AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Statistics

Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The discrete random variable X has the following probability distribution

x	-15	18	29
$P(X = x)$	0.2	0.7	0.1

Find $P(X > 18)$

Circle your answer.

[1 mark]

(0.1) 0.2 0.7 0.8

$P(X > 18) = P(X = 29) = 0.1$

2 The continuous random variable Y has probability density function $f(y)$ where

$$\int_{-\infty}^{\infty} yf(y)\,dy = 16 \quad \text{and} \quad \int_{-\infty}^{\infty} y^2 f(y)\,dy = 1040$$

Find the standard deviation of Y

Circle your answer.

[1 mark]

(28) 32 784 1024

$Var(Y) = 1040 - 16^2 = 784, \quad SD = \sqrt{784} = 28$

Turn over for the next question

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3 The discrete random variable A has the following probability distribution function

$$\mathrm{P}(A = a) = \begin{cases} 0.45 & a = 0 \\ 0.25 & a = 1 \\ 0.3 & a = 2 \\ 0 & \text{otherwise} \end{cases}$$

3 (a) Find the median of A

[1 mark]

$P(A \leq 0) = 0.45 < 0.5, \quad P(A \leq 1) = 0.7 > 0.5$

Median = 1

3 (b) Find the standard deviation of A, giving your answer to three significant figures.

[4 marks]

$E(A) = 0 \times 0.45 + 1 \times 0.25 + 2 \times 0.3$

$= 0.85$

$E(A^2) = 0^2 \times 0.45 + 1^2 \times 0.25 + 2^2 \times 0.3$

$= 1.45$

$Var(A) = E(A^2) - [E(A)]^2 = 1.45 - 0.85^2$

$= 0.7275$

Standard deviation $= \sqrt{0.7275} = 0.852936...$

Standard deviation = 0.853 (3 s.f.)

3 (c) Find $\mathrm{Var}(9A - 2)$

[2 marks]

$Var(9A - 2) = 9^2 \, Var(A)$

$= 81 \times 0.7275$

$= 58.9275$

$= 58.9$ (3 s.f.)

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4 The height of lilac trees, in metres, can be modelled by a normal distribution with variance 0.7

A random sample of n lilac trees is taken and used to construct a 99% confidence interval for the population mean.

This confidence interval is (5.239, 5.429)

4 (a) Find the value of n

[3 marks]

99% confidence $\Rightarrow$ $z = 2.5758$

$\frac{5.429 - 5.239}{2} = 0.095 = 2.5758 \times \frac{\sqrt{0.7}}{\sqrt{n}}$

$\sqrt{n} = \frac{2.5758 \times \sqrt{0.7}}{0.095} = 22.685...$

$n = 22.685...^2 = 514.6...$

$n = 515$

4 (b) Joey claims that the mean height of lilac trees is 5.3 metres.

State, with a reason, whether the confidence interval supports Joey's claim.

[1 mark]

The confidence interval supports Joey's claim, because 5.3 lies within the interval (5.239, 5.429).

Turn over for the next question

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5 The continuous random variable X has probability density function

$$f(x) = \begin{cases} x^3 & 0 < x \leq 1 \\ \frac{9}{1696}x^3(x^2+1) & 1 < x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

5 (a) Find $P(X < 1.8)$, giving your answer to three decimal places.

[3 marks]

$P(X < 1.8) = \int_0^1 x^3\,dx + \frac{9}{1696}\int_1^{1.8} x^3(x^2+1)\,dx$

$= \left[\frac{x^4}{4}\right]_0^1 + \frac{9}{1696}\left[\frac{x^6}{6} + \frac{x^4}{4}\right]_1^{1.8}$

$= \frac{1}{4} + \frac{9}{1696}(8.293104 - \frac{5}{12})$

$= 0.25 + 0.041797...$

$= 0.291797...$

$P(X < 1.8) = 0.292$ (3 d.p.)

5 (b) Find the lower quartile of X

[1 mark]

$P(X \leq 1) = \int_0^1 x^3\,dx = \frac{1}{4} = 0.25$

Lower quartile = 1

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5 (c) Show that $\mathrm{E}\left(\frac{1}{X^2}\right) = \frac{133}{212}$

[4 marks]

$E(\frac{1}{X^2}) = \int_0^1 \frac{x^3}{x^2}\,dx + \int_1^3 \frac{9x^3(x^2+1)}{1696x^2}\,dx$

$= \int_0^1 x\,dx + \frac{9}{1696}\int_1^3 (x^3 + x)\,dx$

$= [\frac{x^2}{2}]_0^1 + \frac{9}{1696}[\frac{x^4}{4} + \frac{x^2}{2}]_1^3$

$= \frac{1}{2} + \frac{9}{1696}[(\frac{81}{4} + \frac{9}{2}) - (\frac{1}{4} + \frac{1}{2})]$

$= \frac{1}{2} + \frac{9}{1696} \times 24 = \frac{1}{2} + \frac{27}{212}$

$= \frac{106}{212} + \frac{27}{212} = \frac{133}{212}$ as required

5 (d) The random variable Y has a discrete uniform distribution, taking the values 1, 2, 3, 4, 5

Find the exact value of $\mathrm{E}\left(\frac{1}{X^2} + Y\right)$

[3 marks]

$E(Y) = \frac{1+2+3+4+5}{5} = 3$

$E(\frac{1}{X^2} + Y) = E(\frac{1}{X^2}) + E(Y)$

$= \frac{133}{212} + 3$

$= \frac{769}{212}$

Turn over ▶

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6 The number of computers sold per day by a shop can be modelled by the random variable Y where $Y \sim \text{Po}(42)$

6 (a) State the variance of Y

[1 mark]

For a Poisson distribution, variance = mean

Var(Y) = 42

6 (b) One month ago, the shop started selling a new model of computer.

On a randomly chosen day in the last month, the shop sold 53 computers.

Carry out a hypothesis test, at the 5% level of significance, to investigate whether the mean number of computers sold per day has increased in the last month.

[6 marks]

Let λ = mean number of computers sold per day

H_0: $\lambda = 42$ H_1: $\lambda > 42$

Under H_0, $Y \sim Po(42)$

p-value = $P(Y \geq 53) = 1 - P(Y \leq 52)$

$= 1 - 0.943366... = 0.056634... = 0.0566$

0.0566 > 0.05, so the result is not significant at the 5% level

Do not reject H_0

There is insufficient evidence at the 5% level to suggest that the mean number of computers sold per day has increased in the last month.

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6 (c) Describe, in the context of the hypothesis test in part **(b)**, what is meant by a Type II error.

[1 mark]

Concluding that the mean number of computers sold per day has not increased, when in fact it has increased.

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7 Wade and Odelia are investigating whether there is an association between the region where a person lives and the brand of washing powder they use.

They decide to conduct a χ^2-test for association and survey a random sample of 200 people.

The expected frequencies for the test have been calculated and are shown in the contingency table below.

		Brand of washing powder		
		A	**B**	**C**
Region	**North**	14.14	9.66	4.2
	South	20.705	14.145	6.15
	East	32.32	22.08	9.6
	West	33.835	23.115	10.05

Wade conducts a hypothesis test where the number of degrees of freedom for the test is 4

Odelia states that the number of degrees of freedom for the test should be 3

7 (a) Explain why **both** Wade's value and Odelia's value for the number of degrees of freedom for the test could be valid.

[3 marks]

The expected frequency for North, brand C is 4.2, which is less than 5, so cells must be combined before carrying out the test.

Wade: merge the North row with another region (e.g. North and South), giving a 3 × 3 table, so $\nu = (3 - 1)(3 - 1) = 4$

Odelia: merge the brand C column with another brand (e.g. B and C), giving a 4 × 2 table, so $\nu = (4 - 1)(2 - 1) = 3$

Both methods remove the expected frequency below 5, so both are valid.

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7 (b) The test statistic for Wade's test is 13.6

Investigate whether there is an association between region and brand of washing powder, using the 1% level of significance.

[5 marks]

H_0: there is no association between region and brand of washing powder

H_1: there is an association between region and brand of washing powder

$\nu = 4$, 1% critical value of $\chi^2(4) = 13.277$

Test statistic 13.6 > 13.277, so the result lies in the critical region (significant at the 1% level)

Reject H_0

There is sufficient evidence at the 1% level to suggest that there is an association between region and brand of washing powder used.

END OF QUESTIONS

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