AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Discrete

Friday 19 May 2023 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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4	
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6	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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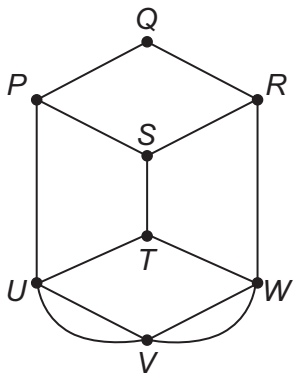
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ANSWER IN THE SPACES PROVIDED

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Answer **all** questions in the spaces provided.

1 The graph *G* has 8 vertices and 13 edges as shown in the diagram below.

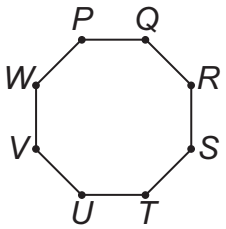


Graph *H* is a simple-connected subgraph of graph *G*

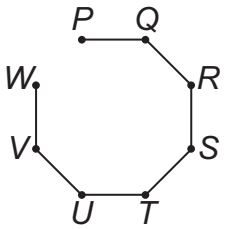
Which of the following diagrams could represent graph *H*?

Tick (✓) **one** box.

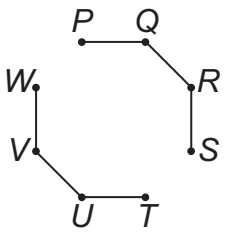
[1 mark]



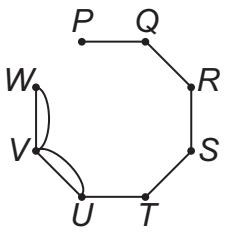
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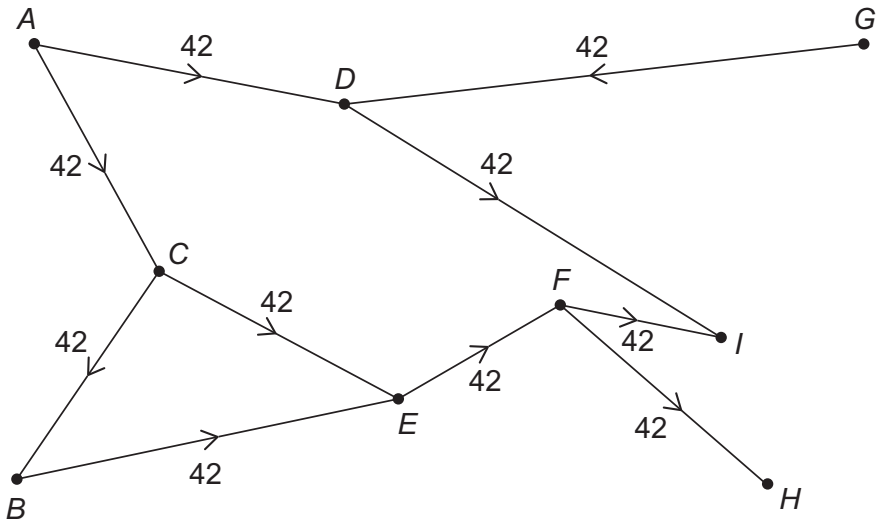
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2 The diagram below shows a network of pipes with their capacities.



A supersource is added to the network.

Which nodes are connected to the supersource?

Tick (✓) **one** box.

[1 mark]

A and *B* ☐

A and *G* ☐

G and *H* ☐

H and *I* ☐

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3 Ben is packing eggs into boxes, labelled Town Box or Country Box.

Each Town Box must contain 10 chicken eggs and 2 duck eggs.
Each Country Box must contain 4 chicken eggs and 8 duck eggs.

Ben has 253 chicken eggs and 151 duck eggs.

Ben wants to pack as many boxes as possible.

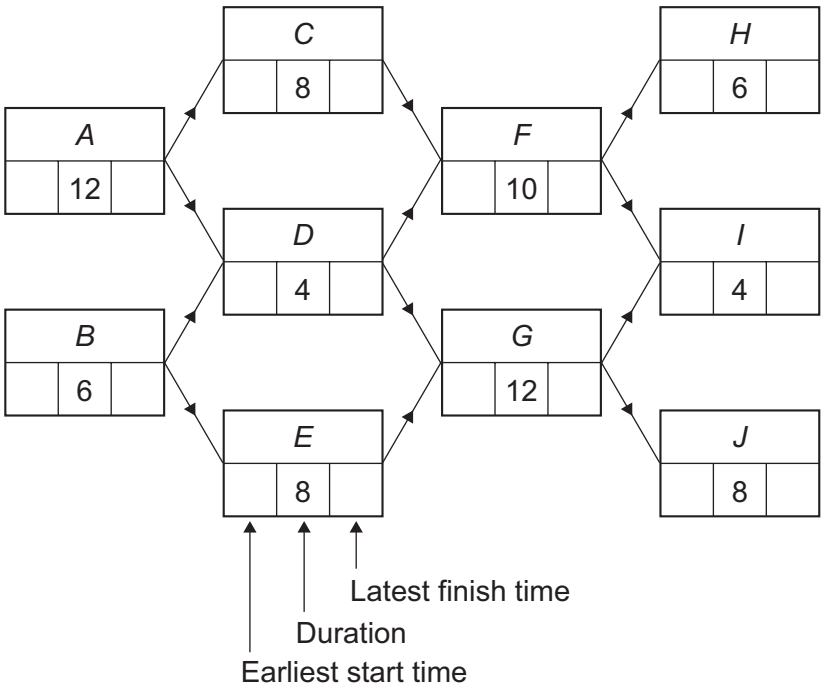
Formulate Ben's situation as a linear programming problem, defining any variables you introduce.

[4 marks]

Turn over for the next question

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4 A community project consists of 10 activities *A*, *B*, …, *J*, as shown in the activity network below.



The duration of each activity is shown in days.

4 (a) (i) Complete the activity network in the diagram above, showing the earliest start time and latest finish time for each activity.

[3 marks]

4 (a) (ii) State the minimum completion time for the community project.

[1 mark]

4 (b) Write down the critical activities of the network.

[1 mark]

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4 (c) Glyn claims that a project's activity network can be used to determine its minimum completion time by adding together the durations of all the project's critical activities.

4 (c) (i) Show that Glyn's claim is false for this community project's activity network.

[1 mark]

4 (c) (ii) Describe a situation in which Glyn's claim would be true.

[1 mark]

Turn over for the next question

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5 (a) The set S is defined as $S = \{0, 1, 2, 3, 4, 5\}$

5 (a) (i) State the identity element of S under the operation multiplication modulo 6

[1 mark]

5 (a) (ii) An element g of a set is said to be self-inverse under a binary operation $*$ if

$$g * g = e$$

where e is the identity element of the set.

Find all the self-inverse elements in S under the operation multiplication modulo 6

[2 marks]

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5 (b) The set T is defined as

$$T = \{a, b, c\}$$

Figure 1 shows a partially completed Cayley table for T under the commutative binary operation ♦

Figure 1

♦	a	b	c
a	a	c	b
b		b	a
c			c

5 (b) (i) Complete the Cayley table in **Figure 1**

[1 mark]

5 (b) (ii) Prove that ♦ is **not** associative when acting on the elements of T

[3 marks]

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6 Xander and Yvonne are playing a zero-sum game.

The game is represented by the pay-off matrix for Xander.

		Yvonne		
	Strategy	Y_1	Y_2	Y_3
Xander	X_1	−4	1	−3
	X_2	4	−3	−3
	X_3	−1	1	−2

6 (a) Show that the game has a stable solution.

[3 marks]

6 (b) State the play-safe strategy for each player.

[1 mark]

Play-safe strategy for Xander is _____

Play-safe strategy for Yvonne is _____

Do not write outside the box

6 (c) The game that Xander and Yvonne are playing is part of a marbles challenge.

The pay-off matrix values represent the number of marbles gained by Xander in each game.

In the challenge, the game is repeated until one player has 24 marbles more than the other player.

Explain why Xander and Yvonne must play at least 3 games to complete the challenge.

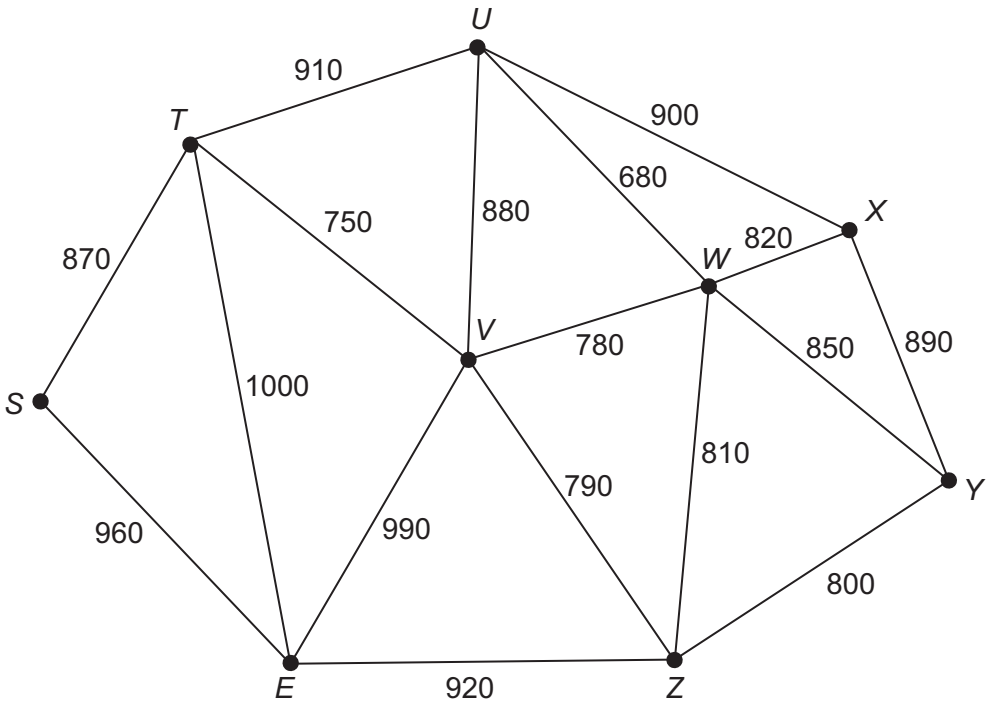
[2 marks]

Turn over for the next question

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7 A construction company has built eight wind turbines on a moorland site.

The network below shows nodes which represent the site entrance, E, and the wind turbine positions, S, T, …, Z



Each arc represents an access track with its length given in metres.
These 17 tracks were created in order to build the wind turbines.

Eight of the tracks are to be retained so that each turbine can be accessed for maintenance, directly or indirectly, from the site entrance.

The other nine tracks will be removed.

7 (a) (i) To save money the construction company wants to **maximise** the total length of the eight tracks to be retained.

Determine which tracks the construction company should retain.

[2 marks]

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7 (a) (ii) Find the total length of the eight tracks that are to be retained.

[2 marks]

7 (b) The total length of the 17 tracks is 14.6 km

The cost of removing all 17 tracks would be £87,600

Using your answer to part **(a)(ii)**, calculate an estimate for the cost of removing the nine tracks that will **not** be retained.

[2 marks]

7 (c) Comment on why the modelling used in part **(b)** may **not** give an accurate estimate for the cost of removing the nine tracks.

[1 mark]

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8 (a) The graph *G* has 2 vertices.

The sum of the degrees of all the vertices of *G* is 6

Draw *G*

[1 mark]

8 (b) The planar graph *P* is Eulerian, with at least one vertex of degree x, where x is a positive integer.

Some of the properties of *P* are shown in the table below.

Number of vertices	Number of edges	Number of faces
$18(x-1)$	$x(5x+1)$	$4x(x-2)$

Find the sum of the degrees of all the vertices of *P*

Fully justify your answer.

[6 marks]

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