AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

AS
FURTHER MATHEMATICS

Paper 2 – Discrete

Thursday 17 May 2018 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 The table shows some of the outcomes of performing a modular arithmetic operation.

	2	3
2		1
3	1	

Which pair are operations that could each be represented by the table?

Tick (✓) **one** box.

- ☐ Addition mod 6 and multiplication mod 5
- ☐ Addition mod 6 and multiplication mod 6
- ☑ Addition mod 4 and multiplication mod 5
- ☐ Addition mod 4 and multiplication mod 6

[1 mark]

2 The binary operation $\otimes$ is given by

$a \otimes b = 3a(5 + b) \pmod 8$

where $a, b \in \mathbb{Z}$

Given that $2 \otimes x = 6$, which of the integers below is a possible value of x?

Circle your answer.

[1 mark]

(0) 1 2 3

Do not write outside the box

3 Alex and Sam are playing a zero-sum game.

The game is represented by the pay-off matrix for Alex.

		Sam		
	Strategy	$\mathbf{S_1}$	$\mathbf{S_2}$	$\mathbf{S_3}$
Alex	$\mathbf{A_1}$	2	2	3
	$\mathbf{A_2}$	0	3	5
	$\mathbf{A_3}$	−1	2	−2

3 (a) Explain why the value of the game is 2

[3 marks]

Row minima: A_1: 2, A_2: 0, A_3: −2

Maximum of the row minima (maximin) = 2

Column maxima: S_1: 2, S_2: 3, S_3: 5

Minimum of the column maxima (minimax) = 2

max(row minima) = 2 = min(column maxima)

so the game has a stable solution (saddle point at A_1, S_1)

and therefore the value of the game is 2

3 (b) Identify the play-safe strategy for each player.

[1 mark]

Alex: play-safe strategy is A_1

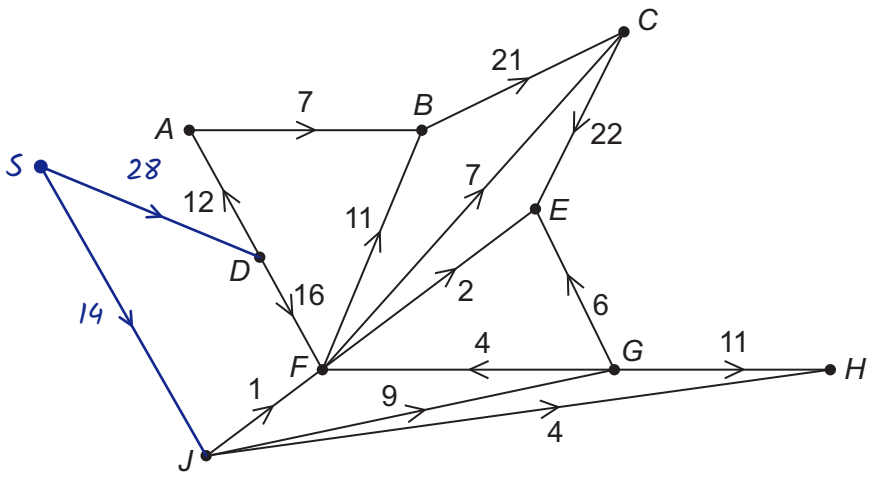
Sam: play-safe strategy is S_1

Turn over ▶

Do not write outside the box

4 The diagram shows a network of pipes.

Each pipe is labelled with its upper capacity in $cm^3\,s^{-1}$



4 (a) (i) Find the value of the cut given by $\{A, B, C, D, F, J\}$ $\{E, G, H\}$.

[1 mark]

Arcs from {A, B, C, D, F, J} to {E, G, H}: CE, FE, JG, JH

(GF goes from the sink side, so it is not counted)

Value of cut = 22 + 2 + 9 + 4 = 37

4 (a) (ii) State what can be deduced about the maximum flow through the network.

[1 mark]

Maximum flow ≤ 37 $cm^3\ s^{-1}$

(the maximum flow cannot be greater than the value of any cut)

Do not write outside the box

4 (b) (i) List the nodes which are sources of the network.

[1 mark]

D and J

4 (b) (ii) Add a supersource S to the network.

[1 mark]

Supersource S drawn on the diagram on page 4:
S→D capacity 28 (= 12 + 16), S→J capacity 14 (= 1 + 9 + 4)

4 (c) (i) List the nodes which are sinks of the network.

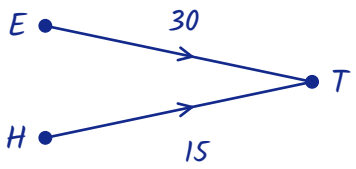
[1 mark]

E and H

4 (c) (ii) Add a supersink T to the network.

[1 mark]

Supersink T added:
E→T capacity 30 (= 22 + 2 + 6)
H→T capacity 15 (= 11 + 4)



Turn over for the next question

Do not write outside the box

5 A group of friends want to prepare a meal. They start preparing the meal at 6:30 pm

Activities to prepare the meal are shown in **Figure 1** below.

Figure 1

Label	Activity	Duration (mins)	Immediate predecessors
A	Weigh rice	1	–
B	Cook rice	18	*A*
C	Drain rice	1	*B*
D	Chop vegetables	10	–
E	Fry vegetables	12	*D*
F	Combine fried vegetables and drained rice	1	*C, E*
G	Prepare sauce ingredients	4	–
H	Boil sauce	12	*G*
I	Serve meal on plates	2	*F, H*

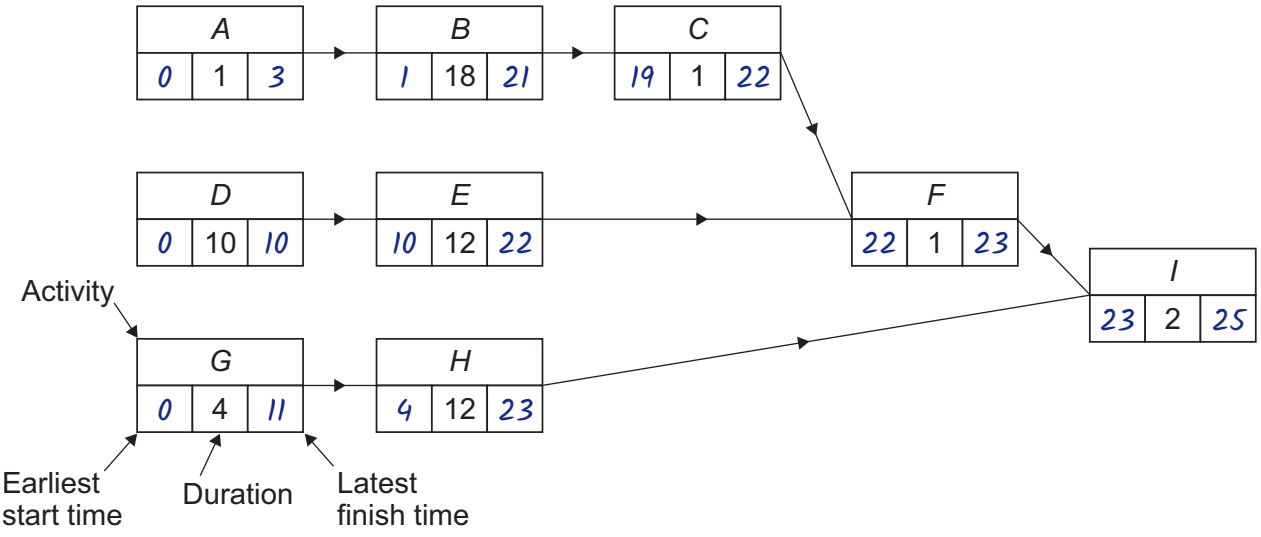
5 (a) (i) Use **Figure 2** shown below to complete **Figure 1** above.

[1 mark]

5 (a) (ii) Complete **Figure 2** showing the earliest start time and latest finishing time for each activity.

[2 marks]

Figure 2



Do not write outside the box

5 (b) (i) State the activity which must be started first so that the meal is served in the shortest possible time.

Fully justify your answer.

[2 marks]

D (chop vegetables)

The critical path is D → E → F → I, length 25 minutes.

D has earliest start 0 and latest finish 10, duration 10,

so its float = 10 − 10 − 0 = 0; D is critical.

D is the first activity on the critical path, so any delay in

starting D delays the meal (A has float 2 and G has float 7).

So D must be started first.

5 (b) (ii) Determine the earliest possible time at which the preparation of the meal can be completed.

[1 mark]

Minimum completion time = 25 minutes

6:30 pm + 25 minutes = 6:55 pm

Question 5 continues on the next page

Do not write outside the box

5 (c) The group of friends want to cook spring rolls so that they are served at the same time as the rest of the meal. This requires the additional activities shown in **Figure 3**.

Figure 3

Label	Activity	Duration	Immediate predecessors
J	Switch on and heat oven	10.25 mins	–
K	Put spring rolls in oven and cook	8.25 mins	J
L	Transfer spring rolls to serving dish	0.5 mins	K

It takes 15 seconds to switch on the oven.

The oven must be allowed to heat up for 10 minutes before the spring rolls are put in the oven.

It takes 15 seconds to put the spring rolls in the oven.

The spring rolls must cook in the hot oven for 8 minutes.

It takes 30 seconds to transfer the spring rolls to a serving dish.

5 (c) (i) Complete **Figure 3** above.

[1 mark]

5 (c) (ii) Determine the latest time at which the oven can be switched on in order for the spring rolls to be served at the same time as the rest of the meal.

[2 marks]

Duration of J, K, L = 10.25 + 8.25 + 0.5 = 19 minutes

The rest of the meal is completed 25 minutes after 6:30 pm

Latest start for J = 25 – 19 = 6 minutes after 6:30 pm

Latest time to switch on the oven = 6:30 pm + 6 minutes

= 6:36 pm

Do not write outside the box

Turn over for the next question

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

6 An animal sanctuary has a rainwater collection site.

The manager of the sanctuary is installing a pipe system to connect the rainwater collection site to five other sites in the sanctuary.

Each site does **not** need to be connected directly to the rainwater collection site.

There are nine possible routes between the sites that are suitable for water pipes.

The distances, in metres, of the nine possible routes are given in the table below.

From/To	**Henhouse (*H*)**	**Goatshed (*G*)**	**Kennels (*K*)**	**Cattery (*C*)**
Rainwater collection site (*R*)	840	810	520	370
Cattery (*C*)	–	680	610	
Duckpond (*D*)	480	310		
Goatshed (*G*)	150			

Water pipe costs 60 pence per metre.

Find the minimum cost of connecting all the sites to the rainwater collection site.

Fully justify your answer.

[5 marks]

Need a minimum spanning tree: use Kruskal's algorithm.

Arcs in order of increasing length:

GH 150 ✓ add

DG 310 ✓ add

RC 370 ✓ add

DH 480 ✗ reject (would form cycle D G H)

RK 520 ✓ add

CK 610 ✗ reject (would form cycle R C K)

CG 680 ✓ add (5 arcs joining 6 sites, so the tree is complete)

Minimum spanning tree: GH, DG, RC, RK, CG

Total length = 150 + 310 + 370 + 520 + 680 = 2030 m

Minimum cost = 2030 × £0.60 = £1218

Do not write outside the box

Turn over ▶

Do not write outside the box

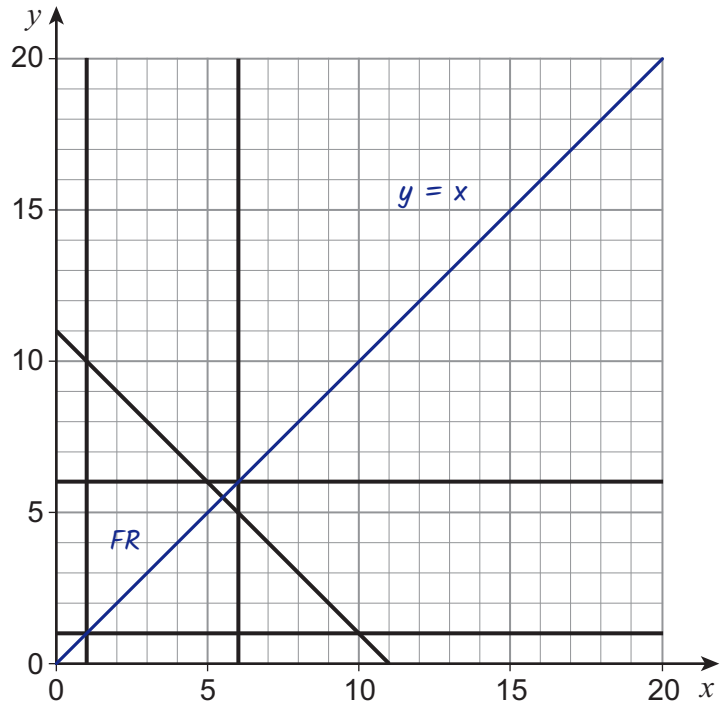
7 A linear programming problem has the constraints

$$1 \leq x \leq 6$$
$$1 \leq y \leq 6$$
$$y \geq x$$
$$x + y \leq 11$$

7 (a) (i) Complete **Figure 4** to identify the feasible region for the problem.

[2 marks]

Figure 4



7 (a) (ii) Determine the maximum value of $5x + 4y$ subject to the constraints.

[2 marks]

Vertices of the feasible region: (1, 1), (1, 6), (5, 6), (5.5, 5.5)

At (1, 1): 5(1) + 4(1) = 9 At (1, 6): 5(1) + 4(6) = 29

At (5, 6): 5(5) + 4(6) = 49

At (5.5, 5.5): 5(5.5) + 4(5.5) = 49.5

Maximum value of 5x + 4y = 49.5 (at x = 5.5, y = 5.5)

Do not write outside the box

7 (b) The simple-connected graph G has seven vertices.

The vertices of G have degree 1, 2, 3, v, w, x and y

7 (b) (i) Explain why $x \geq 1$ and $y \geq 1$

[1 mark]

G is connected, so every vertex is joined to at least one other vertex: no vertex can have degree 0.

Hence $x \geq 1$ and $y \geq 1$

7 (b) (ii) Explain why $x \leq 6$ and $y \leq 6$

[1 mark]

G is simple (no loops or multiple edges), so each vertex can be joined at most once to each of the other 6 vertices.

Hence $x \leq 6$ and $y \leq 6$

7 (b) (iii) Explain why $x + y \leq 11$

[2 marks]

If $x = 6$ and $y = 6$, both of these vertices would be adjacent to all the other 5 vertices, so every other vertex would have degree at least 2. But G has a vertex of degree 1: contradiction.

So x and y cannot both be 6: one of them is at most 6 and the other is at most 5.

Hence $x + y \leq 6 + 5 = 11$

7 (b) (iv) State an additional constraint that applies to the values of x and y in this context.

[1 mark]

x and y must be integers (degrees are whole numbers)

Turn over ▶

Do not write outside the box

7 (c) The graph G also has eight edges. The inequalities used in part **(a)(i)** apply to the graph G.

7 (c) (i) Given that $v + w = 4$, find all the feasible values of x and y.

[3 marks]

Sum of degrees = 2 × number of edges = 2 × 8 = 16

1 + 2 + 3 + v + w + x + y = 16

6 + 4 + x + y = 16 ⇒ x + y = 6

Using y ≥ x, 1 ≤ x ≤ 6, 1 ≤ y ≤ 6, x and y integers:

x = 1, y = 5

x = 2, y = 4

x = 3, y = 3

(x = 4, y = 2 etc. are not feasible since y ≥ x)

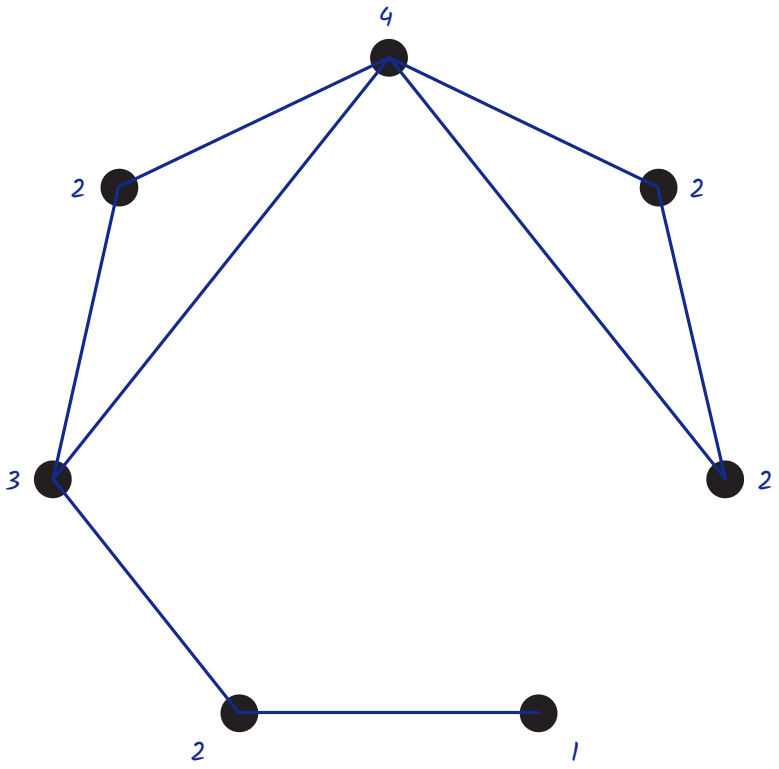
Do not write outside the box

7 (c) (ii) It is also given that the graph G is semi-Eulerian.

On **Figure 5**, draw G.

[2 marks]

Figure 5



Degrees: 1, 2, 2, 2, 2, 3, 4 (two odd vertices, 8 edges)

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Copyright information

For confidentiality purposes, from the November 2015 examination series, acknowledgements of third party copyright material will be published in a separate booklet rather than including them on the examination paper or support materials. This booklet is published after each examination series and is available for free download from www.aqa.org.uk after the live examination series.

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team, AQA, Stag Hill House, Guildford, GU2 7XJ.

Copyright © 2018 AQA and its licensors. All rights reserved.