204890.22}}{\sqrt{100}}$

$= 355.22 \pm 98.2287\ldots$

$= (256.991\ldots, 453.448\ldots)$

97% CI = (257, 453) light years

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5 (b) Mike claims that the population mean is 267 light years.

Rebekah says that the confidence interval supports Mike's claim.

State, with a reason, whether Rebekah is correct.

[1 mark]

Rebekah is correct, because 267 lies within the confidence interval (257, 453), so the interval supports Mike's claim.

Turn over for the next question

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6 An insurance company models the number of motor claims received in 1 day using a Poisson distribution with mean 65

6 (a) Find the probability that the company receives at most 60 motor claims in 1 day.

Give your answer to three decimal places.

[1 mark]

$X \sim Po(65)$

$P(X \leq 60) = 0.29327... = 0.293$

6 (b) The company receives motor claims using a telephone line which is open 24 hours a day.

Find the probability that the company receives exactly 2 motor claims in 1 hour.

Give your answer to three decimal places.

[2 marks]

Y = claims in 1 hour, $Y \sim Po\left(\frac{65}{24}\right) = Po(2.70833...)$

$P(Y = 2) = \frac{e^{-65/24} \times (65/24)^2}{2!}$

$= 0.24443... = 0.244$

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6 (c) The company models the number of property claims received in 1 day using a Poisson distribution with mean 23

Assume that the number of property claims received is independent of the number of motor claims received.

6 (c) (i) Find the standard deviation of the variable that represents the total number of motor claims and property claims received in 1 day.

Give your answer to three significant figures.

[2 marks]

T = total claims in 1 day, $T \sim Po(65 + 23) = Po(88)$

$Var(T) = 88$ (sum of independent Poisson variables)

Standard deviation $= \sqrt{88} = 9.3808... = 9.38$

6 (c) (ii) Find the probability that the company receives a total of more than 90 motor claims and property claims in 1 day.

Give your answer to three significant figures.

[2 marks]

$P(T > 90) = 1 - P(T \leq 90)$

$= 1 - 0.61139...$

$= 0.38860... = 0.389$

Turn over for the next question

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7 A theatre has morning, afternoon and evening shows.

On one particular day, the theatre asks all of its customers to state whether they enjoyed or did not enjoy the show.

The results are summarised in the table.

	Morning show	Afternoon show	Evening show	Total
Enjoyed	62	91	172	325
Not enjoyed	25	35	115	175
Total	87	126	287	500

The theatre claims that there is no association between the show that a customer attends and whether they enjoyed the show.

7 (a) Investigate the theatre's claim, using a 2.5% level of significance.

[8 marks]

H_0: no association between the show a customer attends and whether they enjoyed the show

H_1: there is an association between the show a customer attends and whether they enjoyed the show

Expected frequencies $E = \frac{\text{row total} \times \text{column total}}{500}$

E	Morning	Afternoon	Evening
Enjoyed	56.55	81.90	186.55
Not enjoyed	30.45	44.10	100.45

Contributions $(O - E)^2 / E$:

Enjoyed	0.5252	1.0111	1.1348
Not enjoyed	0.9755	1.8778	2.1075

$\chi^2 = \Sigma (O - E)^2/E = 0.5252 + 1.0111 + 1.1348 + 0.9755 + 1.8778 + 2.1075$

$= 7.6319... = 7.63$

Degrees of freedom $\nu = (2 - 1)(3 - 1) = 2$

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Critical value at 2.5% level: $\chi^2_2(0.025) = 7.378$

$7.63 > 7.378$, so reject H_0

There is sufficient evidence at the 2.5% level to suggest that there is an association between the show a customer attends and whether they enjoyed the show, so the theatre's claim is not supported.

7 (b) By considering observed and expected frequencies, interpret in context the association between the show that a customer attends and whether they enjoyed the show.

[2 marks]

Largest contribution: evening show / not enjoyed, $\frac{(O-E)^2}{E} = 2.11$

Observed 115 > expected 100.45 (and enjoyed 172 < 186.55), so people who attend the evening show did not enjoy the show more often than expected.

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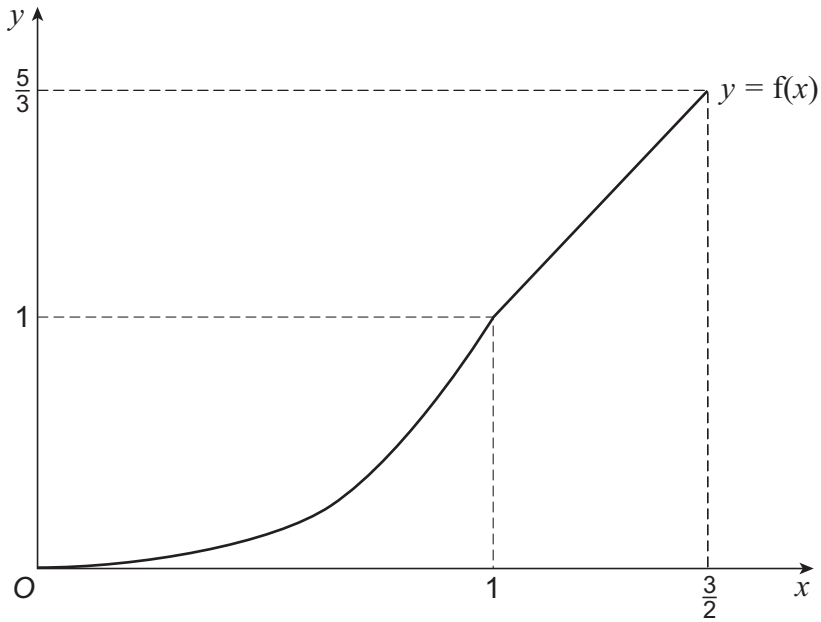
8 The continuous random variable X has probability density function $\mathrm{f}(x)$

It is given that $\mathrm{f}(x) = x^2$ for $0 \leq x \leq 1$

It is also given that $\mathrm{f}(x)$ is a linear function for $1 < x \leq \frac{3}{2}$

For all other values of x, $\mathrm{f}(x) = 0$

A sketch of the graph of $y = \mathrm{f}(x)$ is shown below.



Show that $\mathrm{Var}(X) = 0.0864$ correct to three significant figures.

[8 marks]

Straight-line part passes through (1, 1) and (3/2, 5/3):

$m = \frac{5/3 - 1}{3/2 - 1} = \frac{2/3}{1/2} = \frac{4}{3}$

$f(x) - 1 = \frac{4}{3}(x - 1) \Rightarrow f(x) = \frac{4}{3}x - \frac{1}{3}$ for $1 < x \leq \frac{3}{2}$

$E(X) = \int_0^1 x^3\,dx + \int_1^{3/2} x\left(\frac{4}{3}x - \frac{1}{3}\right)dx$

$= \left[\frac{x^4}{4}\right]_0^1 + \left[\frac{4x^3}{9} - \frac{x^2}{6}\right]_1^{3/2}$

$= \frac{1}{4} + \left(\frac{3}{2} - \frac{3}{8}\right) - \left(\frac{4}{9} - \frac{1}{6}\right)$

$= \frac{1}{4} + \frac{61}{72} = \frac{79}{72}$ $(= 1.0972\ldots)$

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$E(X^2) = \int_0^1 x^4\,dx + \int_1^{3/2} x^2\left(\frac{4}{3}x - \frac{1}{3}\right)dx$

$= \left[\frac{x^5}{5}\right]_0^1 + \left[\frac{x^4}{3} - \frac{x^3}{9}\right]_1^{3/2}$

$= \frac{1}{5} + \left(\frac{27}{16} - \frac{3}{8}\right) - \left(\frac{1}{3} - \frac{1}{9}\right)$

$= \frac{1}{5} + \frac{157}{144} = \frac{929}{720} \quad (= 1.2902\ldots)$

$Var(X) = E(X^2) - (E(X))^2$

$= \frac{929}{720} - \left(\frac{79}{72}\right)^2$

$= 1.290277\ldots - 1.203896\ldots$

$= 0.086381\ldots = 0.0864$ (3 s.f.) as required

END OF QUESTIONS

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