AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Discrete

Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
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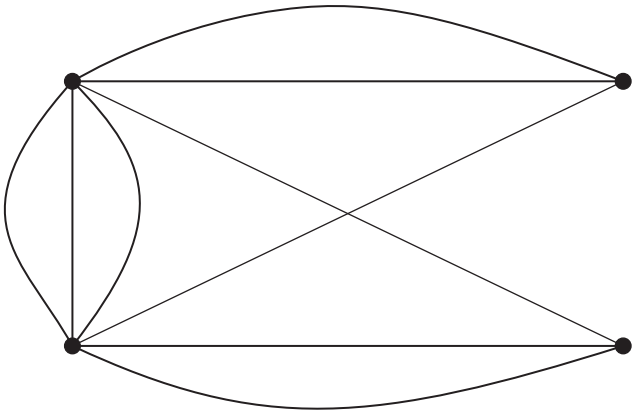
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The connected graph G is shown below.



The graphs A and B are subgraphs of G

Both A and B have four vertices.

1 (a) The graph A is a tree with x edges.

State the value of x

Circle your answer.

[1 mark]

(3) 4 5 9

1 (b) The graph B is simple-connected with y edges.

Find the maximum possible value of y

Circle your answer.

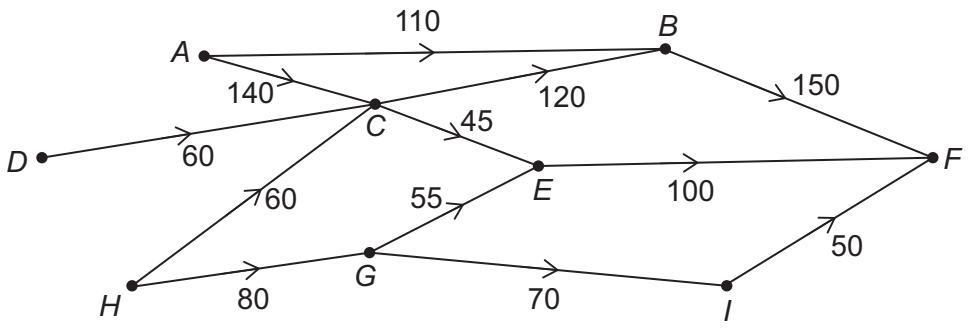
[1 mark]

3 4 (5) 9

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2 The diagram shows a network of pipes.

Each pipe is labelled with its upper capacity in $m^3\,s^{-1}$



2 (a) Find the value of the cut $\{A, C, D, G, H\}$ $\{B, E, F, I\}$

[1 mark]

Cut = 110 + 120 + 45 + 55 + 70 (AB, CB, CE, GE, GI)

= 400 $m^3\,s^{-1}$

2 (b) Write down a cut with a value of 300 $m^3\,s^{-1}$

[1 mark]

{A, B, C, D, E, G, H, I} {F}

(value = 150 + 100 + 50 = 300 $m^3\,s^{-1}$)

2 (c) Using the values from part **(a)** and part **(b)**, state what can be deduced about the maximum flow through the network.

Fully justify your answer.

[2 marks]

300 < 400, so the minimum cut is at most 300 $m^3\,s^{-1}$

By the maximum flow–minimum cut theorem,

maximum flow = value of the minimum cut

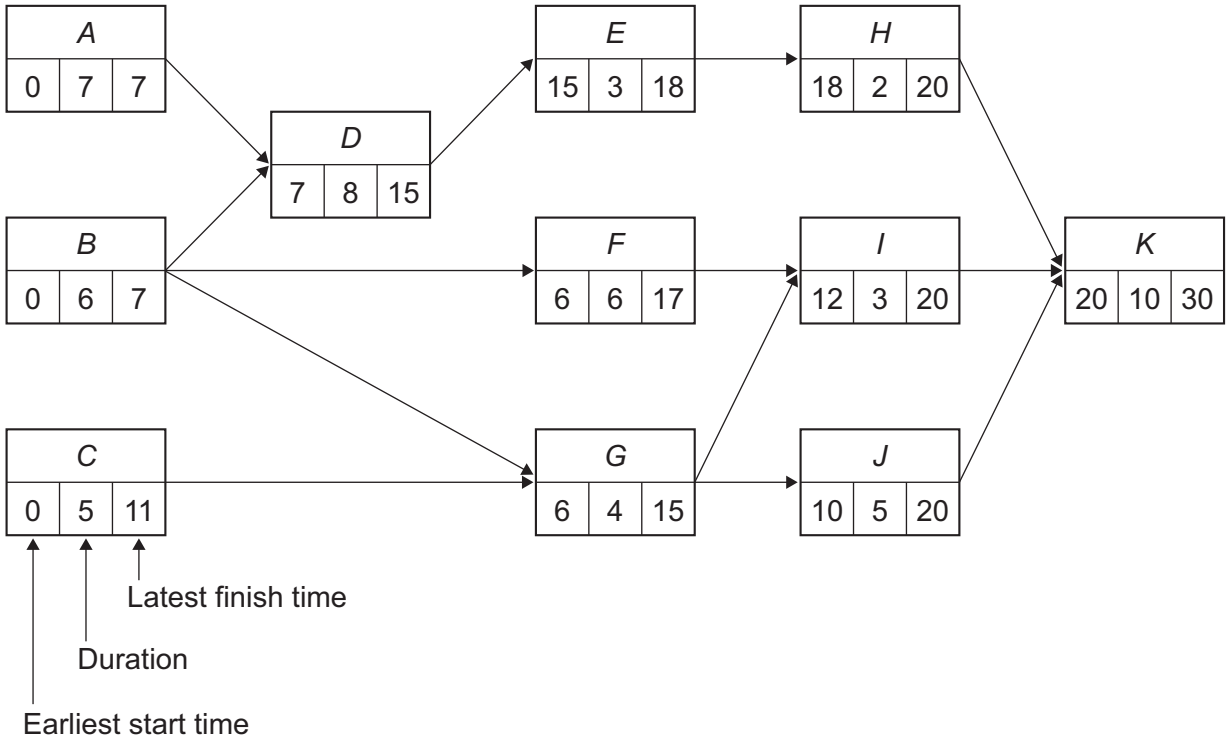
$\therefore$ maximum flow $\leq$ 300 $m^3\,s^{-1}$

(the maximum flow through the network cannot exceed 300)

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3 A project consists of 11 activities *A*, *B*, …, *K*

A completed activity network for the project is shown in the diagram below.



All times on the activity network are given in days.

3 (a) Write down the critical path.

[1 mark]

A D E H K

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3 (b) Due to an issue with the supply of materials, the duration of activity *G* is doubled.

Deduce the effect, if any, that this change will have on the earliest start time and latest finish time for each of the activities *I*, *J* and *K*

[3 marks]

G now takes 8 days, so G finishes at 6 + 8 = 14 (≤ 15, its LFT, so G and the critical path are not affected).

Earliest start of I = max(12, 14) = 14 (increases from 12 to 14)

Earliest start of J = 14 (increases from 10 to 14)

I: 14 + 3 = 17 ≤ 20 and J: 14 + 5 = 19 ≤ 20, so I and J remain non-critical: the latest finish times of I and J are unchanged (20).

The earliest start time (20) and latest finish time (30) of K are unchanged.

Turn over for the next question

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4 Alun, a baker, delivers bread to community shops located in Aber, Bangor, Conwy, and E'bach.

Alun starts and finishes his journey at the bakery, which is located in Deganwy.

The distances, in miles, between the five locations are given in the table below.

	Aber	**Bangor**	**Conwy**	**Deganwy**	**E'bach**
Aber	–	9.1	10.0	12.3	17.1
Bangor	9.1	–	15.5	17.8	22.7
Conwy	10.0	15.5	–	2.4	7.6
Deganwy	12.3	17.8	2.4	–	8.0
E'bach	17.1	22.7	7.6	8.0	–

The minimum total distance that Alun can travel in order to make all four deliveries, starting and finishing at the bakery in Deganwy is x miles.

4 (a) Using the nearest neighbour algorithm starting from Deganwy, find an upper bound for x

[2 marks]

Deganwy → Conwy (2.4) → E'bach (7.6) → Aber (17.1)

→ Bangor (9.1) → Deganwy (17.8)

2.4 + 7.6 + 17.1 + 9.1 + 17.8 = 54.0

Upper bound for x = 54.0 miles

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4 (b) By deleting **Aber**, find a lower bound for x

[3 marks]

MST of Bangor, Conwy, Deganwy, E'bach (Prim's from Conwy):

CD = 2.4, CE = 7.6, BC = 15.5

MST weight = 2.4 + 7.6 + 15.5 = 25.5

Two shortest arcs from Aber: AB = 9.1, AC = 10.0

9.1 + 10.0 = 19.1

Lower bound = 25.5 + 19.1 = 44.6

Lower bound for x = 44.6 miles

Turn over for the next question

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5 (a) A connected planar graph has 9 vertices, 20 edges and f faces.

Use Euler's formula for connected planar graphs to find f

[2 marks]

$v - e + f = 2$

$9 - 20 + f = 2$

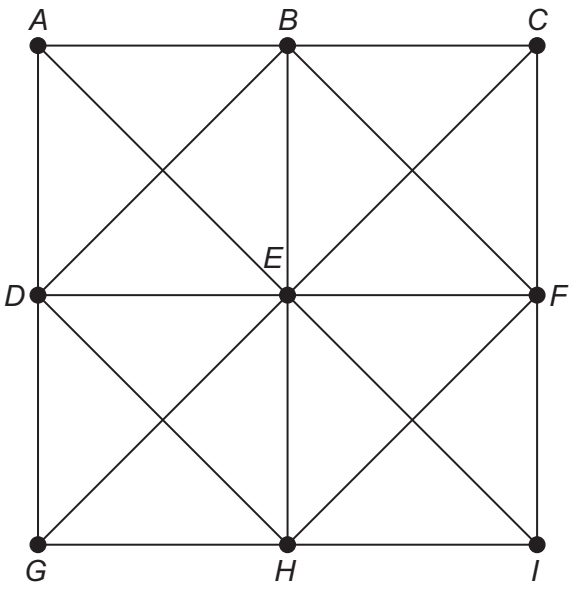
$f = 2 + 11$

$f = 13$

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5 (b) The graph J, shown in **Figure 1**, has 9 vertices and 20 edges.

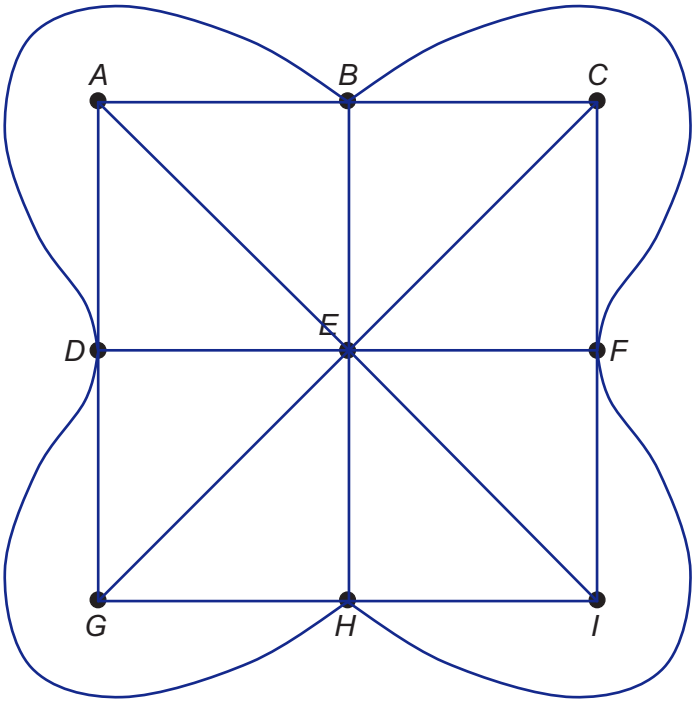
Figure 1



By redrawing the graph J using **Figure 2**, show that J is planar.

[1 mark]

Figure 2



Turn over ▶

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6 The set S is given by $S = \{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$ where

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

6 (a) Complete the Cayley table for S under matrix multiplication.

[2 marks]

	A	**B**	**C**	**D**
A	**A**	*A*	**D**	*D*
B	*A*	**B**	*C*	*D*
C	*D*	*C*	**C**	*D*
D	*D*	*D*	*D*	**D**

6 (b) Using the Cayley table above, explain why **B** is the identity element of S under matrix multiplication.

[1 mark]

The row for B and the column for B are the same as the column and row headings: BX = XB = X for every X in S, so B is the identity.

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6 (c) Sam states that the Cayley table in part **(a)** shows that matrix multiplication is commutative.

Comment on the validity of Sam's statement.

[2 marks]

The Cayley table is symmetrical about the leading diagonal, so matrix multiplication is commutative for the elements of S.

However, matrix multiplication is not commutative in general (in general $MN \neq NM$ for matrices M and N).

Therefore Sam's statement is not valid.

Turn over for the next question

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7 Kez and Lui play a zero-sum game. The game does **not** have a stable solution.

The game is represented by the following pay-off matrix for Kez.

		Lui		
	Strategy	$\mathbf{L_1}$	$\mathbf{L_2}$	$\mathbf{L_3}$
Kez	$\mathbf{K_1}$	4	1	−2
	$\mathbf{K_2}$	−4	−2	0
	$\mathbf{K_3}$	−2	−1	2

7 (a) State, with a reason, why Kez should never play strategy $\mathbf{K_2}$

[1 mark]

K_3 dominates K_2, since $-2 > -4$, $-1 > -2$ and $2 > 0$,

so Kez should never play strategy K_2

7 (b) Kez and Lui play the game 20 times.

Kez plays their optimal mixed strategy.

Find the expected number of times that Kez will play strategy $\mathbf{K_3}$

Fully justify your answer.

[6 marks]

Remove K_2. Let Kez play K_1 with probability p and K_3 with probability $1 - p$.

Expected gain for Kez if Lui plays:

L_1: $4p - 2(1 - p) = 6p - 2$

L_2: $p - (1 - p) = 2p - 1$

L_3: $-2p + 2(1 - p) = 2 - 4p$

Intersections: $6p - 2 = 2p - 1 \Rightarrow p = 0.25$

$2p - 1 = 2 - 4p \Rightarrow p = 0.5$

$6p - 2 = 2 - 4p \Rightarrow p = 0.4$

Kez maximises the minimum expected gain: the highest point of the lowest boundary (graph below) is where L_2 meets L_3

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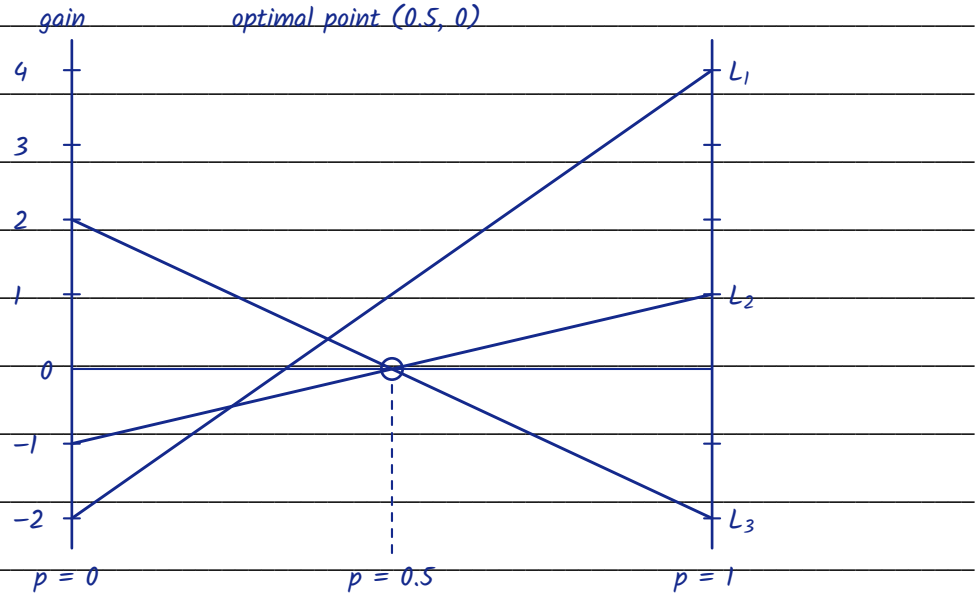
$2p - 1 = 2 - 4p$

$6p = 3 \Rightarrow p = 0.5$ (value of game = 0)

So $P(K_3) = 1 - p = 0.5$

Expected number of times K_3 is played $= 0.5 \times 20 = 10$

Kez is expected to play strategy K_3 10 times.



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8 Alli is planting garlic cloves and leek seedlings in a garden.

The planting density is the number of plants that are planted per m^2

The planting densities and costs are shown in the table below.

	Planting density	**Cost**
Garlic cloves	16 cloves per m^2	15 pence per clove
Leek seedlings	36 seedlings per m^2	10 pence per seedling

Alli has £15 to spend on garlic cloves and leek seedlings.

Alli needs to plant at least 50 leek seedlings.

Alli needs to plant at least the same number of garlic cloves as leek seedlings.

Alli plants x garlic cloves and y leek seedlings.

8 (a) Explain why the total area, in m^2, that Alli plants can be written as $\frac{1}{16}x + \frac{1}{36}y$

[1 mark]

16 cloves are planted per m^2, so each clove needs 1/16 m^2;

x garlic cloves need (1/16)x m^2

36 seedlings are planted per m^2, so each needs 1/36 m^2;

y leek seedlings need (1/36)y m^2

Total area = (1/16)x + (1/36)y m^2

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8 (b) (i) Alli wants to plant garlic cloves and leek seedlings over the largest possible total area.

Formulate a linear programming problem that models Alli's situation.

[3 marks]

Maximise $A = \frac{1}{16}x + \frac{1}{36}y$

subject to:

$15x + 10y \leq 1500$ (cost in pence, £15 = 1500p)

$y \geq 50$

$x \geq y$

$x \geq 0, y \geq 0$

x and y are integers

8 (b) (ii) State a limitation of the linear programming problem formed in part **(b)(i)** to model Alli's situation.

[1 mark]

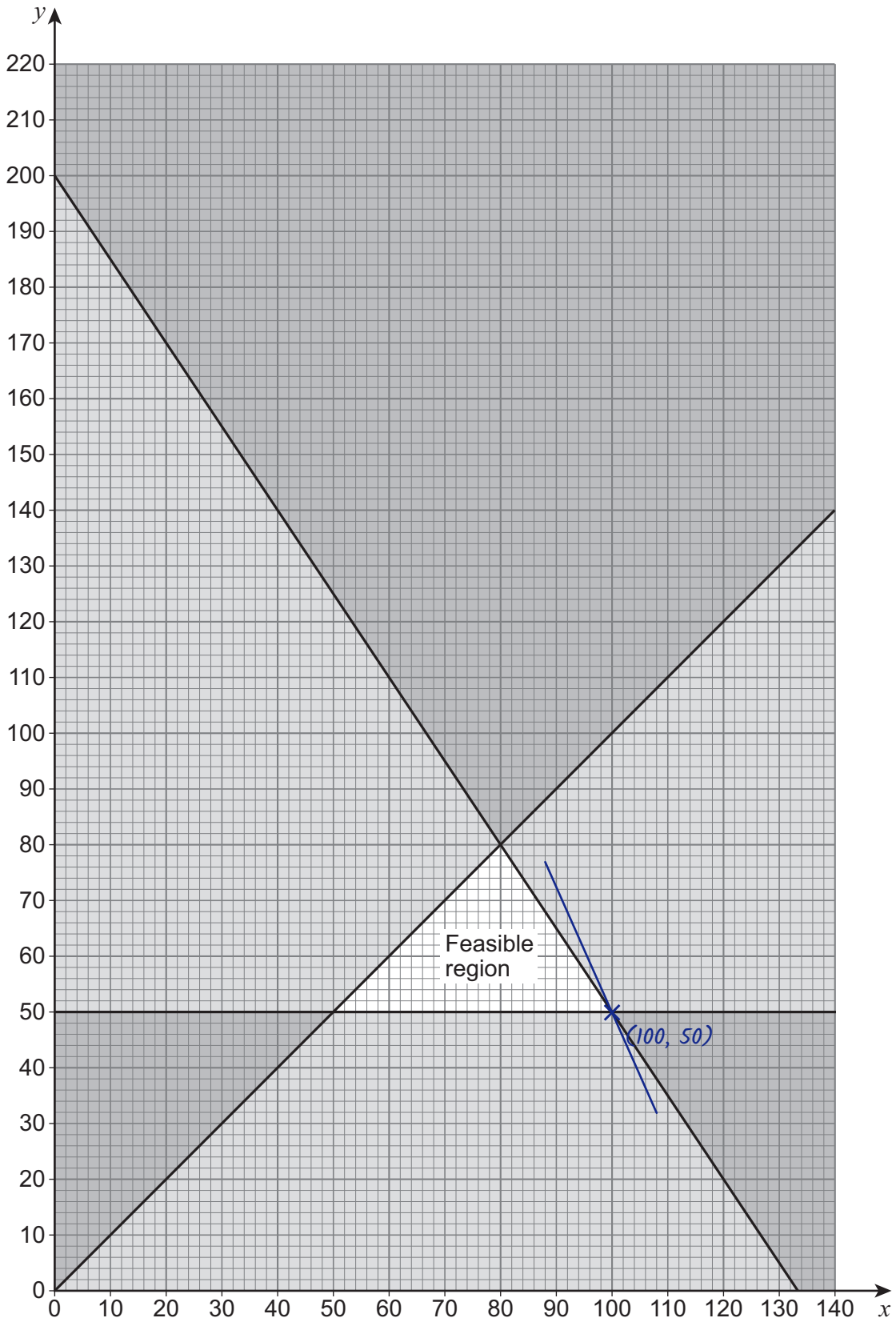
The model does not take into account the area of Alli's garden available for planting (no constraint on the total area).

Question 8 continues on the next page

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8 (c) A change is made to one of the constraints of the linear programming problem that models Alli's situation.

A graphical representation of the new linear programming problem is shown below.



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8 (c) (i) By identifying which constraint has changed, state how Alli's situation may have changed.

[2 marks]

The cost constraint has changed: the line now joins (0, 200) and (133⅓, 0), so it is $15x + 10y \leq 2000$ instead of $15x + 10y \leq 1500$.

Alli now has more money to spend: £20 instead of £15 (so Alli can now buy and plant more garlic cloves and leek seedlings).

8 (c) (ii) Using the graph, find the largest possible total area, in m^2, that Alli plants.

[3 marks]

Vertices of the feasible region: (50, 50), (80, 80), (100, 50)

$A = x/16 + y/36$

(50, 50): $50/16 + 50/36 = 4.51...$

(80, 80): $80/16 + 80/36 = 7.22...$

(100, 50): $100/16 + 50/36 = 7.638...$ (largest)

Optimal point (100, 50): 100 garlic cloves, 50 leek seedlings

Largest possible total area = 7.64 m^2 (3 s.f.)

END OF QUESTIONS

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