AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Mechanics

Friday 19 May 2023 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 A particle moves along the x-axis under the action of a force, F newtons, where

$$F = 3x^2 + 5$$

Find the work done by the force as the particle moves from $x = 0$ metres to $x = 2$ metres.

Circle your answer.

[1 mark]

12 J 17 J (18 J) 34 J

$W = \int_0^2 (3x^2 + 5)\,dx = [x^3 + 5x]_0^2 = 8 + 10 = 18$ J

2 Two particles are moving directly towards each other when they collide.

Given that the collision is perfectly elastic, state the value of the coefficient of restitution.

Circle your answer.

[1 mark]

$e = -1$ $e = 0$ $e = \frac{1}{2}$ ($e = 1$)

3 A stone of mass 0.2 kg is thrown vertically upwards with a speed of 10 m s^{-1}

Find the initial kinetic energy of the stone.

Circle your answer.

[1 mark]

1 J 5 J (10 J) 20 J

$KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.2)(10)^2 = 10$ J

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4 Reena is skating on an ice rink, which has a horizontal surface.

She follows a circular path of radius 5 metres and centre O

She completes 10 full revolutions in 1 minute, moving with a constant angular speed of ω radians per second.

The mass of Reena is 40 kg

4 (a) Find the value of ω

[1 mark]

$\omega = (10 \times 2\pi) \div 60 = \pi/3$ rad s^{-1} $(= 1.047...)$

4 (b) (i) Find the magnitude of the horizontal resultant force acting on Reena.

[2 marks]

$F = mr\omega^2$

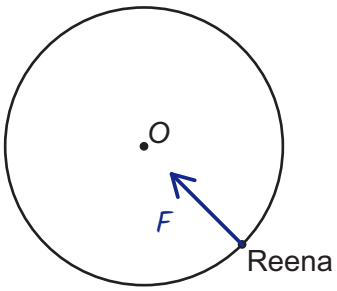
$F = 40 \times 5 \times (\pi/3)^2$

$= 219.32...$

Magnitude of resultant force = 219 N (3 s.f.)

4 (b) (ii) Show the direction of this horizontal resultant force on the diagram below.

[1 mark]



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5 An impulse of $\begin{bmatrix} -5 \\ 12 \end{bmatrix}$ N s is applied to a particle of mass 5 kg which is moving with velocity $\begin{bmatrix} 6 \\ 2 \end{bmatrix}$ m s^{-1}

5 (a) Calculate the magnitude of the impulse.

[1 mark]

$|I| = \sqrt{((-5)^2 + 12^2)} = \sqrt{169} = 13$ N s

5 (b) Find the **speed** of the particle immediately after the impulse is applied.

[3 marks]

$I = mv - mu$

$\begin{bmatrix} -5 \\ 12 \end{bmatrix} = 5v - 5\begin{bmatrix} 6 \\ 2 \end{bmatrix} \Rightarrow 5v = \begin{bmatrix} 25 \\ 22 \end{bmatrix} \Rightarrow v = \begin{bmatrix} 5 \\ 4.4 \end{bmatrix}$ m s^{-1}

Speed $= \sqrt{(5^2 + 4.4^2)} = \sqrt{44.36}$

$= 6.6603...$

Speed $= 6.66$ m s^{-1} (3 s.f.)

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6 A ball is thrown with speed u at an angle of 45° to the horizontal from a point O

When the horizontal displacement of the ball is x, the vertical displacement of the ball above O is y where

$$y = x - \frac{kx^2}{u^2}$$

6 (a) Use dimensional analysis to find the dimensions of k

[3 marks]

$[x] = L, \quad [y] = L, \quad [u] = LT^{-1}$

For the equation to be dimensionally consistent,

$[y] = [x] = [k][x^2] \div [u^2]$

$L = [k]\, L^2 \div (LT^{-1})^2$

$L = [k]\, L^2 \div (L^2T^{-2})$

$L = [k]\, T^2$

$\therefore \quad [k] = LT^{-2}$

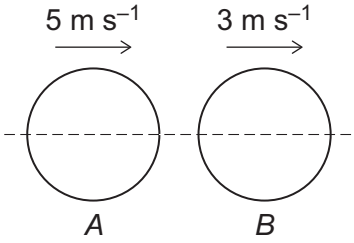
6 (b) State what can be deduced about k from the dimensions that you found in part **(a)**.

[1 mark]

LT^{-2} are the dimensions of acceleration, so k represents an acceleration (in fact k = g, acceleration due to gravity).

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7 Two smooth, equally sized spheres, A and B, are moving in the same direction along a straight line on a smooth horizontal surface, as shown in the diagram below.



The spheres subsequently collide.

Immediately after the collision, A has speed 2.5 m s^{-1} and B has speed 3.5 m s^{-1}

The coefficient of restitution between the spheres is e

7 (a) (i) Show that A does not change its direction of motion as a result of the collision.

[2 marks]

Suppose A reversed: velocities after are $v_A = -2.5$, $v_B = 3.5$

$e = (3.5 - (-2.5)) \div (5 - 3) = 6 \div 2 = 3$

This is impossible since $0 \leq e \leq 1$

$\therefore$ A cannot reverse, so A does not change its direction of motion.

7 (a) (ii) Find the value of e

[1 mark]

$e = (3.5 - 2.5) \div (5 - 3) = 1 \div 2 \Rightarrow e = \frac{1}{2}$

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7 (b) Given that the mass of *B* is 0.6 kg, find the mass of *A*

[3 marks]

Conservation of momentum, mass of A = m kg:

$5m + 0.6(3) = 2.5m + 0.6(3.5)$

$5m + 1.8 = 2.5m + 2.1$

$2.5m = 0.3$

$m = 0.12$

Mass of A = 0.12 kg

Turn over for the next question

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8 **In this question use $g = 9.8$ m s^{-2}**

Omar, a bungee jumper of mass 70 kg, has his ankles attached to one end of an elastic cord.

The other end of the cord is attached to a bridge which is 80 metres above the surface of a river.

Omar steps off the bridge at the point where the cord is attached and falls vertically downwards.

The cord can be modelled as a light elastic string of natural length L metres and modulus of elasticity 2800 N

Model Omar as a particle.

8 (a) Given that Omar just reaches the surface of the river before being pulled back up, find the value of L

Fully justify your answer.

[5 marks]

At the river Omar is momentarily at rest, so all GPE lost is stored as EPE in the cord (KE = 0 at top and bottom).

GPE lost = $mgh = 70 \times 9.8 \times 80 = 54880$ J

Extension of cord at the river: $x = 80 - L$

EPE = $\lambda x^2 \div 2l = 2800(80 - L)^2 \div 2L$

Conservation of energy: $2800(80 - L)^2 \div 2L = 54880$

$1400(80 - L)^2 = 54880L \Rightarrow (80 - L)^2 = 39.2L$

$L^2 - 160L + 6400 = 39.2L$

$L^2 - 199.2L + 6400 = 0$

$L = (199.2 \pm \sqrt{(199.2^2 - 4 \times 6400)}) \div 2 = (199.2 \pm \sqrt{14080.64}) \div 2$

$L = 40.268...$ or $L = 158.93...$

L must be less than 80 (the cord must stretch to reach the river), so reject $L = 158.9$

$\therefore L = 40.3$ (3 s.f.) ≈ 40

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8 (b) If Omar is not modelled as a particle, explain the effect of revising this assumption on your answer to part **(a)**.

[2 marks]

If Omar is not modelled as a particle, his height must be considered: his head (and centre of mass) is below his ankles, so his body reaches the water before the cord has stretched by $80 - L$.

So the length found in part (a) would be too long and Omar would end up in the water. Hence $L < 40$

Turn over for the next question

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9 Christina is investigating two possible models for the resistance force, R newtons, acting on a car when it is moving in a straight line on a horizontal road.

She uses the following information in both models:

- Mass of the car is 1000 kg
- Maximum power of the engine is 51 kW
- With maximum power, the car accelerates at 4.9 m s^{-2} when travelling at 10 m s^{-1}

9 (a) For Model 1, Christina assumes that R is constant.

9 (a) (i) Show that $R = 200$

[3 marks]

$P = Fv \Rightarrow$ driving force $F = 51000 \div 10 = 5100$ N

Newton's second law along the road:

$F - R = ma$

$5100 - R = 1000 \times 4.9$

$R = 5100 - 4900$

$R = 200$ as required

9 (a) (ii) Hence, calculate the maximum possible speed of the car using Model 1.

[2 marks]

At maximum speed, $a = 0$ so driving force = resistance

$51000 \div v = 200$

$v = 51000 \div 200$

Maximum speed = 255 m s^{-1}

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9 (b) For Model 2, Christina assumes that $R = kv$, where k is a constant and v m s^{-1} is the speed of the car.

9 (b) (i) Find the value of k

[2 marks]

When v = 10, R = 200 (resistance found from the given data)

200 = k × 10

k = 20

9 (b) (ii) Hence, calculate the maximum possible speed of the car using Model 2.

[2 marks]

At maximum speed, driving force = resistance

51000 ÷ v = 20v ⇒ v^2 = 2550

v = √2550 = 50.497...

Maximum speed = 50.5 m s^{-1} (3 s.f.)

9 (c) Evaluate each model, giving reasons for your answers.

[2 marks]

Model 1 Not realistic: 255 m s^{-1} is far too high, and resistance is not constant in reality (it increases with speed).

Model 2 More realistic: 50.5 m s^{-1} is a plausible top speed for a car, and it allows for resistance that varies with speed.

9 (d) Suggest a further model for R that Christina could consider.

[1 mark]

Model the resistance as $R = kv^2$, i.e. proportional to the square of the speed.

END OF QUESTIONS

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