AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 1

Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
TOTAL	

Do not write outside the box

Answer **all** questions in the spaces provided.

1 Which of the following exponential expressions is equivalent to $2\sinh x$?

Circle your answer.

[1 mark]

e^{x} $\quad$ $e^{x}+e^{-x}$ $\quad$ $e^{x}-e^{-x}$ $\quad$ e^{-x}

2 The quadratic equation $x^2+px+q=0$ has roots α and β

Which of the following is equal to $\alpha\beta$?

Circle your answer.

[1 mark]

p $\quad$ $-p$ $\quad$ q $\quad$ $-q$

3 Which of the following transformations is represented by the matrix $\begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}$?

Tick (✓) **one** box.

[1 mark]

Rotation of 180° about the x-axis ☐

Reflection in the plane $x=0$ ☐

Rotation of 180° about the y-axis ☐

Reflection in the plane $y=0$ ☐

Do not write outside the box

4 The complex numbers w and z are defined as

$$w = 2(\cos\alpha + \mathrm{i}\sin\alpha)$$

$$z = 3(\cos\beta + \mathrm{i}\sin\beta)$$

Find the product wz

Tick (✓) **one** box.

[1 mark]

$5(\cos(\alpha\beta) + \mathrm{i}\sin(\alpha\beta))$ ☐

$6(\cos(\alpha\beta) + \mathrm{i}\sin(\alpha\beta))$ ☐

$5(\cos(\alpha + \beta) + \mathrm{i}\sin(\alpha + \beta))$ ☐

$6(\cos(\alpha + \beta) + \mathrm{i}\sin(\alpha + \beta))$ ☐

Turn over for the next question

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

5 Show that $(2 + i)^3$ is $2 + 11i$

[3 marks]

Turn over for the next question

Do not write outside the box

6 The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 5 & 2 \\ -3 & 4 \end{bmatrix}$$

6 (a) Find $\det \mathbf{A}$

[1 mark]

6 (b) Find $\mathbf{A}^{-1}$

[1 mark]

Do not write outside the box

6 (c) Given that $\mathbf{AB} = \begin{bmatrix} 9 & 6 \\ 5 & 12 \end{bmatrix}$ and $\mathbf{M} = 2\mathbf{A} + \mathbf{B}$ find the matrix $\mathbf{M}$

[3 marks]

Do not write outside the box

7 The lines l_1 and l_2 have equations

$$l_1 : \mathbf{r} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$$

$$l_2 : \mathbf{r} = \begin{bmatrix} -12 \\ a \\ -3 \end{bmatrix} + \mu \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

7 (a) Show that the point $P(-3, 9, -4)$ lies on l_1

[2 marks]

7 (b) Show that l_1 is perpendicular to l_2

[2 marks]

Do not write outside the box

7 (c) Given that the lines l_1 and l_2 intersect, calculate the value of the constant a

[4 marks]

7 (d) Hence, find the coordinates of the point of intersection of l_1 and l_2

[1 mark]

Do not write outside the box

8 The curve C has the polar equation

$$r = 4 - 2\cos\theta \qquad -\pi < \theta \leq \pi$$

8 (a) Verify that the point with polar coordinates $\left(3, \frac{\pi}{3}\right)$ lies on C

[1 mark]

8 (b) Find the exact polar coordinates of the point on C which is furthest from the pole, O

[3 marks]

Do not write outside the box

8 (c) Find the exact Cartesian coordinates of the point on C where θ is $\frac{\pi}{6}$

[3 marks]

Do not write outside the box

9 (a) Show that, for $r > 0$,

$$\ln(r+2) - \ln r = \ln\left(1 + \frac{2}{r}\right)$$

[1 mark]

9 (b) Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \ln\left(1 + \frac{2}{r}\right) = \ln\left(\frac{1}{2}(n+a)(n+b)\right)$$

where a and b are integers to be found.

[4 marks]

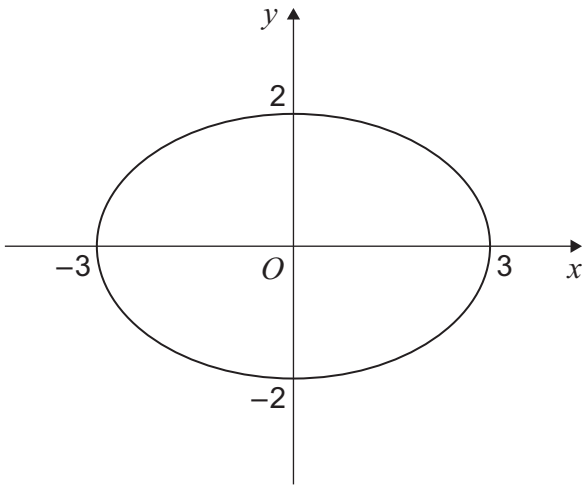
Do not write outside the box

Turn over for the next question

Do not write outside the box

10 The diagram below shows an ellipse E

The coordinate axes are the lines of symmetry of E

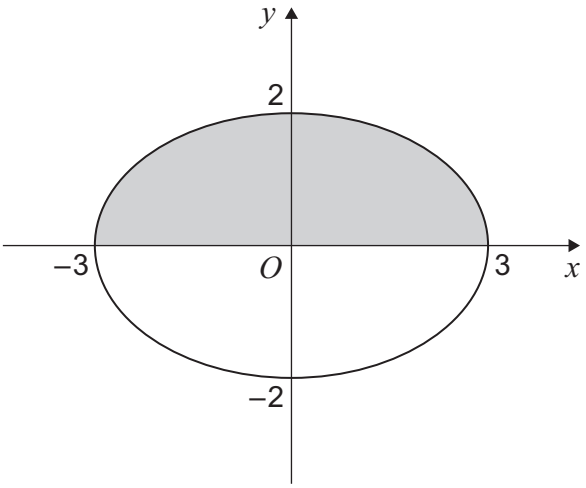


10 (a) Write down an equation of E

[2 marks]

Do not write outside the box

10 (b) The region bounded by the x-axis and the ellipse E for $y \geq 0$ is shaded in the diagram below.



A solid S is formed by rotating the shaded region through 360° about the x-axis.

Show that the volume of S is $a\pi$ where a is an integer to be found.

[4 marks]

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

11 Prove by induction that, for all integers $n \geq 1$,

$$(\mathbf{ABA}^{-1})^n = \mathbf{AB}^n\mathbf{A}^{-1}$$

where **A** and **B** are square matrices of equal dimensions, and **A** is non-singular.

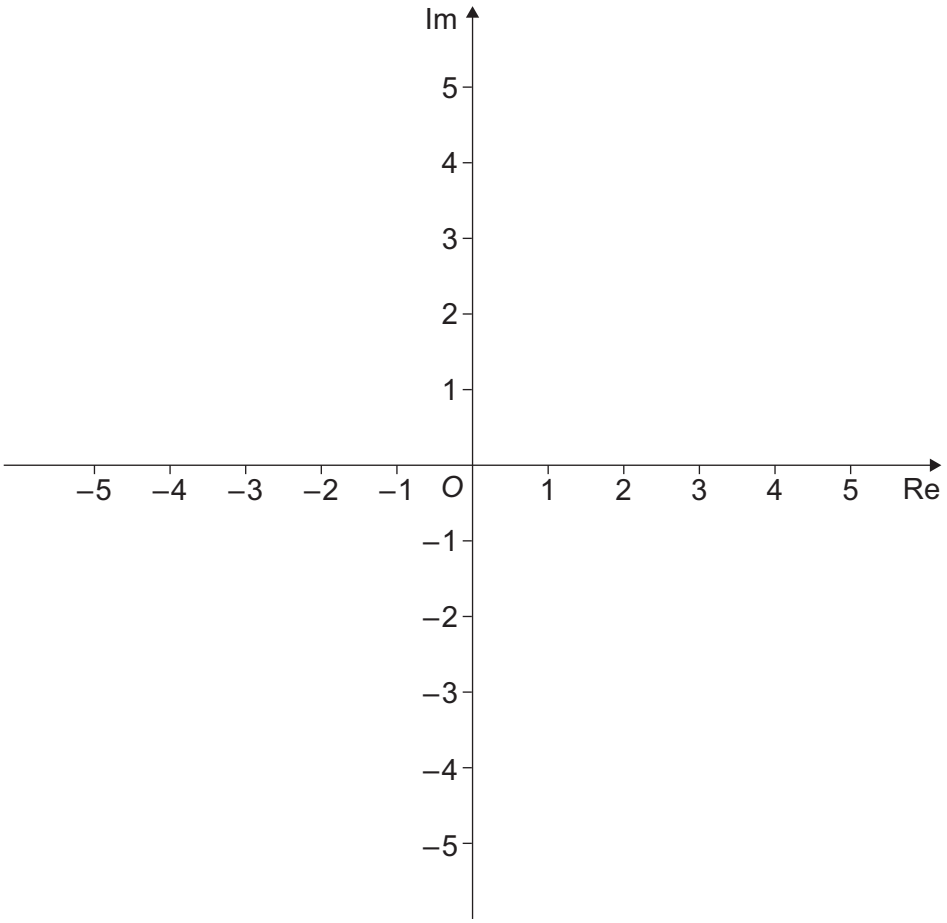
[4 marks]

Do not write outside the box

12 (a) Sketch, on the Argand diagram below, the locus of points satisfying the equation

$$|z - 2\mathrm{i}| = 2$$

[2 marks]



12 (b) Sketch, also on the Argand diagram above, the locus of points satisfying the equation

$$\arg z = \frac{\pi}{3}$$

[1 mark]

Do not write outside the box

12 (c) For the complex number w find the maximum value of $|w|$ such that

$$|w - 2i| \leq 2 \qquad \text{and} \qquad 0 \leq \arg w \leq \frac{\pi}{3}$$

[3 marks]

Turn over for the next question

Do not write outside the box

13 A curve C_1 has equation

$$y = \frac{2x + 7}{3x + 5}$$

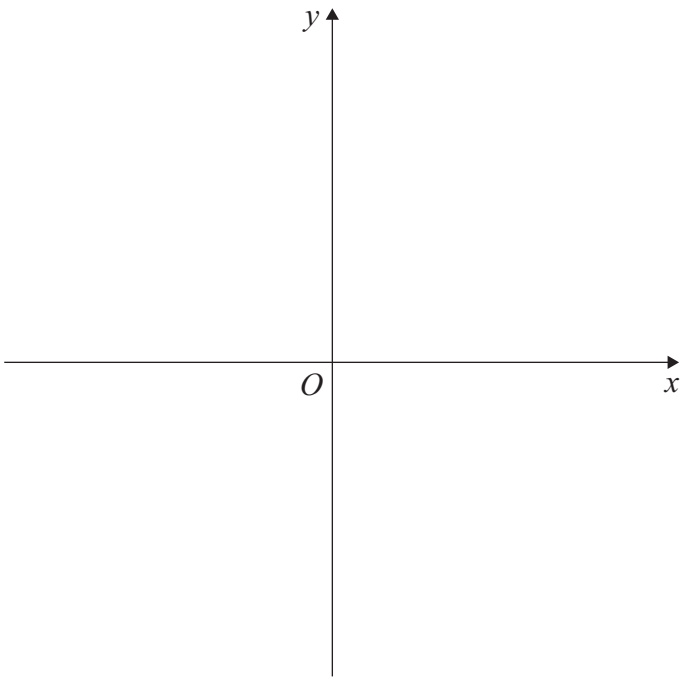
13 (a) Write down the equations of the asymptotes of curve C_1

[2 marks]

13 (b) On the axes below, sketch the graph of curve C_1

Indicate the values of the intercepts of the curve with the axes.

[3 marks]



13 (c) Hence, or otherwise, solve the inequality

$$\frac{2x + 7}{3x + 5} \geq 0$$

[2 marks]

Do not write outside the box

13 (d) Curve C_2 is a reflection of curve C_1 in the line $y = -x$

Find an equation for curve C_2 in the form $y = \mathrm{f}(x)$

[3 marks]

Do not write outside the box

14 The function f is defined by

$$\mathrm{f}(x) = \frac{x^2 - 3}{x^2 + px + 7} \qquad x \in \mathbb{R}$$

where p is a constant.

The graph of $y = \mathrm{f}(x)$ has only one asymptote.

14 (a) Write down the equation of the asymptote.

[1 mark]

14 (b) Find the set of possible values of p

[4 marks]

Do not write outside the box

14 (c) Find the coordinates of the points at which the graph of $y = \mathrm{f}(x)$ intersects the axes.

[3 marks]

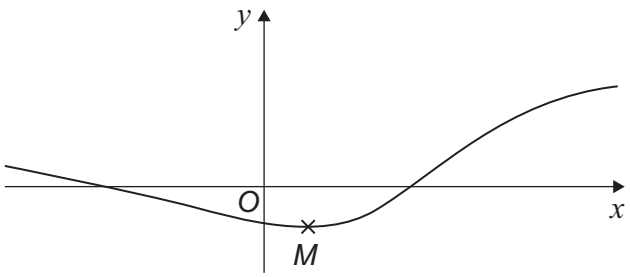
Question 14 continues on the next page

Do not write outside the box

14 (d) A curve C has equation

$$y = \frac{x^2 - 3}{x^2 - 3x + 7}$$

The curve C has a local minimum at the point M as shown in the diagram.



The line $y = k$ intersects curve C

14 (d) (i) Show that

$$19k^2 - 16k - 12 \leq 0$$

[5 marks]

Do not write outside the box

14 (d) (ii) Hence, find the y-coordinate of point M

[2 marks]

Turn over for the next question

Do not write outside the box

15 The two values of θ that satisfy the equation

$$\sinh^2\theta - \sinh\theta - 2 = 0$$

are θ_1 and θ_2

15 (a) Hamzah is asked to find the value of $\theta_1 + \theta_2$

He writes his answer as follows:

The quadratic coefficients are $a = 1,\ b = -1,\ c = -2$

The sum of the roots is $-\frac{b}{a}$

So $\theta_1 + \theta_2 = -\frac{-1}{1} = 1$

Explain Hamzah's error.

[1 mark]

Do not write outside the box

15 (b) Find the correct value of $\theta_1 + \theta_2$

Give your answer as a single logarithm.

[5 marks]

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Copyright information

For confidentiality purposes, all acknowledgements of third-party copyright material are published in a separate booklet. This booklet is published after each live examination series and is available for free download from www.aqa.org.uk.

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team.

Copyright © 2022 AQA and its licensors. All rights reserved.