AQA

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Centre number | | | | | | Candidate number | | | | |

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AS
FURTHER MATHEMATICS

Paper 2 – Mechanics

Thursday 17 May 2018 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 A particle A, of mass 0.2 kg, collides with a particle B, of mass 0.3 kg

Immediately before the collision, the velocity of A is $\begin{bmatrix} 4 \\ 12 \end{bmatrix}$ m s^{-1}

and the velocity of B is $\begin{bmatrix} -1 \\ -3 \end{bmatrix}$ m s^{-1}

As a result of the collision the particles coalesce to become a single particle.

Find the velocity of the single particle.

Circle your answer.

[1 mark]

$\begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix}$ m s^{-1} $\quad$ $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ m s^{-1} $\quad$ (circled) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ m s^{-1} $\quad$ $\begin{bmatrix} 3 \\ 9 \end{bmatrix}$ m s^{-1}

$v = [0.2(4, 12) + 0.3(-1, -3)] \div 0.5 = (0.5, 1.5) \div 0.5 = (1, 3)$ m s^{-1}

2 A train is travelling at maximum speed with its engine using its maximum power of 1800 kW

When travelling at this speed the train experiences a total resistive force of 40 000 N

Find the maximum speed of the train.

Circle your answer.

[1 mark]

22 m s^{-1} $\quad$ (circled) 45 m s^{-1} $\quad$ 54 m s^{-1} $\quad$ 90 m s^{-1}

$P = Fv \Rightarrow v = 1\,800\,000 \div 40\,000 = 45$ m s^{-1}

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3 The kinetic energy, E, of a compound pendulum is given by

$$E = \frac{1}{2} I\omega^2$$

where ω is the angular speed and I is a quantity called the moment of inertia.

3 (a) Show that for this formula to be dimensionally consistent then I must have dimensions ML^2, where M represents mass and L represents length.

[2 marks]

$[E] = ML^2T^{-2}$ (energy = force × distance), $[\omega] = T^{-1}$

$E = \frac{1}{2}I\omega^2 \Rightarrow [I] = [E] \div [\omega]^2$ ($\frac{1}{2}$ is dimensionless)

$[I] = ML^2T^{-2} \div (T^{-1})^2 = ML^2T^{-2} \div T^{-2}$

$\therefore [I] = ML^2$ as required

3 (b) The time, T, taken for one complete swing of a pendulum is thought to depend on its moment of inertia, I, its weight, W, and the distance, h, of the centre of mass of the pendulum from the point of suspension.

The formula being proposed is

$$T = kI^{\alpha}W^{\beta}h^{\gamma}$$

where k is a dimensionless constant.

Determine the values of α, β and γ.

[3 marks]

$[T] = T$, $[I] = ML^2$, $[W] = MLT^{-2}$, $[h] = L$, k dimensionless

$T = (ML^2)^{\alpha}(MLT^{-2})^{\beta}(L)^{\gamma} = M^{\alpha+\beta}\, L^{2\alpha+\beta+\gamma}\, T^{-2\beta}$

M: $\alpha + \beta = 0$

T: $-2\beta = 1 \Rightarrow \beta = -\frac{1}{2}$

M: $\alpha = -\beta \Rightarrow \alpha = \frac{1}{2}$

L: $2\alpha + \beta + \gamma = 0 \Rightarrow 1 - \frac{1}{2} + \gamma = 0 \Rightarrow \gamma = -\frac{1}{2}$

$\therefore \alpha = \frac{1}{2}, \quad \beta = -\frac{1}{2}, \quad \gamma = -\frac{1}{2}$

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4 Two smooth spheres A and B of equal radius are free to move on a smooth horizontal surface.

The masses of A and B are m and $4m$ respectively.

The coefficient of restitution between the spheres is e.

The spheres are projected directly towards each other, each with speed u, and subsequently collide.

4 (a) Show that the speed of B immediately after the impact with A is

$$\frac{u(3-2e)}{5}$$

[4 marks]

Take the direction of B's initial motion as positive.

Before: velocity of A = $-u$, velocity of B = u. After: v_A, v_B

CoM: $4mu - mu = mv_A + 4mv_B \Rightarrow 3u = v_A + 4v_B$ (1)

NLR: $v_A - v_B = e(u - (-u)) = 2ue$ (2)

(1) − (2): $5v_B = 3u - 2ue$

$v_B = u(3 - 2e)/5$

Since $0 \leq e \leq 1$, $3 - 2e > 0$ so $v_B > 0$

∴ speed of B = $u(3 - 2e)/5$ as required

4 (b) Find the speed of A in terms of u and e.

[2 marks]

$v_A = v_B + 2ue = u(3 - 2e)/5 + 10ue/5$

$v_A = u(3 + 8e)/5$

Speed of A = $u(3 + 8e)/5$

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4 (c) Comment on the direction of motion of the spheres after the collision, justifying your answer.

[2 marks]

Since $0 \leq e \leq 1$, both $3 - 2e > 0$ and $3 + 8e > 0$, so $v_A > 0$ and $v_B > 0$.

Hence both spheres move in the same direction after the collision (the original direction of B), so A reverses its direction and B keeps moving the same way.

4 (d) The magnitude of the impulse on B due to the collision is I.

Deduce that

$$\frac{8mu}{5} \leq I \leq \frac{16mu}{5}$$

[3 marks]

Impulse on B = change in momentum of B

$I = |4mv_B - 4mu| = |4mu(3 - 2e)/5 - 4mu|$

$= |4mu(3 - 2e - 5)/5| = |4mu(-2 - 2e)/5|$

$I = 8mu(1 + e)/5$

I increases with e, and $0 \leq e \leq 1$:

$e = 0$: $I = 8mu/5$

$e = 1$: $I = 8mu(2)/5 = 16mu/5$

$\therefore$ $8mu/5 \leq I \leq 16mu/5$ as required

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5 A car travels around a roundabout at a constant speed. The surface of the roundabout is horizontal.

The car has mass 990 kg and the path of the car is a circular arc of radius 48 metres.

A simple model assumes that the car is a particle and the only horizontal force acting on it as it travels around the roundabout is friction.

On a dry day typical values of friction, F, between the surface of the roundabout and the tyres of the car are

$$7300\,\text{N} \leq F \leq 9200\,\text{N}$$

5 (a) Using this model calculate a safe speed limit, **in miles per hour**, for the car as it travels around the roundabout.

Explain your reasoning fully.

Note that there are 1600 metres in one mile.

[4 marks]

Friction provides the force towards the centre of the circle:

$F = mv^2/r = 990v^2/48$

For safety the car must not skid for any typical value of friction, so use the smallest value, $F = 7300$ N.

$990v^2/48 = 7300$

$v^2 = 48 \times 7300 \div 990 = 353.93...$

$v = \sqrt{353.93...} = 18.813...$ m s^{-1}

In mph: $18.813... \times 3600 \div 1600 = 42.33...$ mph

Safe speed limit ≈ 42 mph

(In practice a speed limit a little below this, e.g. 40 mph, would be set.)

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5 (b) Gary assumes that on a wet day typical values for friction, F, are

$$5400\,\text{N} \leq F \leq 10\,000\,\text{N}$$

Comment on the validity of Gary's revised assumption.

[2 marks]

Wet conditions reduce friction, so the values should be lower than on a dry day.

The lower value 5400 N < 7300 N is reasonable, but 10000 N > 9200 N is not,

so Gary's assumption is wrong (invalid).

Turn over for the next question

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6 At a fairground a dodgem car is moving in a straight horizontal line towards a side wall that is perpendicular to the velocity of the car.

The speed of the car is $1.8\,\text{m}\,\text{s}^{-1}$

It collides with the side wall and rebounds along its original path with a speed of $1.2\,\text{m}\,\text{s}^{-1}$

The total mass of the dodgem car and the passengers is 250 kg

6 (a) Find the magnitude of the impulse on the car during the collision with the side wall.

[2 marks]

Taking the rebound direction as positive:

$I = mv - mu = 250(1.2) - 250(-1.8) = 300 + 450$

$I = 750$ N s

6 (b) A possible model for the magnitude of the force, F newtons, acting on the dodgem car due to its collision with the side wall is given by

$$F = kt(4 - 5t) \qquad \text{for } 0 \leq t \leq 0.8$$

6 (b) (i) Find the value of k.

[3 marks]

Impulse $= \int F\,dt \Rightarrow 750 = \int_0^{0.8} kt(4 - 5t)\,dt$

$\int_0^{0.8} (4t - 5t^2)\,dt = [2t^2 - 5t^3/3]_0^{0.8}$

$= 2(0.64) - 5(0.512)/3 = 1.28 - 0.8533\ldots = 32/75$

$750 = 32k/75$

$k = 750 \times 75 \div 32 = 28125/16 = 1757.8\ldots$

$k = 1800$ (2 s.f.)

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6 (b) (ii) Determine the maximum magnitude of the force predicted by the model.

[2 marks]

$F = k(4t - 5t^2)$, $dF/dt = k(4 - 10t) = 0 \Rightarrow t = 0.4$

(maximum since the graph of F is an inverted parabola)

$F_{max} = (28125/16) \times 0.4 \times (4 - 5(0.4)) = (28125/16) \times 0.8$

$F_{max} = 5625/4 = 1406.25$ N

Maximum force = 1400 N (2 s.f.)

Turn over for the next question

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7 **Use g as 9.8 m s^{-2} in this question.**

Dominic, a bungee jumper of mass 75 kg, has his ankles attached to one end of a cord. The other end of the cord is attached to a bridge which is 50 metres above the surface of a river.

The cord can be modelled as a light elastic cord of natural length 25 metres and modulus of elasticity 3200 N. Dominic is modelled as a particle.

Dominic steps off the bridge at the point where the cord is attached and falls vertically downwards.

7 (a) Find Dominic's speed at the point when the cord initially becomes taut.

[2 marks]

Free fall of 25 m until the cord becomes taut:

KE gained = PE lost: $\frac{1}{2}mv^2 = mgh$

$\frac{1}{2}(75)v^2 = 75(9.8)(25)$

$v^2 = 2 \times 9.8 \times 25 = 490$

$v = \sqrt{490} = 22.13... = 22$ m s^{-1} (2 s.f.)

7 (b) Determine whether or not Dominic enters the river and gets wet.

[5 marks]

Let x m be the maximum extension. At the lowest point

he is momentarily at rest, so PE lost = EPE gained:

$mg(25 + x) = \lambda x^2/(2l)$

$75(9.8)(25 + x) = 3200x^2/(2 \times 25)$

$735(25 + x) = 64x^2$

$64x^2 - 735x - 18375 = 0$

$x = [735 + \sqrt{(735^2 + 4(64)(18375))}] \div (2 \times 64)$

$= (735 + \sqrt{5244225}) \div 128 = (735 + 2290.0...) \div 128$

$x = 23.63...$ (reject the negative root)

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Q7(b) continued

Greatest distance below the bridge = 25 + 23.63...

= 48.6 m (3 s.f.)

48.6 m < 50 m

∴ Dominic does not reach the river and does not get wet.

7 (c) One limitation of this model is that Dominic is not a particle.

Explain the effect of revising this assumption on your answer to part **(b)**.

[2 marks]

Dominic has size: his ankles reach 48.6 m below the bridge but his body hangs below his ankles, so the lowest point of his body descends further than 48.6 m.

He has 50 − 48.6 = 1.4 m to spare, so if he is taller than about 1.4 m (which is likely) his head would enter the river and he would get wet.

END OF QUESTIONS

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