AQA

Please write clearly in block capitals.

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Surname StemUp Model Solution

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AS
FURTHER MATHEMATICS

Paper 2 Statistics

Thursday 16 May 2019 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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7	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The discrete random variable X has the following probability distribution function

$$\mathrm{P}(X = x) = \begin{cases} \dfrac{5 - x}{10} & x = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

Find $\mathrm{P}(X \geq 3)$

Circle your answer.

[1 mark]

0.1 0.15 0.2 (0.3)

$P(X \geq 3) = P(3) + P(4) = 2/10 + 1/10 = 0.3$

2 A binomial hypothesis test was carried out at the 5% level of significance with the hypotheses

$\mathrm{H}_0 : p = 0.6$

$\mathrm{H}_1 : p > 0.6$

A sample of size 30 was used to carry out the test.

Find the probability that a Type I error was made.

Circle your answer.

[1 mark]

(4.4%) 4.8% 5.0% 9.4%

$X \sim B(30, 0.6)$: $P(X \geq 22) = 0.0940 > 0.05$, $P(X \geq 23) = 0.0435 < 0.05$

$\therefore$ critical region $X \geq 23$, $P(\text{Type I error}) = P(X \geq 23) = 0.0435 = 4.4\%$

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3 Fiona is studying the heights of corn plants on a farm.

She measures the height, x cm, of a random sample of 200 corn plants on the farm.

The summarised results are as follows:

$$\sum x = 60\,255 \qquad \text{and} \qquad \sum (x - \bar{x})^2 = 995$$

Calculate a 96% confidence interval for the population mean of heights of corn plants on the farm, giving your values to one decimal place.

[5 marks]

$\bar{x} = \frac{60255}{200} = 301.275$

$s^2 = \frac{995}{199} = 5 \qquad (s = \sqrt{5} = 2.236...)$

$z = 2.0537... \quad (P(Z < z) = 0.98)$

$CI = \bar{x} \pm z \frac{s}{\sqrt{n}} = 301.275 \pm 2.0537 \times \frac{\sqrt{5}}{\sqrt{200}}$

$= 301.275 \pm 0.3247...$

$= (300.950..., 301.599...)$

96% confidence interval = (301.0, 301.6)

Turn over for the next question

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4 The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{4}{99}(12x - x^2 - x^3) & 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

4 (a) Find $P(X > 1)$

[3 marks]

$P(X > 1) = \frac{4}{99}\int_1^3 (12x - x^2 - x^3)\,dx$

$= \frac{4}{99}\left[6x^2 - \frac{x^3}{3} - \frac{x^4}{4}\right]_1^3$

$= \frac{4}{99}\left[\left(54 - 9 - \frac{81}{4}\right) - \left(6 - \frac{1}{3} - \frac{1}{4}\right)\right]$

$= \frac{4}{99} \times \frac{58}{3} = \frac{232}{297}$ $(= 0.781)$

4 (b) Show that $E(X^{-1}) = \frac{10}{11}$

[3 marks]

$E(X^{-1}) = \frac{4}{99}\int_0^3 x^{-1}(12x - x^2 - x^3)\,dx$

$= \frac{4}{99}\int_0^3 (12 - x - x^2)\,dx$

$= \frac{4}{99}\left[12x - \frac{x^2}{2} - \frac{x^3}{3}\right]_0^3$

$= \frac{4}{99}\left(36 - \frac{9}{2} - 9 - 0\right) = \frac{4}{99} \times \frac{45}{2} = \frac{90}{99}$

$\therefore E(X^{-1}) = \frac{10}{11}$ as required

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4 (c) Find $E(2X^{-1} - 3)$

[2 marks]

$E(2X^{-1} - 3) = 2E(X^{-1}) - 3$

$= 2 \times \frac{10}{11} - 3$

$= -\frac{13}{11}$ $(= -1.18)$

Turn over for the next question

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5 The discrete random variable X has the following probability distribution function

$$P(X = x) = \begin{cases} \frac{1}{n} & x = 1, 2, ..., n \\ 0 & \text{otherwise} \end{cases}$$

5 (a) (i) Prove that $E(X) = \frac{n+1}{2}$

[3 marks]

$E(X) = \sum_{x=1}^{n} x \times \frac{1}{n} = \frac{1}{n} \sum_{x=1}^{n} x$

$= \frac{1}{n} \times \frac{n(n+1)}{2}$

$= \frac{n+1}{2}$ as required

5 (a) (ii) Prove that $\text{Var}(X) = \frac{n^2 - 1}{12}$

[4 marks]

$E(X^2) = \sum_{x=1}^{n} x^2 \times \frac{1}{n} = \frac{1}{n} \sum_{x=1}^{n} x^2$

$= \frac{1}{n} \times \frac{n(n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{6}$

$Var(X) = E(X^2) - (E(X))^2$

$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$

$= \frac{2n^2 + 3n + 1}{6} - \frac{n^2 + 2n + 1}{4}$

$= \frac{4n^2 + 6n + 2 - 3n^2 - 6n - 3}{12}$

$= \frac{n^2 - 1}{12}$ as required

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5 (b) State **two** conditions under which a discrete uniform distribution can be used to model the score when a cubic dice is rolled.

[2 marks]

The dice is fair (unbiased), so each score is equally likely.

The faces are numbered 1, 2, 3, 4, 5, 6 (so $n = 6$).

Turn over ▶

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6 A company owns two machines, *A* and *B*, which make toys. Both machines run continuously and independently.

Machine *A* makes an average of 2 errors per hour.

6 (a) Using a Poisson model, find the probability that the machine makes exactly 5 errors in 4 hours, giving your answer to three significant figures.

[2 marks]

Mean in 4 hours = $2 \times 4 = 8$, $X \sim Po(8)$

$P(X = 5) = \frac{e^{-8} \times 8^5}{5!} = 0.091603... = 0.0916$

6 (b) Machine *B* makes an average of 5 errors per hour. Both machines are switched on and run for 1 hour.

The company finds the probability that the total number of errors made by machines *A* and *B* in 1 hour is greater than 8.

If the probability is greater than 0.4, a new machine will be purchased.

Using a Poisson model, determine whether or not the toy company will purchase a new machine.

[3 marks]

Let Y = errors by B in 1 hour. Mean of $X + Y = 2 + 5 = 7$

$X + Y \sim Po(7)$ (A and B independent)

$P(X + Y > 8) = 1 - P(X + Y \leq 8)$

$= 1 - 0.72909... = 0.27090... = 0.271$

$0.271 < 0.4$, so the company will not purchase a new machine.

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6 (c) After investigation, the standard deviation of errors made by machine *A* is found to be 0.5 errors per hour and the standard deviation of errors made by machine *B* is also found to be 0.5 errors per hour.

Explain whether or not the use of Poisson models in parts **(a)** and **(b)** is appropriate.

[2 marks]

Variance = 0.5^2 = 0.25 for each machine.

For a Poisson distribution mean = variance, but the means 2 and 5

are not equal to the variance 0.25, so the Poisson models are

not appropriate.

Turn over for the next question

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7 Mohammed is conducting a medical trial to study the effect of two drugs, *A* and *B*, on the amount of time it takes to recover from a particular illness.

Drug *A* is used by one group of 60 patients and drug *B* is used by a second group of 60 patients.

The results are summarised in the table:

		Recovery time			
		1 week	**2 weeks**	**3 weeks**	**Total**
Drug	***A***	36	19	5	60
	B	21	24	15	60
	Total	57	43	20	120

Mohammed claims that there is an association between recovery time and drug used.

7 (a) Investigate Mohammed's claim using the 1% level of significance.

[7 marks]

H_0: there is no association between recovery time and drug used

H_1: there is an association between recovery time and drug used

Expected values $E = \frac{\text{row total} \times \text{column total}}{120}$:

E	1 week	2 weeks	3 weeks
A	28.5	21.5	10
B	28.5	21.5	10

$$\chi^2 = \Sigma \frac{(O-E)^2}{E} = \frac{(36-28.5)^2}{28.5} + \frac{(19-21.5)^2}{21.5} + \frac{(5-10)^2}{10} + \frac{(21-28.5)^2}{28.5} + \frac{(24-21.5)^2}{21.5} + \frac{(15-10)^2}{10}$$

$$= 1.974 + 0.291 + 2.5 + 1.974 + 0.291 + 2.5$$

$$= 9.528\ldots = 9.53$$

Degrees of freedom $= (2-1)(3-1) = 2$

Critical value at 1% level: $\chi^2_2(0.01) = 9.210$

$9.53 > 9.210$, so reject H_0

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There is sufficient evidence at the 1% level to suggest that there is an association between recovery time and drug used, so the evidence supports Mohammed's claim.

7 (b) Interpret any association between recovery time and the drug used.

[2 marks]

Largest contributions are 3 weeks for both drugs, $\frac{(O-E)^2}{E} = 2.5$ each.

Fewer patients than expected using drug A took 3 weeks to recover (5 v 10), and more than expected using drug B took 3 weeks (15 v 10), so drug A is associated with faster recovery than drug B.

END OF QUESTIONS

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