AQA

# AS
# Further Mathematics

## 7366/2S - Statistics
## Mark scheme

7366

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Copyright © 2018 AQA and its licensors. All rights reserved.
AQA retains the copyright on all its publications. However, registered schools/colleges for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

# Mark scheme instructions to examiners

## General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

**Key to mark types**

| | |
|---|---|
| M | mark is for method |
| dM | mark is dependent on one or more M marks and is for method |
| R | mark is for reasoning |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| F | follow through from previous incorrect result |

**Key to mark scheme abbreviations**

| | |
|---|---|
| CAO | correct answer only |
| CSO | correct solution only |
| ft | follow through from previous incorrect result |
| 'their' | Indicates that credit can be given from previous incorrect result |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| sf | significant figure(s) |
| dp | decimal place(s) |

Examiners should consistently apply the following general marking principles

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**Diagrams**

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

**Work erased or crossed out**

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

**Choice**

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer."

## AS/A-level Maths/Further Maths assessment objectives

| AO | | Description |
|---|---|---|
| **AO1** | AO1.1a | Select routine procedures |
| | AO1.1b | Correctly carry out routine procedures |
| | AO1.2 | Accurately recall facts, terminology and definitions |
| **AO2** | AO2.1 | Construct rigorous mathematical arguments (including proofs) |
| | AO2.2a | Make deductions |
| | AO2.2b | Make inferences |
| | AO2.3 | Assess the validity of mathematical arguments |
| | AO2.4 | Explain their reasoning |
| | AO2.5 | Use mathematical language and notation correctly |
| **AO3** | AO3.1a | Translate problems in mathematical contexts into mathematical processes |
| | AO3.1b | Translate problems in non-mathematical contexts into mathematical processes |
| | AO3.2a | Interpret solutions to problems in their original context |
| | AO3.2b | Where appropriate, evaluate the accuracy and limitations of solutions to problems |
| | AO3.3 | Translate situations in context into mathematical models |
| | AO3.4 | Use mathematical models |
| | AO3.5a | Evaluate the outcomes of modelling in context |
| | AO3.5b | Recognise the limitations of models |
| | AO3.5c | Where appropriate, explain how to refine models |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **1** | Circles correct answer | AO1.2 | B1 | 0 |
| | **Total** | | **1** | |
| **2** | Circles correct answer | AO1.1b | B1 | 0.801 |
| | **Total** | | **1** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **3** | Evaluates $E(X^2)$ by calculating $1^2 \times 0.2 + 2^2 \times 0.4 + 4^2 \times 0.35 + 9^2 \times 0.05$ | AO1.1a | M1 | $E(X^2) = 1^2 \times 0.2 + 2^2 \times 0.4 + 4^2 \times 0.35 + 9^2 \times 0.05$<br>$= 11.45$ or $\frac{229}{20}$ |
| | Evaluates $E(Y^2)$ by integrating $\int y^2 \times f(y) \ \ dy$<br>Must see the integral | AO1.1a | M1 | $E(Y^2) = \int_0^4 y^2 \times \frac{1}{64} y^3 \ \ dy$<br>$= \frac{1}{64}\left[\frac{1}{6} y^6\right]_0^4 = \left[\frac{1}{384} y^6\right]_0^4 = \frac{32}{3}$ |
| | Finds $E(X^2) = 11.45$ and $E(Y^2) = \frac{32}{3}$ | AO1.1b | A1 | $E(X^2 + Y^2) = E(X^2) + E(Y^2)$<br>$= 11.45 + \frac{32}{3}$ |
| | Uses $E(X^2 + Y^2) = E(X^2) + E(Y^2)$ to show that $E(X^2 + Y^2) = \frac{1327}{60}$ AG<br>Mark awarded if they have a completely correct solution, which is clear, easy to follow and contains no slips | AO2.1 | R1 | $= \frac{1327}{60}$ |
| | **Total** | | **4** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **4(a)** | Calculates the correct sample mean | AO1.1b | B1 | $\bar{x} = \frac{3540}{100}$<br>= 35.4 |
| | Finds correct *z* value to at least 3 significant figures.<br>Condone -2.326…<br>Can be implied by a correct confidence interval. | AO1.1a | M1 | z = 2.32634787<br><br>$\bar{x} \pm z\sqrt{\frac{\sigma^2}{n}}$ |
| | Uses formula for confidence interval with $\sqrt{\frac{10}{100}}$ | AO1.1a | M1 | = $35.4 \pm 2.326\sqrt{\frac{10}{100}}$<br><br>= (34.66, 36.14) or 35.4 ± 0.7357 |
| | Obtains confidence interval<br>CAO | AO1.1b | A1 | |
| **4(b)** | States null hypothesis is rejected as the confidence interval does not contain 38. OE<br>Follow through their confidence interval. | AO3.5a | E1F | Dante rejects the null hypothesis because 38 is outside the confidence interval. |
| | **Total** | | **5** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **5(a)** | States the mode | AO1.2 | B1 | 1.75 |
| **5(b)** | Uses the area under a p.d.f. is equal to 1 to form an equation to find the y-coordinate of the maximum point of the p.d.f. | AO3.1a | M1 | $\frac{2k}{2} = 1$<br>$k = 1$<br>Maximum point of the p.d.f. is (1.75, 1) |
| | Finds the equation of one line using their k, the y-coordinate of the maximum point of the p.d.f.<br>May be in terms of k | AO1.1a | M1 | $y = \frac{k}{1.75}x$<br>$y = \frac{1}{1.75}x$<br>$y = \frac{4}{7}x$ |
| | Finds the equation of both lines<br>Follow through their k<br>May be in terms of k | AO1.1b | A1F | $y = -\frac{k}{0.25}(x-2)$<br>$y = -\frac{1}{0.25}(x-2)$<br>$y = -4x + 8$ |
| | States fully defined probability density function (OE) CAO | AO3.2a | A1 | $f(x) = \begin{cases} \frac{4}{7}x & 0 \le x < 1.75 \\ -4x+8 & 1.75 \le x < 2 \\ 0 & \text{otherwise} \end{cases}$ |
| | **Total** | | **5** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **6** | Uses sum of probabilities = 1 to correctly find k | AO1.1b | B1 | $2k + 4k + 6k + 8k = 1$<br>$20k = 1$<br>$k = \frac{1}{20}$ or 0.05 |
| | Calculates $E(Y)$ or $E(Y^2)$ correctly | AO1.1a | M1 | $E(Y) = 1 \times 2k + 2 \times 4k + 3 \times 6k + 4 \times 8k$<br>$= 60k$ or 3 |
| | Calculates both E(Y) and $E(Y^2)$ correctly | AO1.1b | A1 | $E(Y^2) = 1^2 \times 2k + 2^2 \times 4k + 3^2 \times 6k + 4^2 \times 8k$<br>$= 200k$ or 10 |
| | Calculates Var ($Y$) | AO1.1a | M1 | Var $(Y) = E(Y^2) - E(Y)^2$<br>$= 200k - (60k)^2$ or $10 - 3^2$<br>$= 200k - 3600k^2$ or 1 |
| | Uses formula for Var ($aY \pm b$) | AO1.1a | M1 | Var $(5Y - 2) = 5^2$ Var $(Y)$<br>$= 5^2 \times 1$<br>$= 25$ |
| | Completes a fully correct, logical argument to show that Var $(5Y - 2) = 25$ AG | AO2.1 | R1 | |

<table>
<tr><td rowspan="6">6<br>ALT</td><td>Uses sum of probabilities = 1 to correctly find k</td><td>AO1.1b</td><td>B1</td><td>$2k + 4k + 6k + 8k = 1$<br>$20k = 1$<br>$k = \frac{1}{20}$ or 0.05</td></tr>
<tr><td>Finds values of $5Y – 2$ for each value of $y$</td><td>AO1.1a</td><td>M1</td><td><table><tr><td>$y$</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$5y-2$</td><td>3</td><td>8</td><td>13</td><td>18</td></tr></table></td></tr>
<tr><td>Calculates $E(5Y – 2)$ or $E((5Y – 2)^2)$ correctly</td><td>AO1.1a</td><td>M1</td><td>$E(5Y – 2) = 3 \times 2k + 8 \times 4k + 13 \times 6k + 18 \times 8k$<br>$= 260k$ or 13</td></tr>
<tr><td>Calculates $E(5Y – 2)$ and $E((5Y – 2)^2)$ correctly</td><td>AO1.1b</td><td>A1</td><td>$E((5Y – 2)^2) = 3^2 \times 2k + 8^2 \times 4k + 13^2 \times 6k + 18^2 \times 8k$<br>$= 3880k$ or 194</td></tr>
<tr><td>Calculates Var $(5Y – 2)$</td><td>AO1.1a</td><td>M1</td><td>Var $(5Y – 2) = E((5Y–2)^2) – E(5Y–2)^2$<br>$= 3880k – (260k)^2$ or $194 – 13^2$<br>$= 3880k – 67600k^2$</td></tr>
<tr><td>Completes a fully correct, logical argument to show that<br>Var $(5Y – 2) = 25$ AG</td><td>AO2.1</td><td>R1</td><td>= 25</td></tr>
<tr><td></td><td>Total</td><td></td><td>6</td><td></td></tr>
</table>

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **7(a)** | States both hypotheses using correct language | AO2.5 | B1 | $H_0$: $\lambda = 50$<br>$H_1$: $\lambda < 50$<br><br>$X \sim Po(50)$<br><br>$P(X \leq 30) = 0.002$<br>($P(X \leq 33) = 0.007$ and<br>$P(X \leq 34) = 0.0108$)<br><br>$p$-value = $0.002 < 0.01$<br>($30 < 34$ = critical value)<br><br>Reject $H_0$ in favour of $H_1$<br><br>Significant evidence to suggest that the mean number of vehicles passing the service station per minute has reduced |
| | Selects and uses Poisson model with $\lambda = 50$ to find<br>P(total vehicles ≤ 30) or<br>P(total vehicles < 30) or<br>P(total vehicles ≤ 33) and P(total vehicles ≤ 34) | AO3.3 | M1 | |
| | Obtains AWRT $P(X \leq 30) = 0.002$ (0.001594) or<br>AWRT $P(X \leq 33) = 0.007$ (0.0069788) and AWRT<br>$P(X \leq 34) = 0.0108$ (0.01078145) | AO1.1b | A1 | |
| | Evaluates the Poisson model by comparing 'their' $p$-value with 0.01 or 30 with their critical value | AO3.5a | R1 | |
| | Infers $H_0$ rejected | AO2.2b | E1 | |
| | Concludes in context<br>(not definite) | AO3.2a | E1 | |
| **7(b)** | States meaning in context of Type I error | AO3.2a | E1 | Type I error is to conclude that the mean number of vehicles passing the service station per minute using the junction has reduced when it has not. |
| **7(c)** | States that rate at which events occur is unlikely to be constant over time in context<br>or<br>States that the events do not occur independently in context<br>Must be consistent | AO3.5b | E1 | The rate at which vehicles pass the service station is unlikely to be constant over time.<br>or<br>Vehicles may not pass the service station independently, eg convoy |
| | **Total** | | **8** | |

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td rowspan="2">8(a)</td><td>Obtains one correct missing expected value<br>e.g. 0 and car = $\frac{28\times 35}{80}$</td><td>AO1.1a</td><td>M1</td><td rowspan="2"><table><tr><th></th><th>0</th><th>1</th><th>2</th><th>3+</th></tr><tr><td>C</td><td>12.25</td><td>10.0625</td><td>8.3125</td><td>4.375</td></tr><tr><td>MB</td><td>15.75</td><td>12.9375</td><td>10.6875</td><td>5.625</td></tr></table></td></tr>
<tr><td>Obtains all expected values<br>Must not be rounded CAO<br>Condone 12.937 for 12.9375</td><td>AO1.1b</td><td>A1</td></tr>
</table>

<table>
<tr><td rowspan="8">8(b)</td><td>States both hypotheses using correct language<br>Accept equivalent wording</td><td>AO2.5</td><td>B1</td><td rowspan="8">$H_0$: There is no association between number of claims and type of insurance policy<br>$H_1$: There is an association between number of claims and type of insurance policy<br><br><table><tr><td></td><td colspan="2">2 or more</td></tr><tr><td></td><td>Observed</td><td>Expected</td></tr><tr><td>C</td><td>16</td><td>12.6875</td></tr><tr><td>MB</td><td>13</td><td>16.3125</td></tr></table><br>$\sum \frac{(O-E)^2}{E}$ =<br>$\frac{(9-12.25)^2}{12.25} + \frac{(19-15.75)^2}{15.75}$ +<br>$\frac{(10-10.0625)^2}{10.0625} + \frac{(13-12.9375)^2}{12.9375}$ +<br>+ $\frac{(16-12.6875)^2}{12.6875} + \frac{(13-16.3125)^2}{16.3125}$<br>= 3.07<br><br>$\chi^2$ cv for 2 df = 4.605<br>(p = 0.215)<br><br>3.07 < 4.605<br>(0.215 > 0.1)<br><br>Accept $H_0$<br><br>No significant evidence to suggest that there is an association between number of claims and type of insurance policy</td></tr>
<tr><td>Combines expected and observed values for 2 and 3 or more correctly<br>Follow through their answers to part (a)</td><td>AO1.1b</td><td>B1F</td></tr>
<tr><td>Calculates $\chi^2$-test statistic<br>Can be awarded even if 2 and 3 or more are not combined</td><td>AO1.1a</td><td>M1</td></tr>
<tr><td>Calculates $\chi^2$-test statistic correctly.<br>Condone given to 1 s.f.<br>CAO</td><td>AO1.1b</td><td>A1</td></tr>
<tr><td>States critical value (or p-value, follow through their $\chi^2$ value)<br>Condone $\chi^2$ cv for 3 df = 6.251<br>If 2 and 3 or more are not combined</td><td>AO1.1b</td><td>B1F</td></tr>
<tr><td>Evaluates $\chi^2$-test statistic by comparing the cv with their ts (or their p value with 0.1)</td><td>AO3.5a</td><td>R1</td></tr>
<tr><td>Infers $H_0$ not rejected (OE)</td><td>AO2.2b</td><td>E1</td></tr>
<tr><td>Concludes in context<br>(not definite)<br>Accept equivalent wording</td><td>AO3.2a</td><td>E1</td></tr>
<tr><td></td><td>Total</td><td></td><td>10</td><td></td></tr>
<tr><td></td><td>TOTAL</td><td></td><td>40</td><td></td></tr>
</table>