AQA

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Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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AS
FURTHER MATHEMATICS

Paper 2 Mechanics

Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 A box is being pushed in a straight line along horizontal ground by a force.

The force is applied in the direction of motion and has magnitude 10 newtons.

The box moves 5 metres in 2 seconds.

Calculate the work done by the force.

Circle your answer.

[1 mark]

20 J 25 J (50 J) 100 J

Work done = $Fd = 10 \times 5 = 50$ J

2 Two particles of equal mass are moving on a horizontal surface when they collide.

Immediately before the collision, their velocities are $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ m s^{-1} and $\begin{bmatrix} 6 \\ -2 \end{bmatrix}$ m s^{-1}

As a result of the collision the particles coalesce to become a single particle.

Find the velocity of the single particle, immediately after the collision.

Circle your answer.

[1 mark]

$\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ m s^{-1} $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ m s^{-1} $\begin{bmatrix} 8 \\ 2 \end{bmatrix}$ m s^{-1} $\begin{bmatrix} 8 \\ 6 \end{bmatrix}$ m s^{-1}

$v = \frac{1}{2}[(2, 4) + (6, -2)] = \frac{1}{2}(8, 2) = (4, 1)$ m s^{-1}

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3 **In this question use $g = 9.8\,\text{m s}^{-2}$**

A ball of mass of 0.75 kg is thrown vertically upwards with an initial speed of $12\,\text{m s}^{-1}$

The ball is thrown from ground level.

3 (a) Calculate the initial kinetic energy of the ball.

[1 mark]

$KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.75)(12)^2 = 54$ J

3 (b) The maximum height of the ball above the ground is h metres.

Jeff and Gurjas use an energy method to find h

Jeff concludes that $h = 7.3$

Gurjas concludes that $h < 7.3$

Explain the reasoning that they have used, showing any calculations that you make.

[3 marks]

Energy method: initial KE = gain in GPE at the top

$mgh = 54 \Rightarrow 0.75(9.8)h = 54$

$h = 54 \div (0.75 \times 9.8) = 7.3469... \approx 7.3$

Jeff has assumed there is no air resistance, so all the kinetic energy is converted to potential energy and he obtains h = 7.3

Gurjas has taken air resistance into account: some energy is lost as work done against air resistance, so less than 54 J becomes PE and the ball does not reach 7.3 m, so h < 7.3

Turn over ▶

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4 Wavelength is defined as the distance from the highest point on one wave to the highest point on the next wave.

Surfers classify waves into one of several types related to their wavelengths.

Two of these classifications are deep water waves and shallow water waves.

4 (a) The wavelength w of a deep water wave is given by

$$w = \frac{gt^2}{k}$$

where g is the acceleration due to gravity and t is the time period between consecutive waves.

Given that the formula for a deep water wave is dimensionally consistent, show that k is a dimensionless constant.

[2 marks]

$[w] = L, \quad [g] = LT^{-2}, \quad [t] = T$

$[gt^2] = LT^{-2} \times T^2 = L$

$k = gt^2/w \quad \Rightarrow \quad [k] = [gt^2] \div [w] = L \div L = 1$

$\therefore$ k has no dimensions, so k is a dimensionless constant

4 (b) The wavelength w of a shallow water wave is given by

$$w = (gd)^{\alpha} t^{\beta}$$

where g is the acceleration due to gravity, d is the depth of water and t is the time period between consecutive waves.

Use dimensional analysis to find the values of α and β

[3 marks]

$[w] = L, \quad [gd] = LT^{-2} \times L = L^2T^{-2}, \quad [t] = T$

$[(gd)^{\alpha}t^{\beta}] = (L^2T^{-2})^{\alpha}T^{\beta} = L^{2\alpha}T^{-2\alpha+\beta}$

$L^{2\alpha}T^{-2\alpha+\beta} = L^1T^0$

L: $2\alpha = 1 \quad \Rightarrow \quad \alpha = \frac{1}{2}$

T: $-2\alpha + \beta = 0 \quad \Rightarrow \quad \beta = 2\alpha = 1$

$\therefore \quad \alpha = \frac{1}{2}, \quad \beta = 1$

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5 A car, of mass 1000 kg, is travelling on a straight horizontal road.

When the car travels at a speed of v m s^{-1}, it experiences a resistance force of magnitude $25v$ newtons.

The car has a maximum speed of 72 km h^{-1} on the straight road.

Find the maximum power output of the car.

Fully justify your answer.

[5 marks]

At maximum speed the acceleration is zero, so the driving force F equals the resistance force R.

$v = 72$ km h^{-1} $= 72 \times 1000 \div (60 \times 60) = 20$ m s^{-1}

$R = 25v = 25(20) = 500$ N

$F = R = 500$ N

$P = Fv = 500 \times 20 = 10\,000$ W

Maximum power output = 10 kW

Turn over for the next question

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6 An ice hockey puck, of mass 0.2 kg, is moving in a straight line on a horizontal ice rink under the action of a single force which acts in the direction of motion.

At time t seconds, the force has magnitude $(2t + 3)$ newtons.

The force acts on the puck from $t = 0$ to $t = T$

6 (a) Show that the magnitude of the impulse of the force is $aT^2 + bT$, where a and b are integers to be found.

[3 marks]

Impulse $= \int_0^T F\,dt = \int_0^T (2t + 3)\,dt$

$= [t^2 + 3t]_0^T$

$= (T^2 + 3T) - 0$

$= T^2 + 3T$

$\therefore\ a = 1,\ b = 3$

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6 (b) While the force acts on the puck, its speed increases from $1\,\text{m}\,\text{s}^{-1}$ to $4\,\text{m}\,\text{s}^{-1}$

Use your answer from part **(a)** to find T, giving your answer to three significant figures.

Fully justify your answer.

[4 marks]

Impulse = change in momentum

$I = mv - mu = 0.2(4) - 0.2(1) = 0.8 - 0.2 = 0.6$ N s

$T^2 + 3T = 0.6$

$T^2 + 3T - 0.6 = 0$

$T = [-3 \pm \sqrt{(3^2 + 4(0.6))}] \div 2 = (-3 \pm \sqrt{11.4}) \div 2$

$T = 0.18819...$ or $T = -3.18819...$

T must be positive (the force acts from t = 0 to t = T),

so reject $T = -3.19$

$\therefore$ $T = 0.188$ s (3 s.f.)

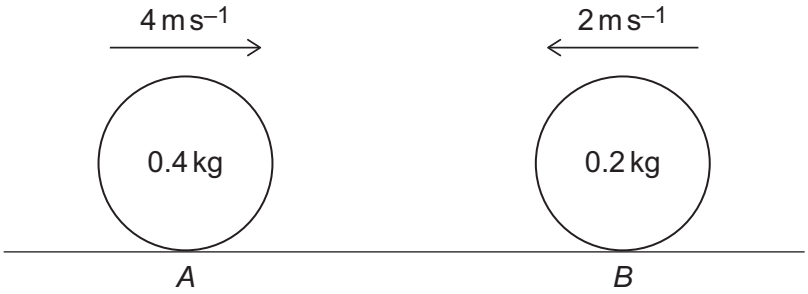
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7 The particles A and B are moving on a smooth horizontal surface directly towards each other.

Particle A has mass 0.4 kg and particle B has mass 0.2 kg

Particle A has speed $4\,\text{m s}^{-1}$ and particle B has speed $2\,\text{m s}^{-1}$ when they collide, as shown in the diagram below.



The coefficient of restitution between the particles is e

7 (a) Find the magnitude of the total momentum of the particles before the collision.

[2 marks]

Take the direction of A's motion as positive.

Momentum of A = 0.4 × 4 = 1.6 N s

Momentum of B = 0.2 × (−2) = −0.4 N s

Total momentum = 1.6 − 0.4 = 1.2 N s

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7 (b) (i) Show that the speed of B immediately after the collision is $(4e + 2)\,\text{m s}^{-1}$

[3 marks]

After: velocity of A = u, velocity of B = v (both positive to the right)

CoM: $1.2 = 0.4u + 0.2v \Rightarrow 12 = 4u + 2v$ (1)

NLR: $v - u = e(4 - (-2)) = 6e \Rightarrow u = v - 6e$ (2)

Sub (2) into (1): $12 = 4(v - 6e) + 2v$

$12 = 6v - 24e$

$2 = v - 4e$

$v = 4e + 2 > 0$, so speed of B = $(4e + 2)$ m s^{-1} as required

7 (b) (ii) Find an expression, in terms of e, for the speed of A immediately after the collision.

[2 marks]

$u = v - 6e = 4e + 2 - 6e$

$u = 2 - 2e$

Speed of A = $(2 - 2e)$ m s^{-1}

7 (c) Explain what happens to particle A when the collision is perfectly elastic.

[2 marks]

Perfectly elastic collision $\Rightarrow e = 1$

$u = 2 - 2(1) = 0$

So the velocity of A after the collision is zero: particle A comes to rest (it stops) immediately after the collision.

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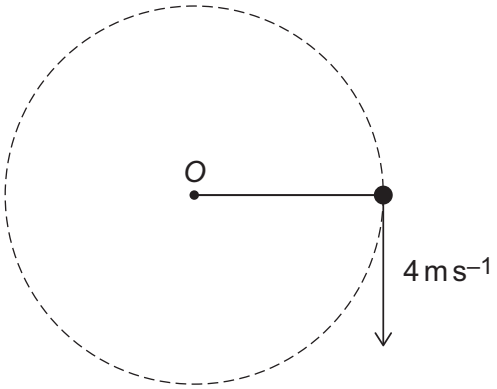
8 A particle, of mass 3 kg, is attached to one end of an elastic string.

The particle is placed on a smooth horizontal table.

The other end of the string is attached to a fixed point O on the horizontal table.

The elastic string has natural length 1 metre and modulus of elasticity 200 N

The particle is set in motion so that it moves in a horizontal circle, centre O, with a constant speed of $4\,\text{m}\,\text{s}^{-1}$, as shown in the diagram below.



Throughout the motion, the extension of the string is x metres and the tension is T newtons.

8 (a) Show that $T = 200x$

[1 mark]

Hooke's law: $T = \lambda x/l = 200x/1 = 200x$ as required

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8 (b) By considering the circular motion of the particle, show that

$$25x^2 + 25x - 6 = 0$$

[4 marks]

The string is stretched, so radius $r = l + x = 1 + x$

Tension provides the force towards O (centre of circle):

$T = mv^2/r$

$T = 3(4)^2/(1 + x) = 48/(1 + x)$

With $T = 200x$: $200x = 48/(1 + x)$

$200x(1 + x) = 48$

$200x^2 + 200x - 48 = 0$

$\div 8$: $25x^2 + 25x - 6 = 0$ as required

8 (c) Hence deduce the radius of the circle.

[2 marks]

$(5x - 1)(5x + 6) = 0 \Rightarrow x = 0.2$ or $x = -1.2$; $x > 0$ so $x = 0.2$

Radius $= 1 + x = 1 + 0.2 = 1.2$ m

8 (d) Describe one limitation of the model that you have used.

[1 mark]

The string is modelled as light, but in reality it has mass.

END OF QUESTIONS

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