AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

AS
FURTHER MATHEMATICS

Paper 2 Discrete

Thursday 16 May 2019 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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4	
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6	
7	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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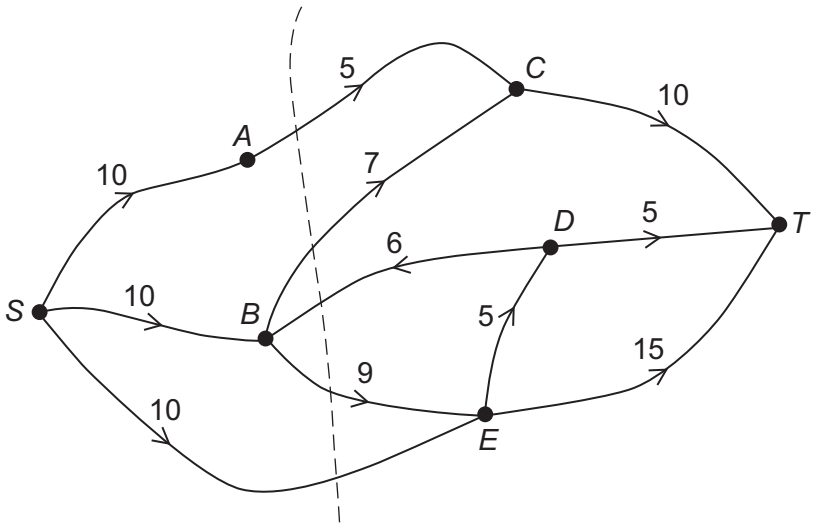
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Answer **all** questions in the spaces provided.

1 The network represents a system of pipes.

The number on each arc represents the upper capacity for each pipe in $cm^3\,s^{-1}$



The value of the cut $\{S, A, B\}$ $\{C, D, E, T\}$ is $V\,cm^3\,s^{-1}$

Find V.

Circle your answer.

[1 mark]

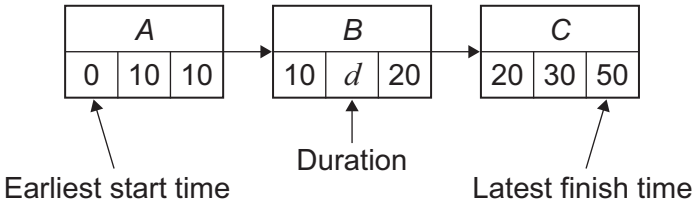
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2 Part of an activity network is shown in the diagram below.

ABC is part of the critical path of the activity network.



The duration of activity *B* is d.

Which of the following statements about d is correct?

Circle your answer.

[1 mark]

$0 < d < 10$ $\quad$ $d = 10$ $\quad$ $10 < d < 20$ $\quad$ $d = 20$

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3 Manon makes apple cakes and banana cakes.

Each apple cake is made with 3 eggs and 100 grams of flour.
Each banana cake is made with 2 eggs and 150 grams of flour.

Manon has 36 eggs and 1500 grams of flour.

Manon wants to make as many cakes as possible.

Formulate Manon's situation as a linear programming problem, clearly defining any variables you introduce.

[4 marks]

Let x = number of apple cakes and y = number of banana cakes.

Maximise $P = x + y$

subject to

$3x + 2y \leq 36$ (eggs)

$100x + 150y \leq 1500$ (flour, in grams)

$x \geq 0, \quad y \geq 0$

x and y are integers

Turn over for the next question

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4 (a) State the definition of a bipartite graph.

[1 mark]

A bipartite graph is a graph whose vertices can be split into two sets such that no edge joins two vertices in the same set, i.e. every edge joins a vertex in one set to a vertex in the other set.

4 (b) A jazz quintet has five musical instruments: bassoon, clarinet, flute, oboe and violin.

Jay, Kay, Lee, Mel and Nish are musicians and each plays a musical instrument in the jazz quintet.

Jay knows how to play the bassoon and the clarinet.

Kay knows how to play the bassoon, the oboe and the violin.

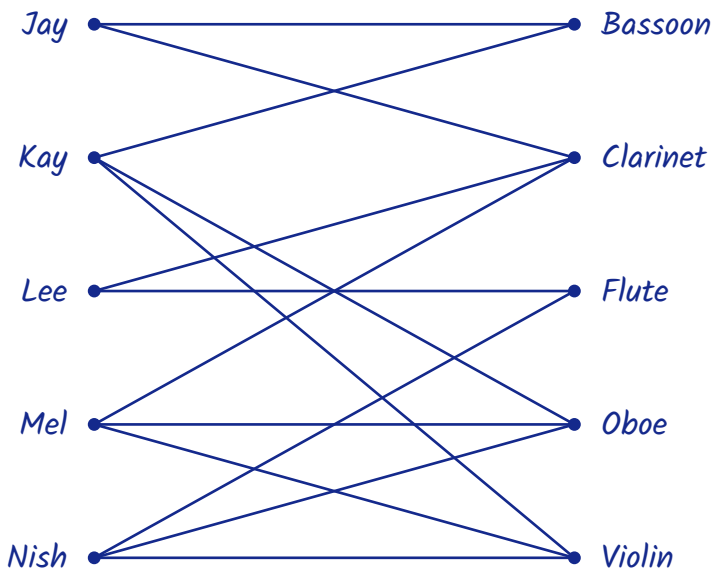
Lee knows how to play the clarinet and the flute.

Mel knows how to play the clarinet, the oboe and the violin.

Nish knows how to play the flute, the oboe and the violin.

4 (b) (i) Draw a graph to show which musicians know how to play which instruments.

[2 marks]



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4 (b) (ii) Nish arrives late to a jazz quintet rehearsal.

Each of the other four musicians is already playing an instrument:

Jay is playing the clarinet
Kay is playing the oboe
Lee is playing the flute
Mel is playing the violin.

Explain how the graph in part **(b)(i)** shows that there is no instrument available that Nish knows how to play.

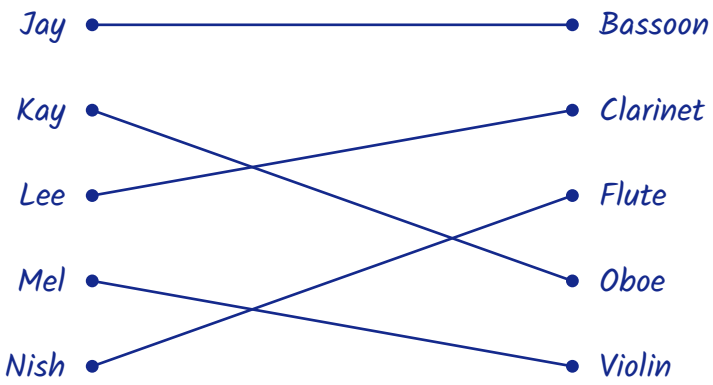
[1 mark]

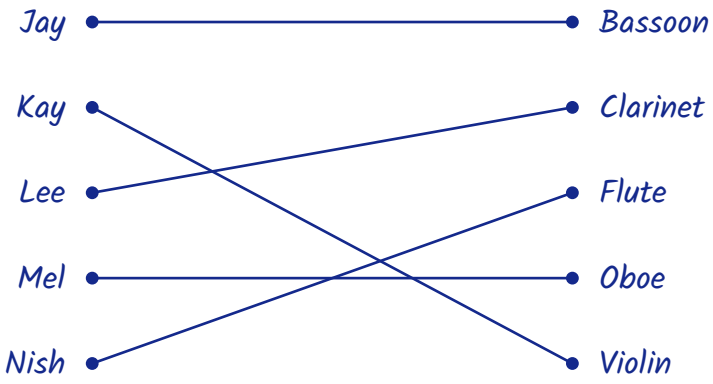
The vertex for Nish is only connected to the vertices for flute, oboe and violin, which are all being played. The vertex for Nish is not connected to the vertex for bassoon, which is the only instrument available.

4 (b) (iii) When Nish arrives the rehearsal stops. When they restart the rehearsal, Nish is playing the flute.

Draw all possible subgraphs of the graph in part **(b)(i)** that show how Jay, Kay, Lee and Mel can each be assigned a unique musical instrument they know how to play.

[2 marks]





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5 (a) Complete the Cayley table in **Figure 1** for multiplication modulo 4

[2 marks]

Figure 1

$\times_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

5 (b) The set S is defined as

$$S = \{a, b, c, d\}$$

Figure 2 shows an incomplete Cayley table for S under the commutative binary operation $\bullet$

Figure 2

$\bullet$	a	b	c	d
a	b	a	a	c
b	a	c	d	c
c	a	d	d	d
d	c	c	d	d

5 (b) (i) Complete the Cayley table in **Figure 2**.

[1 mark]

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5 (b) (ii) Determine whether the binary operation • is associative when acting on the elements of S.

Fully justify your answer.

[2 marks]

Test with the elements a, b and c:

$a \cdot (b \cdot c) = a \cdot d = c$

$(a \cdot b) \cdot c = a \cdot c = a$

$c \neq a$, so $a \cdot (b \cdot c) \neq (a \cdot b) \cdot c$

Therefore the binary operation $\cdot$ is not associative on S.

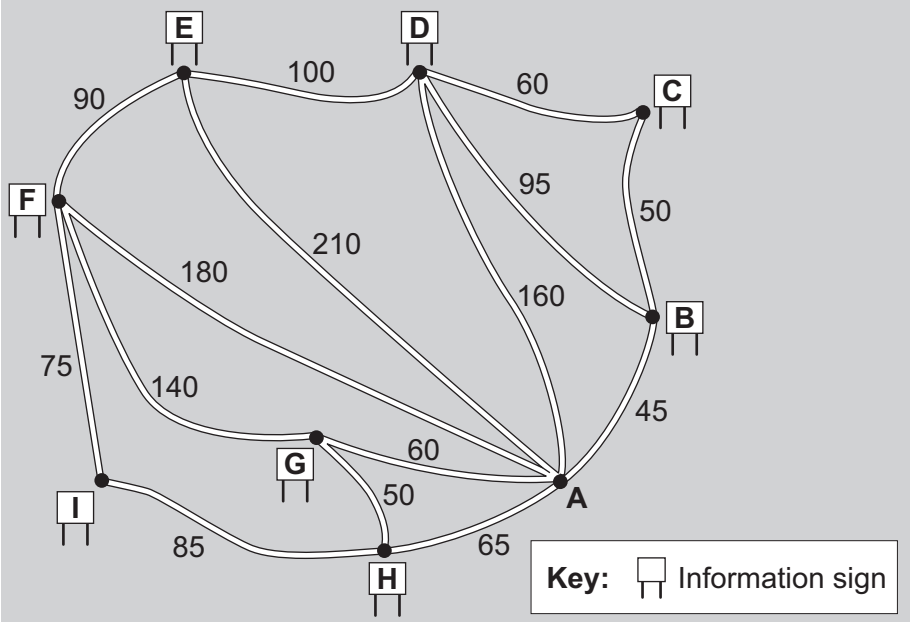
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6 The diagram shows a nature reserve which has its entrance at *A*, eight information signs at *B*, *C*, ..., *I*, and fifteen grass paths.

The length of each grass path is given in metres.

The total length of the grass paths is 1465 metres.



To cut the grass, Ashley starts at the entrance and drives a mower along every grass path in the nature reserve.

The mower moves at 7 kilometres per hour.

6 (a) Find the least possible time that it takes for Ashley to cut the grass on all fifteen paths in the nature reserve and return to the entrance.

Fully justify your answer.

[5 marks]

Route inspection: odd degree vertices are B, E, G and H

Shortest distances between odd vertices:

BE = 95 + 100 = 195 (BDE) BG = 45 + 60 = 105 (BAG)

BH = 45 + 65 = 110 (BAH) EH = 90 + 75 + 85 = 250 (EFIH)

EG = 90 + 140 = 230 (EFG) GH = 50

Pairings: BE + GH = 195 + 50 = 245 (least)

BG + EH = 105 + 250 = 355

BH + EG = 110 + 230 = 340

Repeat BD, DE and GH: extra 245 m

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Least distance = 1465 + 245 = 1710 m = 1.71 km

Time = 1.71 ÷ 7 = 0.244285... hours

Least possible time = 0.244 hours (about 14.7 minutes)

6 (b) Brook visits every information sign in the nature reserve to update them, **starting and finishing at the entrance**.

For the eight information signs, the minimum connecting distance of the grass paths is 510 metres.

6 (b) (i) Determine a lower bound for the distance Brook walks to visit every information sign.

Fully justify your answer.

[2 marks]

Remove the entrance A: minimum connecting distance of the eight information signs = 510 m

Add the two shortest arcs from A: AB = 45 and AG = 60

Lower bound = 510 + 45 + 60

= 615 metres

6 (b) (ii) Using the nearest neighbour algorithm **starting from the entrance**, determine an upper bound for the distance Brook walks to visit every information sign.

[2 marks]

Nearest neighbour from A:

A → B → C → D → E → F → I → H → G → A

45 + 50 + 60 + 100 + 90 + 75 + 85 + 50 + 60

Upper bound = 615 metres

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6 (c) Brook takes one minute to update the information at one information sign.
Brook walks on the grass paths at an average speed of 5 kilometres per hour.
Ashley and Brook start from the entrance at the same time.

6 (c) (i) Use your answers from parts **(a)** and **(b)** to show that Ashley and Brook will return to the entrance at approximately the same time.

Fully justify your answer.

[3 marks]

Lower bound = upper bound = 615 m, so Brook's optimal distance is exactly 615 m.

Brook walking: $0.615 \div 5 = 0.123$ hours

Updating 8 signs: $8 \times 1 = 8$ minutes $= 0.1333...$ hours

Brook total $= 0.123 + 0.1333... = 0.2563...$ hours (≈ 15.4 minutes)

Ashley total $= 0.244$ hours (≈ 14.7 minutes)

Both times are approximately 0.25 hours (a quarter of an hour), so Ashley and Brook return to the entrance at approximately the same time.

6 (c) (ii) State an assumption that you have used in part **(c)(i)**.

[1 mark]

Ashley and Brook each move at a constant speed with no breaks (the speed is the same whether or not the grass has been cut).

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7 Ali and Bex play a zero-sum game.
The game is represented by the following pay-off matrix for Ali.

		Bex		
	Strategy	**B_1**	**B_2**	**B_3**
Ali	**A_1**	2	−1	3
	A_2	−4	−2	2
	A_3	0	1	1
	A_4	−3	2	−2

7 (a) (i) Write down the pay-off matrix for Bex.

[2 marks]

		Ali			
	Strategy	A_1	A_2	A_3	A_4
Bex	B_1	−2	4	0	3
	B_2	1	2	−1	−2
	B_3	−3	−2	−1	2

7 (a) (ii) Explain why the pay-off matrix for Bex can be written as

−2	0	3
1	−1	−2

[2 marks]

Compare B_1 with B_3: $-2 > -3$, $4 > -2$, $0 > -1$, $3 > 2$, so strategy B_1 dominates strategy B_3: Bex never plays B_3 (remove row B_3).

Ali wants Bex's pay-off small: $-2 < 4$, $1 < 2$, $-3 < -2$, so strategy A_1 dominates strategy A_2: Ali never plays A_2 (remove column A_2).

This leaves rows B_1, B_2 and columns A_1, A_3, A_4: the matrix shown.

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7 (b) The game does **not** have a stable solution.

Bex plays her optimal mixed strategy.

Find the value of the game **for Ali**.

[6 marks]

Let Bex play B_1 with probability p and B_2 with probability $1 - p$.

Expected gains for Bex:

Ali plays A_1: $-2p + 1(1 - p) = 1 - 3p$

Ali plays A_3: $0p - 1(1 - p) = p - 1$

Ali plays A_4: $3p - 2(1 - p) = 5p - 2$

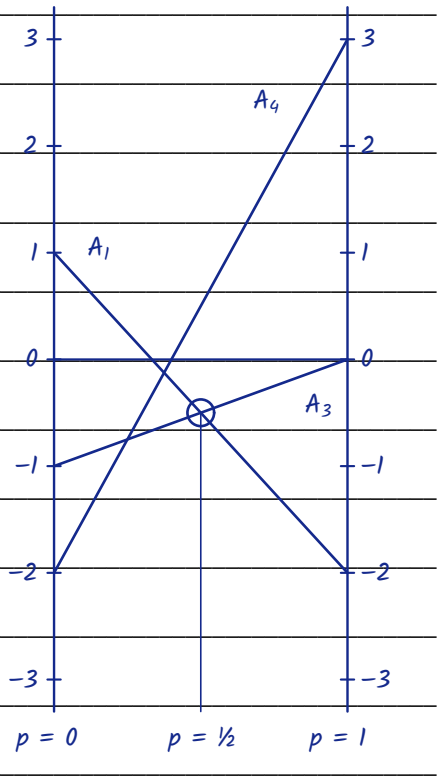
Bex chooses p to maximise her minimum expected gain: the highest point of the lowest boundary is where A_1 and A_3 meet.

$p - 1 = 1 - 3p$

$4p = 2 \Rightarrow p = 1/2$

Value of game for Bex $= 1/2 - 1 = -1/2$

Zero-sum game, so value of the game for Ali $= 1/2$



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