

AQA A-LEVEL PHYSICS • 7408

PHYSICS



EXAM QUESTION PACK

3.11.1 ROTATIONAL DYNAMICS

SCORE

/97

PERCENTAGE

%

STEMUP

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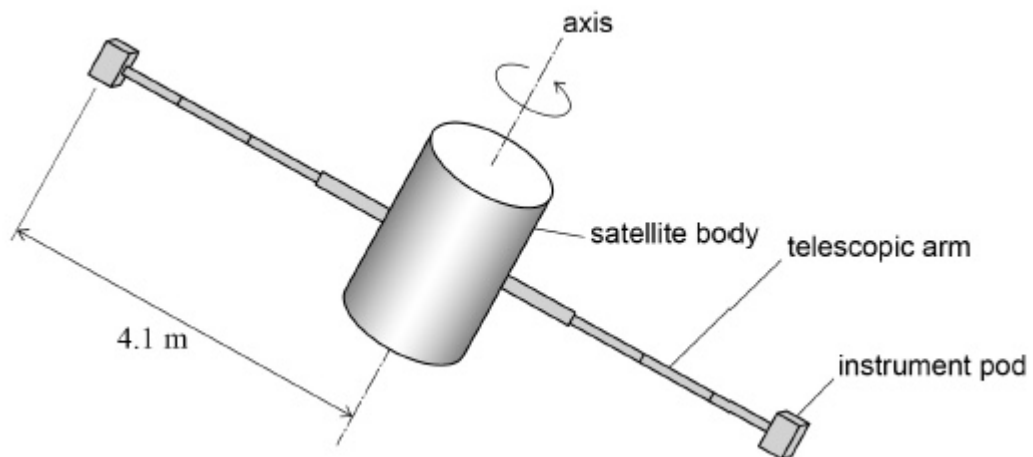
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Q1.

- (a) State the law of conservation of angular momentum.

(1)

- (b) The diagram shows an orbiting satellite fitted with two small instrument pods attached to the ends of telescopic arms. The arms can be extended or retracted by a motor in the body of the satellite.



With the telescopic arms fully extended, the centre of mass of each instrument pod is at a radius of 4.1 m from the axis of rotation.

moment of inertia of satellite body about axis = 71 kg m^2

mass of each instrument pod = 5.0 kg

The mass of the telescopic arms is negligible.

Show that the total moment of inertia of the satellite with the arms fully extended is 240 kg m^2

(2)

- (c) The satellite is initially rotating slowly about its axis with the arms fully extended. The arms are slowly retracted so that the instrument pods move closer to the body of the satellite.

State and explain the change in the angular speed of the satellite as the arms are retracted.

(3)

- (d) The satellite is initially rotating at 1.3 rad s^{-1} with the telescopic arms fully extended.
When fully retracted the instrument pods are at a radius of 0.74 m from the axis.
The satellite contains sensitive equipment that may be damaged if the rotational speed exceeds 4.2 rad s^{-1}

Deduce whether the arms can be retracted fully without the satellite exceeding its maximum permitted angular speed.

(3)

(Total 9 marks)

Q2.

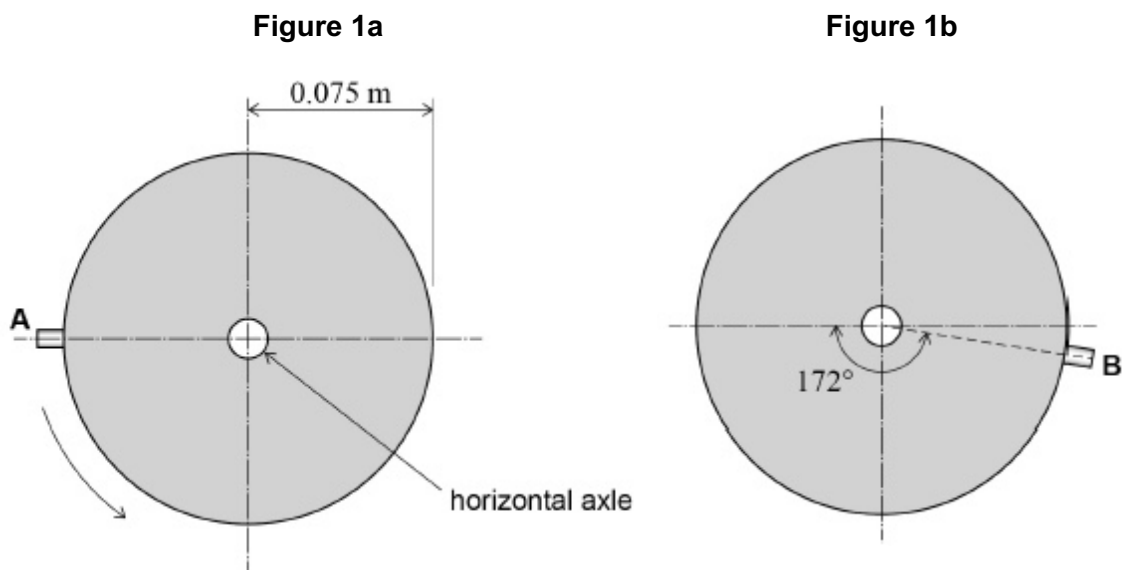
- (a) State **one** function of a flywheel.

(1)

- (b) A student does an experiment to determine the frictional torque acting at the bearings of a steel flywheel. The flywheel has a radius of 0.075 m and is perfectly balanced.

The student places a small magnet of mass 0.020 kg at point **A** on the

circumference of the flywheel on a horizontal line through the axis of rotation as shown in **Figure 1a**. The student releases the flywheel. The flywheel first comes to rest when it has moved through an angle of 3.00 rad (172°), with the magnet now in position **B** as shown in **Figure 1b**.



The loss in gravitational potential energy of the magnet equals the work done against the frictional torque acting at the bearings.

Show that the frictional torque is about $7 \times 10^{-4} \text{ N m}$

(3)

- (c) The student goes on to determine the moment of inertia of the flywheel. The magnet is removed and the flywheel is made to spin. Measurements show that the flywheel makes 573 rotations as its angular speed reduces uniformly from 25.0 rad s^{-1} to zero. Assume the frictional torque at the bearings is constant and the same as in question (b).

Determine the moment of inertia of the flywheel about its axis of rotation.

moment of inertia = _____ kg m²

(3)

(Total 7 marks)

Q3.

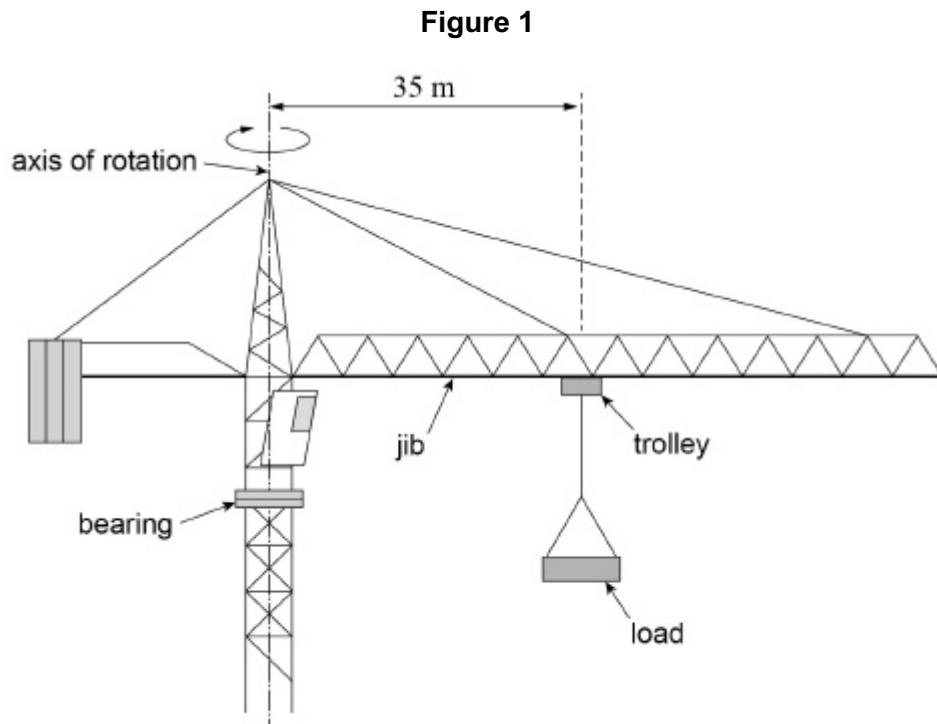
- (a) There is an analogy between quantities in rotational and translational dynamics.

Complete the table, stating in words the quantities in rotational dynamics that are analogous to force and mass in translational dynamics.

Translational dynamics	Rotational dynamics
force	
mass	

(2)

Figure 1 shows a side view of the jib of a tower crane. The load is supported by a trolley which can move along the jib. The jib consists of all the parts of the crane above the bearing, but excluding the trolley and load.



The moment of inertia of the jib about the axis of rotation = $2.6 \times 10^7 \text{ kg m}^2$

Mass of trolley and load = $2.2 \times 10^3 \text{ kg}$

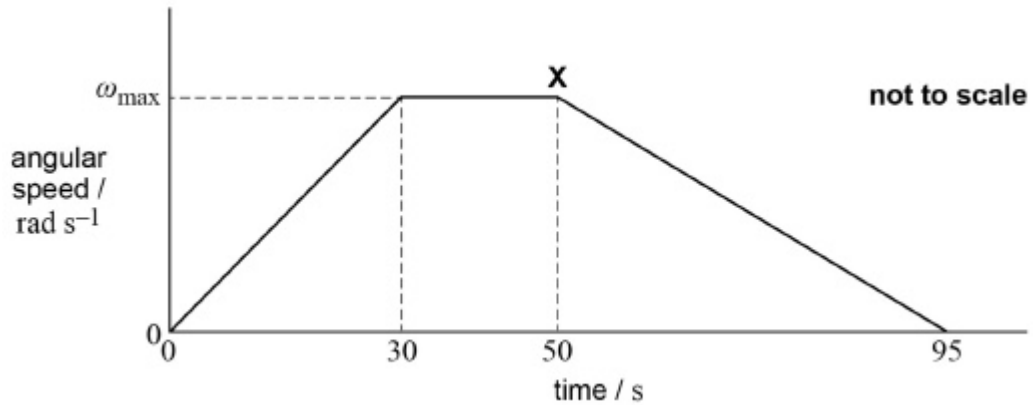
- (b) The load is at a distance of 35 m from the axis of rotation.

Show that the total moment of inertia of the jib, and the trolley and load, about the axis of rotation is about $3 \times 10^7 \text{ kg m}^2$.

(1)

- (c) **Figure 2** shows the variation of angular speed of the jib as it turns through an angle of 4.7 rad (270°) in a total time of 95 s . The trolley and load remain at a distance of 35 m from the axis.

Figure 2



Calculate the maximum angular speed ω_{max} of the jib.

maximum angular speed = _____ rad s^{-1}

(2)

- (d) At time **X** in **Figure 2** the motor that is driving the jib is disengaged. A constant braking torque is then applied to bring the jib to a standstill from its maximum angular speed.

The crane driver repeats the movement of the jib with the same load at 35 m from the axis of rotation. Up to time **X** the motion is the same as before. From time **X** the trolley is driven at a steady speed away from the axis as the jib continues to rotate until the jib comes to a standstill.

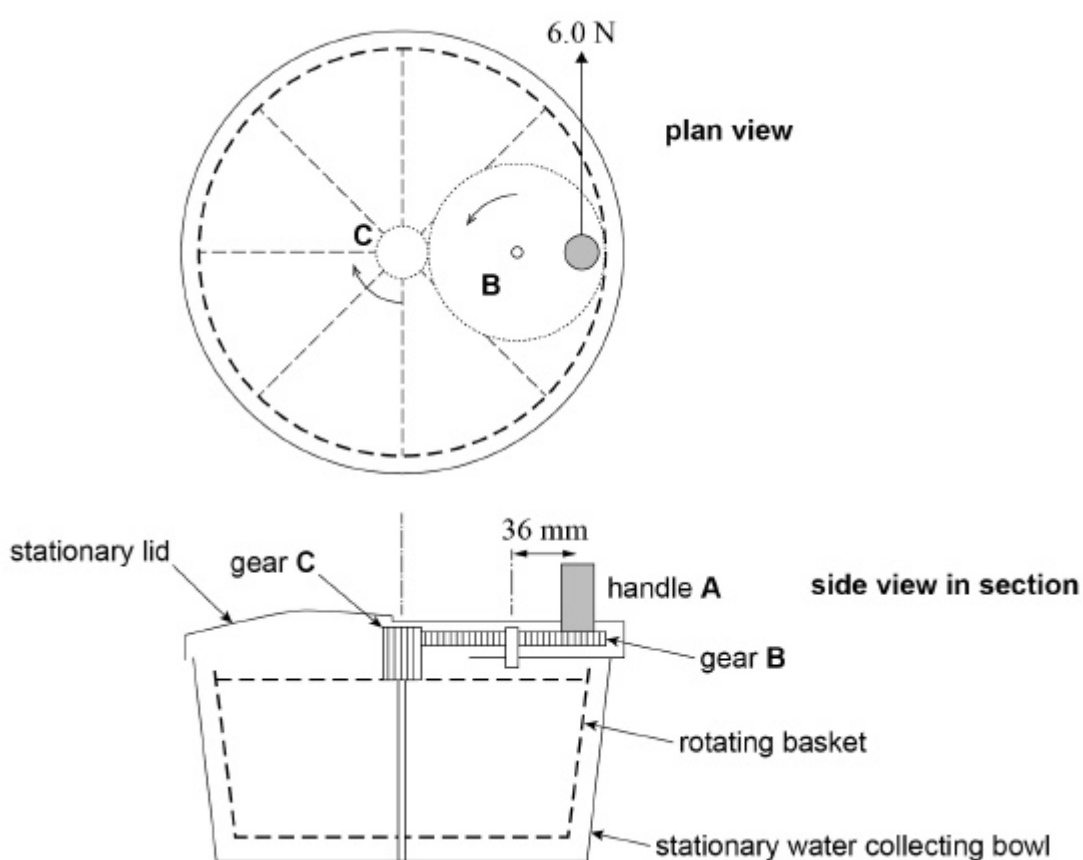
Assume the braking torque remains the same as before.

Discuss how the motion of the trolley affects the time taken for the jib to come to a standstill.

(3)
(Total 8 marks)

Q4.

The diagram shows the basic principle of operation of a hand-operated salad spinner used to dry washed salads.



The salad is placed in the basket and the lid is attached.

When handle **A** is turned the basket and its contents spin rapidly. Water on the salad is driven through holes in the basket into the stationary water collecting bowl. The pivot for gear **B** is fixed to the lid. This pivot and the lid do not move. When gear **B** rotates, gear **C** also rotates but at a greater angular speed. Gear **C** is fixed to the basket and rotates it.

A force of 6.0 N is applied to handle **A** as shown. Handle **A** is at a radius of 36 mm from its centre of rotation.

- (a) Calculate the input torque.

torque = _____ N m

(1)

- (b) Gear **C** rotates four times for every one revolution of gear **B**.

Deduce whether it is possible for the torque on gear **C** to be greater than one quarter of the input torque.

(2)

- (c) It takes 2.1 s for the empty basket to reach an angular speed of 76 rad s^{-1} . The torque on gear **C** is a constant 0.054 N m during this time. Frictional losses are negligible.

Calculate the moment of inertia of the basket about its axis of rotation.

moment of inertia = _____ kg m^2

(2)

- (d) The gears are made from polymer (plastic). An early version of this salad spinner suffered from damaged gear teeth.

Explain with reference to angular impulse why a great force is put on the gear teeth if the user tries to stop the loaded basket too quickly using the handle.

(3)

(Total 8 marks)

Q5.

- (a) A metal flywheel is rotating on frictionless bearings. The temperature is increased so that the flywheel expands.

Consider **each** of the following statements and indicate with a tick (✓) if it is correct.

	✓ if correct
The moment of inertia will decrease.	
The angular velocity will decrease.	
The angular momentum will be unaltered.	

(1)

An electric motor drives a machine which stamps out shapes from sheet steel. The machine is fitted with a flywheel of moment of inertia 25 kg m^2 which is accelerated uniformly until it is rotating at 640 rev min^{-1} . The machine then starts a stamping operation which reduces the flywheel's angular speed to 360 rev min^{-1} .

- (b) Explain why a flywheel is fitted between the motor and the stamping machine.

(2)

- (c) Calculate the energy needed for the stamping operation.

energy _____ J

(2)

- (d) Immediately after the stamping operation the flywheel is accelerated to its initial speed of 640 rev min^{-1} in a time of 5.0 s. The next stamping operation then begins.

Calculate the constant torque provided by the motor during this 5.0 s. Assume that the bearing frictional torque is negligible.

torque = _____ N m

(2)

- (e) Calculate the minimum power output of the electric motor required.

power _____ W

(2)

- (f) The flywheel is a solid disc. It is to be replaced with a flywheel which gives a smaller angular speed change for each stamping operation.

Two replacement flywheels, **A** and **B**, are available and information about them and the original flywheel is given in the table below.

flywheel	density of material / kg m ⁻³	thickness of disc / m	outer radius / m
original	7800	0.10	0.38
A	8800	0.20	0.30
B	2900	0.10	0.50

Deduce which flywheel, **A** or **B**, would be more suitable. Explain your choice.

The moment of inertia I of a solid disc of mass m and outer radius r about an axis through the centre is given by

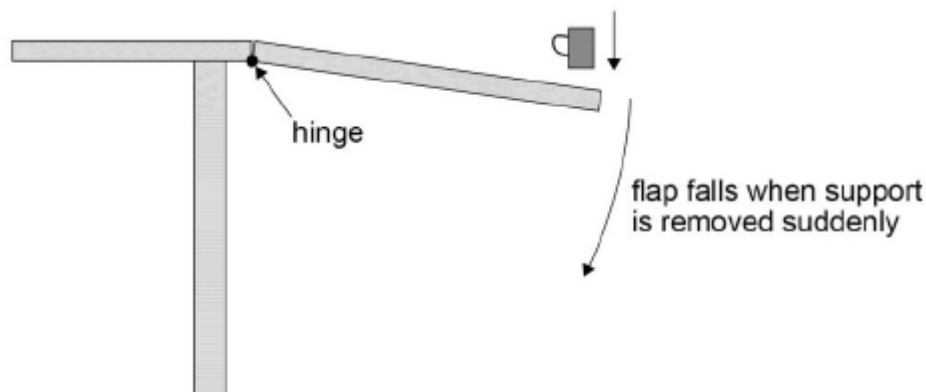
$$I = \frac{1}{2} m r^2$$

(3)

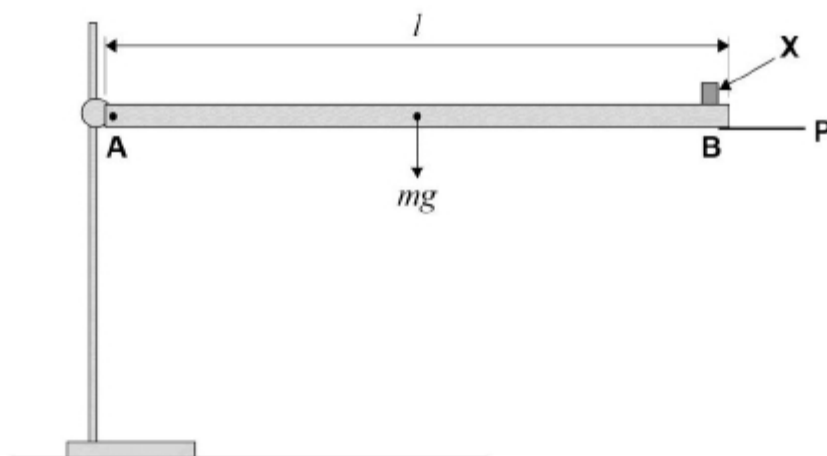
(Total 12 marks)

Q6.

A student is told that if a small cup of coffee is placed near the edge of a table flap, the cup and the flap will lose contact if the flap support is suddenly removed. This is shown in **Figure 1**.

Figure 1

The student does not believe this to be true so decides to model the arrangement using a metre ruler free to pivot about one end **A**, with a small mass **X** resting on the ruler at the other end **B**. The arrangement is shown in **Figure 2**. The mass of **X** is negligible compared to the mass of the ruler. The metre ruler is held in the horizontal position by a support **P** which is quickly removed. A video is taken of the subsequent motion of the ruler and mass.

Figure 2

Assume the ruler is a thin uniform beam of mass m and length l .

- (a) Derive an expression for the torque T acting on the ruler at the moment of release.

(1)

- (b) The moment of inertia I of the metre ruler about the axis through **A** is given by

$$I = \frac{m}{3} l^2$$

Show that the angular acceleration α of the ruler at the moment of release is given by

$$\alpha = \frac{3g}{2l}$$

(2)

- (c) The linear acceleration a of a point on a rotating rigid body at a distance r from the axis of rotation is related to the angular acceleration α by

$$a = r \times \alpha$$

Explain why this causes the small mass to lose contact with the metre ruler as soon as the ruler is released.

(2)

- (d) Estimate how far from **A** the small mass must be placed to ensure it just maintains contact with the ruler when the ruler is released.

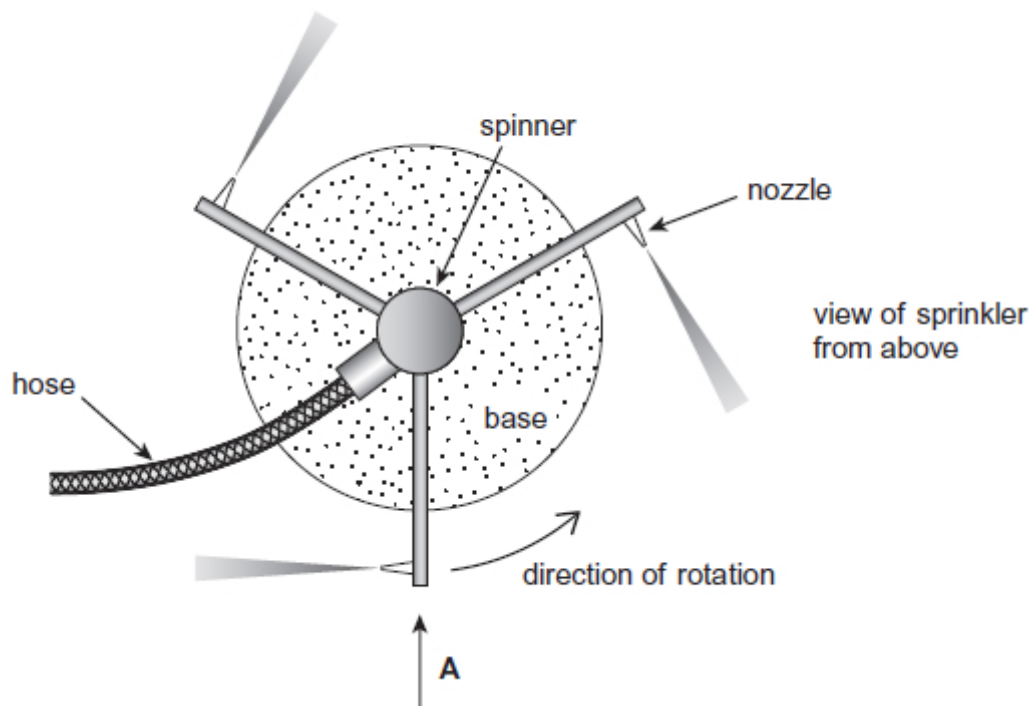
(1)

(Total 6 marks)

Q9.

A garden sprinkler consists of a fixed base and a rotating spinner having three arms as shown in **Figure 1**.

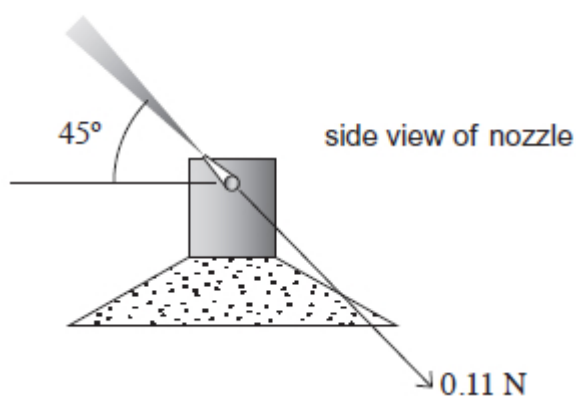
Figure 1



At the end of each arm is a nozzle at 90° to the arm and inclined at 45° to the horizontal. Water flows in jets at a constant rate from these nozzles when the hose water tap is turned on.

Figure 2 shows a side view of one of the nozzles viewed in the direction of arrow **A** in **Figure 1**.

Figure 2



The water jets produce reaction forces that act on the arms which cause the spinner to rotate. The base remains fixed in position. In operation, the spinner rotates at a constant rate of 240 rev min^{-1} . The nozzles rotate in a horizontal circle of radius 120 mm.

- (a) Each water jet exerts a constant force of 0.11 N on its arm at 45° to the horizontal.

Show that the torque exerted on the spinner by the jets of water is about $3 \times 10^{-2} \text{ N m}$.

(2)

- (b) (i) Explain why, when the water tap is turned on, the spinner accelerates initially but then reaches a constant angular speed. Assume that, when the tap is turned on, the flow-rate of the water from the jets is constant.

(2)

- (ii) Calculate the power dissipated by the frictional torque acting between the spinner and the fixed base when the sprinkler is rotating at 240 rev min^{-1} .

power = _____ W

(2)

- (c) When the water is suddenly turned off all the kinetic energy of the spinner and arms is dissipated as heat due to work done by the frictional torque and the spinner makes a further 13 rotations before coming to rest. Assume uniform deceleration.

- (i) Calculate the time taken for the spinner to come to rest.

time = _____ s

(2)

- (ii) Show that the kinetic energy of the spinner when rotating at its operating speed is about 2 J.

(1)

- (iii) Determine the moment of inertia of the spinner about its axis of rotation.

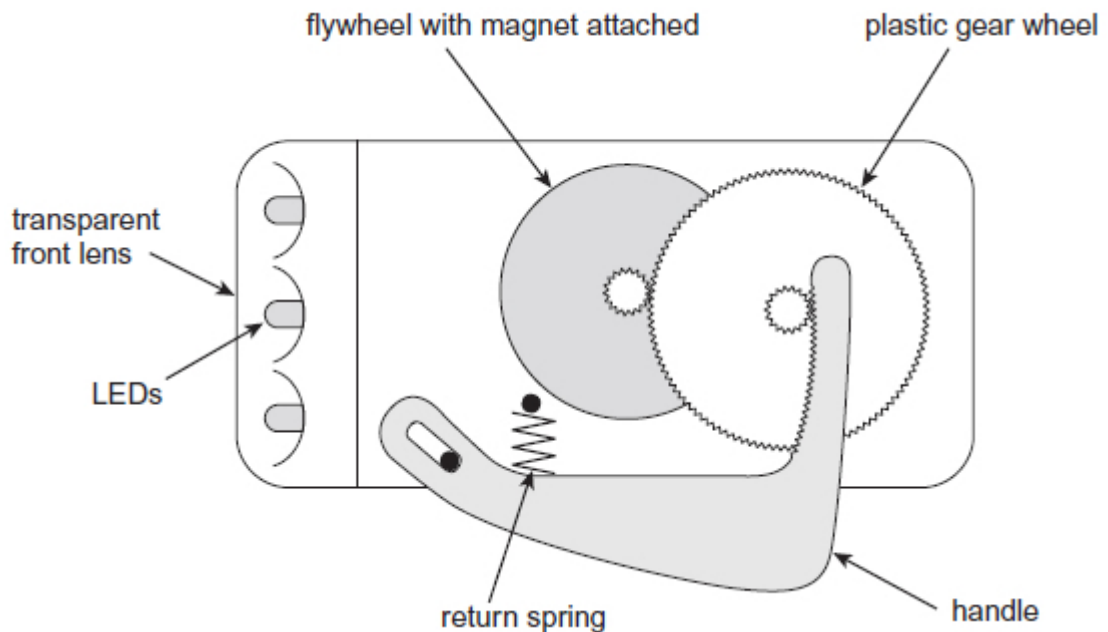
moment of inertia = _____ kg m²

(1)

(Total 10 marks)

Q10.

The diagram shows the mechanism of a dynamo torch for providing light without the use of batteries. When the handle is squeezed the gears rotate the flywheel increasing the kinetic energy stored in the flywheel. On the same axle as the flywheel is a magnet which generates an emf in surrounding coils as it rotates. The magnet, coils and electrical connections are not shown.



When the handle is released the mechanism disengages and a spring returns the handle to its original position. During this time the gears continue to turn. The energy of the flywheel enables it to continue to rotate for several seconds.

A designer wishes to enable the flywheel to store more energy and thus rotate for longer after the handle has been released.

The radius of the flywheel is limited by the overall size of the torch and cannot be increased.

Describe and explain other changes that can be made to the mechanism and flywheel to store more energy. Your answer should include consideration of:

- the flywheel's shape
- the material from which it is made
- changes to the mechanism.

The quality of your written communication will be assessed in your answer.

(Total 6 marks)

Q11.

A roundabout in a fairground requires an input power of 2.5 kW when operating at a constant angular velocity of 0.47 rad s^{-1} .

- (a) Show that the frictional torque in the system is about 5 kN m.

(3)

- (b) When the power is switched off, the roundabout decelerates uniformly because the frictional torque remains constant. The roundabout takes a time of 34 s to come to rest.

- (i) Calculate the moment of inertia of the roundabout.
Give an appropriate unit for your answer.

moment of inertia _____ unit _____

(3)

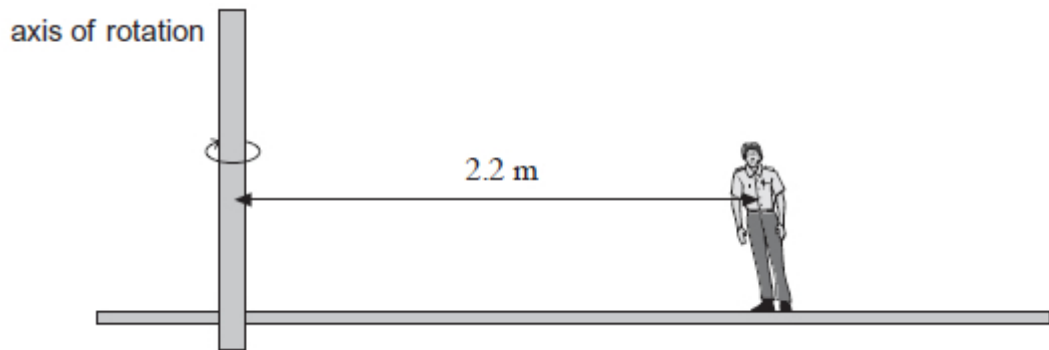
- (ii) Calculate the number of revolutions that are made before the roundabout comes to rest.

number of revolutions _____

(3)

- (c) An operator of mass 65 kg is standing on the roundabout when the roundabout is rotating at an angular velocity of 0.47 rad s^{-1} . His centre of mass is 2.2 m from the axis of rotation. The diagram shows that his body leans towards the centre of the

path.



- (i) Calculate the centripetal force needed for the operator to remain at this radius on the roundabout.

centripetal force _____ N

(2)

- (ii) State the origin of this centripetal force and suggest why the operator has to incline his body towards the centre of rotation to avoid falling over.

You may draw the forces that act on the operator in the diagram to help your answer.

(2)

- (iii) While the roundabout is moving, the operator drops a coin.

Which statement correctly describes and explains what happens to the coin?
Tick (✓) the correct answer in the right-hand column.

	Tick (✓)
There is no longer a centripetal force acting, so the coin falls vertically downwards and lands on the roundabout directly below the point at which it was dropped.	
The centripetal force causes the coin to have a horizontal component of velocity towards the centre of the roundabout, so that it follows a trajectory towards	

the centre of the roundabout.	
There is no longer a centripetal force acting, so there is a horizontal component of the coin's velocity directed away from the centre of the roundabout and it follows a trajectory directly away from the centre.	
There is no longer a centripetal force acting, so the coin has a horizontal component of its velocity tangential to its original path on the roundabout and it follows a trajectory along this tangent.	

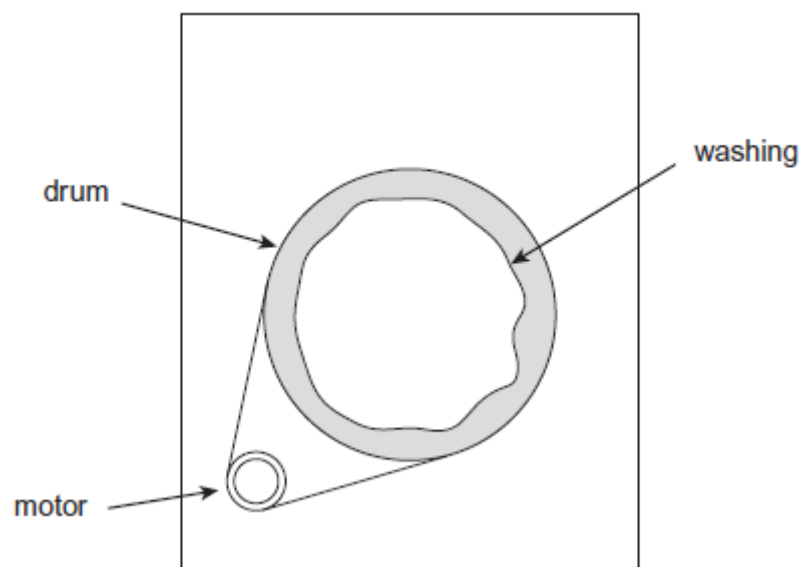
(1)

(Total 14 marks)

Q12.

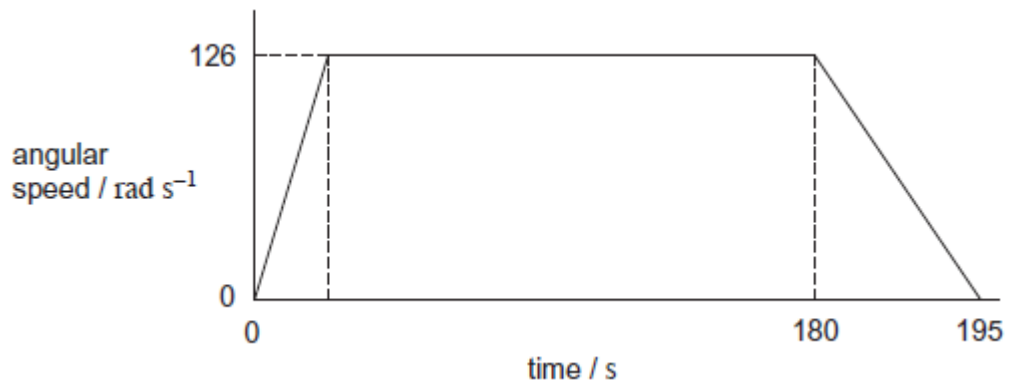
During the spin part of a washing machine programme the maximum rotational speed is 126 rad s^{-1} ($1200 \text{ rev min}^{-1}$). The moment of inertia of the drum and washing at the start of the spin part of the cycle is 0.565 kg m^2 . Assume that the wet washing is evenly distributed around the drum as shown in **Figure 1**.

Figure 1



- (a) **Figure 2** shows the variation of angular speed with time for the spin part of the cycle. The graph is not drawn to scale.

Figure 2



The motor provides a torque of 8.80 N m to accelerate the drum to 126 rad s^{-1} . The drum rotates at 126 rad s^{-1} until near the end of the spin cycle, when the drum decelerates uniformly to zero angular speed in 15 seconds. Friction at the bearings may be neglected. Assume that during acceleration the moment of inertia of the drum and washing remains constant.

- (i) Show that the drum accelerates for about 8 s.

(2)

- (ii) Calculate the total number of revolutions made by the drum during the 195 s shown in **Figure 2**.

number of revolutions = _____

(3)

- (b) (i) In practice, at the start of the cycle the acceleration will not remain constant. Draw on **Figure 2** a line to show how the initial part of the graph will change.

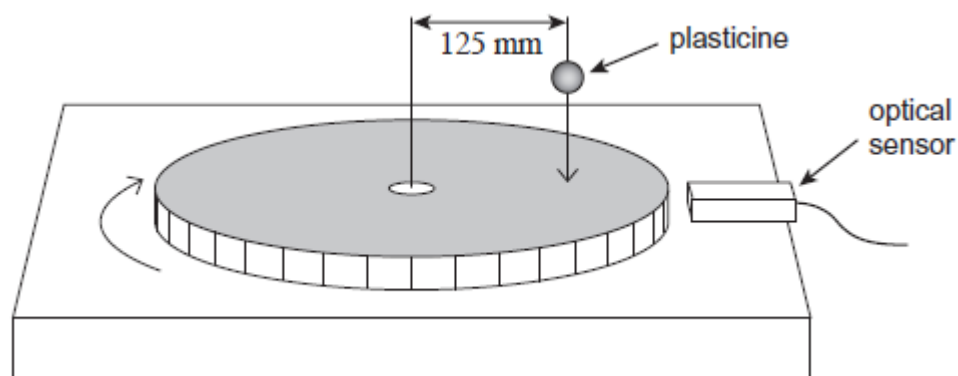
(1)

- (ii) Explain your reasons for the line you have drawn.

(2)
(Total 8 marks)

Q13.

A student carries out an experiment to determine the moment of inertia of a turntable. The diagram shows the turntable with a small lump of plasticine held above it. An optical sensor connected to a data recorder measures the angular speed of the turntable.



The turntable is made to rotate and then it rotates freely. The lump of plasticine is dropped from a small height above the turntable and sticks to it. Results from the experiment are as follows.

mass of plasticine = 16.0 g

radius at which plasticine sticks to the turntable = 125 mm

angular speed of turntable immediately before plasticine is dropped = 3.46 rad s^{-1}

angular speed of turntable immediately after plasticine is dropped = 3.31 rad s^{-1}

The student treats the plasticine as a point mass.

- (a) Explain why the turntable speed decreases when the plasticine sticks to it.

(3)

- (b) Use the results of the experiment to determine the moment of inertia of the turntable.

Give your answer to an appropriate number of significant figures.

moment of inertia _____ kg m² (3)

- (c) (i) Calculate the change in rotational kinetic energy of the turntable and plasticine from the instant before the plasticine is dropped until immediately after it sticks to the turntable.

change in kinetic energy _____ J (2)

- (ii) Explain the change in rotational kinetic energy.

(1)
(Total 9 marks)

Mark schemes

Q1.

- (a) The (total) angular momentum (of a system) remains constant provided no external torque acts (on the system) ✓

Must see 'angular'. Condone 'is conserved' for 'is constant'

Allow ang momtm before equals/is same as ang momtm after OR initial ang momtm = final ang momtm

Allow $I\omega$ is constant if symbols explained

Do not allow 'force' in place of 'torque'

1

- (b) Use of $I = I_{\text{BODY}} + 2 \times mr^2$ ✓

$$I_1 = (71 + 2 \times 5.0 \times 4.1^2) = 239 \text{ kg m}^2 \text{ ✓}$$

$$(\approx 240 \text{ kg m}^2)$$

For 2 marks 239 must be seen

2

- (c) M of I decreases ✓

Because more mass closer to axis **OR** (for pods) $I = (\Sigma)mr^2$ with r less ✓

$I\omega$ / angular momentum remains constant/is conserved

(So as I decreases) ω must increase ✓

Condone 'inertia' for 'moment of inertia'

2nd mark is for the reason why I is decreasing. Answer must relate to pods or masses getting closer to the axis. 'radius decreasing' on its own is not enough. Accept: pods get closer to axis/body as this implies mass is getting closer.

Both points needed for 3rd mark

3

- (d) (Applies conservation of angular momentum/ $I_1\omega_1 = I_2\omega_2$)

$$I_1\omega_1 = 240 \times 1.3 = (312 \text{ (N m s)}) \text{ ✓}$$

$$312 = (71 + 2 \times 5.0 \times 0.74^2) \omega_2$$

$$\omega_2 = 4.08 \text{ rad s}^{-1} \text{ ✓}$$

Therefore max speed not reached OR arms can be retracted safely ✓

OR

$$I_1\omega_1 = 240 \times 1.3 = (312 \text{ (N m s)}) \text{ ✓}$$

$$312 = (71 + 2 \times 5.0 \times r_2^2) 4.2$$

$$r_2 = 0.57 \text{ m} \checkmark$$

So with r at circumference max speed not reached OR arms can be retracted safely $\checkmark$

OR

$$I_1\omega_1 = 240 \times 1.3 = (312 \text{ (N m s)}) \checkmark$$

$$312 = 4.2 I_2 \text{ at safety limit}$$

$$I_2 = 74(.3) \text{ kg m}^2 \checkmark$$

$$\text{Actual } I_2 = 76.5 \text{ kg m}^2$$

Therefore max speed not reached OR arms can be retracted safely $\checkmark$

Using 239 kg m² instead of 240 kg m² leads to

$$\omega^2 = 4.06 \text{ rad s}^{-1}$$

$$\text{Useful: } I_2 = 76.5 \text{ kg m}^2$$

Only credit last mark if conservation of angular momentum is used

Allow a judgement based on incorrect working (eg AE) provided conservation of angular momentum is used

Using 239 kg m² instead of 240 kg m² leads to

$$r_2 = 0.55 \text{ m}$$

3

[9]

Q2.

- (a) To smooth out (fluctuations in) rotational speed $\checkmark$

OR to store (rotational kinetic) energy $\checkmark$

OR to smooth (fluctuations in) torque/power $\checkmark$

Any named form of energy must be (rotational) kinetic

Do not allow an application (eg regenerative braking) unless one of the answers shown alongside is included

1

- (b) Use of $0.075 \sin 8^\circ$ **OR** $0.075 \tan 8^\circ$ **OR** $0.075 (\pi - 3)$ to calculate h

$$h = 1.04 \times 10^{-2} \text{ (m)} \text{ **OR** } 1.05 \times 10^{-2} \text{ (m)} \text{ **OR** } 1.06 \times 10^{-2} \text{ (m)} \checkmark$$

$$mgh = 0.020 \times 9.81 \times 1.04 \times 10^{-2} = 2.04 \times 10^{-3} \text{ (J)} \checkmark$$

$$T\theta = 2.04 \times 10^{-3} \text{ (J)}$$

$$T = 2.04 \times 10^{-3} / 3.00 = 6.80 \times 10^{-4} \text{ Nm} \checkmark$$

1st mark for calculating h

2nd mark for calculating mgh .

3rd mark for dividing mgh by 3.00 rad.

Use of $\tan$ gives $h = 1.05 \times 10^{-2} \text{ (m)}$
 Use of arc length gives $h = 1.06 \times 10^{-2} \text{ (m)}$
 3rd mark only awarded for arriving at correct answer to more than 1 sig fig

3

- (c) Attempt to use $0 = \omega_1^2 - 2\alpha\theta$ ✓

Or $\theta = 573 \times 2\pi = 3600 \text{ rad}$ ✓

Leading to $\alpha = 0.087 \text{ (rad s}^{-2}\text{)}$ ✓

$$I = \frac{\tau}{\alpha} = 7.82 \times 10^{-3} \text{ kg m}^2 \text{ ✓}$$

OR

Attempt to use $T\theta = \frac{1}{2} I (\omega_2^2 - \omega_1^2)$ ✓

or $\theta = 573 \times 2\pi = 3600 \text{ rad}$ ✓

$$(I = 2T\theta/\omega_1^2)$$

$$= 2 \times 6.8 \times 10^{-4} \times 573 \times 2\pi/25^2 \text{ ✓}$$

$$= 7.82 \times 10^{-3} \text{ kg m}^2 \text{ ✓}$$

1st mark for **either** use of equation **or** converting rotations to rad

ECF for 3rd mark

The value of torque used must be a correctly calculated answer to part (b) or $7 \times 10^{-4} \text{ N m}$

For 2nd method

2nd mark for correct substitution

3rd mark for calculating answer

3

[7]

Q3.

(a)

Translational dynamics	Rotational dynamics
force	torque ✓
mass	moment of inertia ✓

Do not allow 'inertia'

2

(b) $I_T = 2.6 \times 10^7 + (2.2 \times 10^3 \times 35^2) = 2.9 \times 10^7 \text{ (kg m}^2\text{)} \text{ ✓}$

Mark only awarded for arriving at correct answer to more than 1 sf.

1

- (c) Use of (total) area under graph = (angular) displacement/distance ✓

$$\omega_{\max}((\frac{1}{2} \times 30) + 20 + (\frac{1}{2} \times 45)) = 4.7$$

$$\omega_{\max} = 0.082 \text{ (rad s}^{-1}\text{)} \text{ ✓}$$

Alternative route is area of trapezium

$$\frac{1}{2} \omega_{\max} (20 + 95) = 4.7$$

- (d) moment of inertia of rotating jib + load increases as trolley moves outwards ✓

reference to $T = I\alpha$ with T constant, so α decreases ✓

decreased α means longer time to stop (than 95 s) ✓

3

[8]

Q4.

- (a) $T = 6.0 \times 0.036 = 0.22 \text{ (N m)}$ ✓

1

- (b) power cannot increase ✓

$P = T\omega$ so if ω is 4x greater, T cannot be more than $1/4$ ✓

OR

Work done by (torque) on C cannot be greater than work done (by torque) on B ✓

$W = T\theta$, if θ is 4x greater, T cannot be more than $1/4$ ✓

Or $T_C \times 4\theta_B = T_B\theta_B$ so $T_C = T_B/4$

OR

Force same on both/force cannot increase/ r_C is $1/4 r_B$ ✓

$F \times r_C = F \times r_B/4$ so $T_C = T_B/4$

Or Because radius is $1/4$, torque on C must be $1/4$ ✓

Accept other valid argument e.g. using knowledge that radius of C is $1/4$ radius of B, or velocity v at point of mesh of gears is the same for both.

Do not allow 'it is not possible' (WTTE) unless backed up by valid argument.

2

- (c) $\alpha = 76/2.1 = 36 \text{ (rad s}^{-1}\text{)} (36.2 \text{ rad s}^{-1})$ ✓

$$I = T/\alpha = \frac{0.054}{36} = 1.5 \times 10^{-3} \text{ (kg m}^2\text{)} \quad \checkmark$$

ECF for 2nd mark for AE or transposing error.

2

- (d) angular impulse = ang. momentum change = $T\Delta t$ ✓

1st mark for statement defining angular impulse

Reference to (large) $\Delta(I\omega)$ in small Δt gives large T ✓

2nd for relating momentum change in small t to high T

($T = F \times r$) so large F on gear teeth. ✓
 3rd for relating high T to high force

3

[8]

Q5.

(a)

	Place a tick or ticks in this column
The moment of inertia will decrease	
The angular velocity will decrease	✓
The angular momentum will be unaltered	✓

1

(b) Flywheel acts as store of energy ✓

Able to deliver large amount of E_K in short time ✓

Without flywheel motor would stall during stamping (as load torque excessively high) ✓

max 2

(c) Converts rev min^{-1} to rad s^{-1} : 67.0 and 37.7 rad s^{-1} ✓

$$E_K = \frac{1}{2} \times 25 \times (67.0^2 - 37.7^2) = 3.83 \times 10^4 \text{ J} \quad \checkmark$$

2

(d) $\alpha = (\omega_2 - \omega_1)/t$

$$= (67.0 - 37.7)/5.0 = 5.86 \text{ rad s}^{-2} \quad \checkmark$$

$$T = I \alpha$$

$$= 25 \times 5.86 = 147 \quad \checkmark$$

If (67 - 0)/5.0 used, leading to $T = 335$ give 1 mark

*Allow approach using angular displacement θ and
change in energy
 θ*

2

(e) $P = \Delta E_K / t = 7.7 \text{ kW} \quad \checkmark$

$$\text{OR } P = T \times \omega_{\text{AVE}} = 147 \times 52.4 = 7.7 \text{ kW} \quad \checkmark$$

Allow CE from 01.4

2

(f) For smaller speed variation, greater I required ✓

I proportional to $\rho t r^4$ ($I = \frac{1}{2} \pi r^2 t \rho r^2$) ✓

Shows that greatest $\rho t r^4$ is for flywheel B ✓ (hence B)

OR discusses qualitatively:

A has smaller r but compensated for by greater t and ρ (hence greater m) ✓

B has smaller mass because of low ρ and t but much greater r
 r is squared twice ✓

$$m_A = 498 \text{ kg}, m_B = 228 \text{ kg}$$

$$\text{i.e. } m_A \text{ roughly } 2 \times m_B$$

$$\text{but } r_B^2 \text{ is roughly } 3 \times r_A^2$$

$$\text{hence } I_B > I_A$$

3

[12]

Q6.

(a) $T = mg \times l/2 = mgl/2$

1

(b) $\alpha = T/I$ ✓

Substitutes $T = mgl/2$ and $I = \frac{m}{3} l^2$ ✓

Leading to $\alpha = \frac{3g}{2l}$

Substitution and cancelling must be seen.

$$\alpha = \frac{mgl/2}{\frac{m}{3} l^2}$$

2

(c) $a = r \times \alpha$

$$r = l$$

$$a = l \times \frac{3g}{2l} = \frac{3g}{2} \text{ ✓}$$

This $> g$, so rule falls with linear accltn $> g$ ✓

2

(d) $a = r \times \frac{3g}{2l}$

$$\text{If } r = 2l/3, a = g$$

So mass placed about 67 cm mark ✓

1

Q9.

- (a) $F = 0.11 \times \cos 45 (= 0.078 \text{ N}) \checkmark$
 $T = F \times 0.12 \text{ on each arm} = 9.4 \times 10^{-3} \text{ N m}$
 Total $T = 3 \times 9.4 \times 10^{-3} = 0.028 \text{ N m} \checkmark$

Do not allow sin 45 instead of cos 45 but give CE for 2nd mark.

No CE for 2nd mark if no attempt to resolve force i.e. no cos or sin.

2

- (b) (i) Initially friction torque < applied torque so spinner accelerates / frictional torque increases with speed $\checkmark$
 Eventually applied torque = friction / resistive torque and spinner angular speed remains constant $\checkmark$

For a full answer is in terms of 'force' rather than 'torque' give max 1 mark.

2

- (ii) $240 \text{ rev min}^{-1} = 25 \text{ rad s}^{-1}$ OR use of $P = T\omega$ with attempt to convert rev min^{-1} to $\text{rad s}^{-1} \checkmark$

$$P = T\omega$$

$$P = 0.028 \times 25 = 0.70 \text{ W} \checkmark$$

Accept attempt such as multiplying by π or dividing by π or 2π , or dividing 240 by 60 and not using 2π .

No CE for incorrect ω

T must be either 0.028 or 0.03 Nm

0.75 W if 3×10^{-2} used

2

- (c) (i) Use of $\theta = \frac{1}{2}(\omega_1 + \omega_2) t$ OR $\theta = 13 \times 2\pi \checkmark$
 $= 0.5 \times 25 \times t$
 $t = 6.5 \text{ s} \checkmark$

CE for ω^1 from b ii

2

- (ii) $E_k = \text{mean power} \times \text{time}$
 $= 0.35 \times 6.5 = 2.3 \text{ J}$

OR

$$E_k = T\theta = 0.028 \times 26\pi = 2.3 \text{ J} \checkmark$$

CE for values of θ and t from c i and P from b ii

Allow use of $E_k = \frac{1}{2} I\omega^2$ with I from part c iii

If $T = 0.03 \text{ N m}$ used, $E_k = 2.4(5) \text{ J}$

1

- (iii) $E_k = \frac{1}{2} I\omega^2$
 $I = 2 E_k / \omega^2 = 7.3 \times 10^{-3} \text{ kg m}^2 \checkmark$

If 2 J used for E_k , $I = 6.3 \times 10^{-3} \text{ kg m}^2$

Alternative: use of $I = T / \alpha$

Where $\alpha = 25/\text{ans c i}$

When $T = 3 \times 10^{-2}$, $I = 7.76 \times 10^{-3} \text{ kg m}^2$
CE for ω

1

[10]

Q10.

Marks awarded for this answer will be determined by the Quality of Written Communication (QWC) as well as the standard of the scientific response.

Level 3 (5-6 marks)

The information conveyed by the answer is clearly organized, logical and coherent, using appropriate specialist vocabulary correctly. The form and style of writing is appropriate to answer the question.

All three bullet points will be addressed. Answers will relate E_k to the factors that give high M of I , and means by which the angular speed can be increased, with sensible suggestions concerning the mechanism and / or reducing friction.

Level 2 (3-4 marks)

The information conveyed in the answer may be less well organized and not fully coherent. There is less use of specialist vocabulary or specialist vocabulary may be used or spelled incorrectly. The form and style of writing is less appropriate.

The candidate may not tackle all of the bullet points in the question fully, but should have a fairly good idea of the factors that affect energy storage. Candidates are likely to relate E_k to angular speed² and to the way the mass is distributed around the axis.

Level 1 (1-2 marks)

The information conveyed by the answer is poorly organised and may not be relevant or coherent. There is little correct use of specialist vocabulary.

The candidate shows little understanding of the factors which affect energy storage in a flywheel. They may relate rotational E_k to angular speed and / or M of I , but not confidently cover aspects of mass, and distribution of mass around the axis, and may not relate their answers well to the context of the question.

0 marks

The information conveyed by the answer is sketchy, and neither relevant or coherent.

The candidate shows inadequate understanding of the concept of moment of inertia. Formulae may be quoted from the Formulae booklet, but the candidate is unable to apply their meaning to the question.

examples of the points made in the response

- E_k proportional to ω^2
- E_k proportional to I

Shape

- I depends on mass and distribution of mass around axis.

- $I = \Sigma mr^2$ so arrange more m at outer edge of flywheel
- By using heavy rim and spokes / thin centre web
- Increase thickness (to increase m or I)

Material

- Higher density material
- Gives greater mass for given size

Mechanism

Increase ω by:

- changing gear ratio / reduce size of small gears / give higher ω per push
- Longer rack segment or more teeth on rack
- Reduce friction at bearings or between gears
- use lubrication or roller bearings
- small increase in ω gives large increase in E_k (because ω^2)

extra information

Also allow

- *use material of higher tensile strength*
- *for higher speeds without bursting / to withstand rotational stresses*
- *use magnet of greater mass*
- *answers in terms of lowering electromagnetic drag*
- *sketches which convey correct info clearly*
- *use of 'depends on' for 'proportional to'*

Level 3: 1st 2 points plus 6 other points from shape, material and mechanism

Level 2: between 4 and 7 points

Level 1 fewer than 4 points.

Quoting formulae alone is not enough.

Ignore references to strength of spring.

[6]

Q11.

(a) $T = \text{power}/\omega$

Torque = $2500/0.47$

5320 N m value to 2 or more sf needed

(b) (i) Deceleration = $0.47/34 = 0.0138 \text{ (rad s}^{-2}\text{)}$

moment of inertia = torque / angular deceleration

$$=5000/0.0138 = 3.57 \times 10^5$$

kg m² (Allow N m s²)

3.8 × 10⁵ if 5320 used

3

- (ii) Suitable equation of motion used with correct data but omitted minus sign

8.0 radian Allow (their $\omega/2\pi$)

1.27 revolutions

Condone 1 revolution

(allowed for thinking question refers to complete revolutions)

3

- (c) (i) $F = 65 \times 2.2 \times 0.47^2$

32(31.6 N)

2

- (ii) Force produced by friction between the feet and the roundabout

Centripetal force has to act through the centre of mass of the operator

or

The resultant of the frictional force and normal reaction has to pass through the centre of mass

Any indication (eg on diagram) of wrong direction = 0

2

- (iii) Ticks 4th box

1

[14]

Q12.

- (a) (i) $\alpha = T / I = 8.80 / 0.565 (= 15.6 \text{ rad s}^{-2}) \checkmark$

use of $t = (\omega_2 - \omega_1) / \alpha$

leading to $t = 8.08 \text{ s} \checkmark$

2

- (ii) $\theta = \frac{1}{2} (172 + 195) \times 126 \checkmark$

= 23100 rad

$23100 / 2\pi = 3680 \text{ rev} \checkmark$

OR

rev per s = $1200 / 60 (=20) \checkmark$

$\theta = \frac{1}{2} (172 + 195) \times 20 \checkmark$

= 3670 rev $\checkmark$

Accept alternative ways of calculating area under graph

Areas are: 504 rad or 80 rev

21670 rad or 3450 rev

945 rad or 150 rev

Numbers will vary if 8.1 s used for acceleration period

Last mark: give CE for wrong θ

- (b) (i) Shows curve of increasing gradient up to first vertical dotted line ✓
OR Shows curve of decreasing gradient up to first vertical dotted line ✓

MARK ii BEFORE i

Answer must match the answer given in part ii

i.e. α increasing: decreasing gradient

α decreasing: increasing gradient

Mark awarded for shape only; ignore any changes to the height of the graph or where curve reaches 126 rad s^{-1}

1

- (ii) Mass of washing will decrease as it loses water, so M of I will decrease ✓

(T constant) so α increases ✓

OR washing moves closer to drum, increasing M of I ✓

(T constant) so α decreases ✓

OR friction (torque) increases with speed ✓

so α decreases ✓

Do not credit answers in terms of conservation of angular momentum

2

[8]

Q13.

- (a) Law of conservation of angular momentum applies and $I_1 \omega_1 = I_2 \omega_2$
OR Law of conservation of angular momentum applies and angular momentum = $I \omega$ ✓
(because no external torque acts)

Adding plasticine increases I ✓

So ω must decrease to maintain $I \omega$ constant / to conserve angular momentum ✓

3

- (b) $I \times 3.46 = (I + 0.016 \times 0.125^2) \times 3.31$ ✓

$I = 0.00552 \text{ kg m}^2$ ✓ 3 sf ✓

Useful: $mr^2 = 2.5 \times 10^{-4}$

Sig fig mark s an independent mark

If method correct but incorrect conversion of g to kg or mm to m, award 1 mark out of first 2 marks

3

- (c) (i) $\Delta E = \frac{1}{2} I \omega_1^2 - \frac{1}{2} (I + mr^2) \omega_2^2$
 $= [\frac{1}{2} \times 5.52 \times 10^{-3} \times 3.46^2] -$
 $[\frac{1}{2} \times 5.77 \times 10^{-3} \times 3.31^2]$ ✓
 $= 1.39 \times 10^{-3} \text{ J}$ ✓

CE for I of turntable or I of plasticine from 2b

Answers will vary depending on rounding e.g. accept 1.43×10^{-3}

- (ii) Work done against friction / deforming plasticine as it collides with turntable / to move or accelerate plasticine ✓

Allow heat loss on collision

Do not allow energy to sound

Examiner reports

Q1.

- (a) Only 41.4% of students scored the one mark for this question. Nearly all answers referred to angular momentum remaining constant, but many failed to include the proviso that there must be no external torque acting. Many wrote 'force' for 'torque' and so failed to gain the mark. Examiners accepted momentum 'before' and 'after', even though students did not say what the before or after referred to. A significant number of students did not take the hint suggested by part (a) that the whole of question 1 might be about conservation of angular momentum, and went on to answer (c) and (d) in terms of conservation of rotational kinetic energy.
- (b) Students were well-prepared for this; 95.1% scored the full two marks. The 2.6% who scored one mark are likely to have missed out '239' and gone straight from the calculation to the 'show that' answer of 240 (kg m²).
- (c) There were some excellent answers (47.1% of students gained all three marks), but although nearly all students knew that the moment of inertia would decrease, a number did not get the second mark for *explaining* the decrease. It was not enough to simply state that the radius was decreasing. Examiners were looking for reference to the change in distribution of the mass of the pods, not just the arms. (Students were told the mass of the arms was negligible.) It was possible to score the third mark by reference to conservation of angular momentum, without scoring the second. A small number of students explained the change in angular speed as being a consequence of conservation of kinetic energy, and so failed to gain the last mark.
- (d) The calculations here were usually done well, and most came to a sensible written conclusion. Those who used the conservation of kinetic energy could score no marks for this question (just over one fifth of the students).

Q2.

- (a) The question asked for one function of a flywheel. Only a brief answer was required and 73.5% of students were able to score the mark. Storing energy was the most common answer, followed by the smoothing out of torque. Answers which only gave an application of a flywheel (for example, kinetic energy recovery systems or a potter's wheel) were not credited.
- (b) In this question, students were asked to work out the frictional torque acting on a flywheel by first finding the loss in potential energy of a magnet attached to the rim. Despite the heavy hints given in the question, 36.9% of students scored no marks. Students who knew how to start usually ended up with three or two marks. It was expected that students would use $r \sin 8^\circ$ to calculate the vertical distance of position B below the horizontal. Because the angle was small, the first mark was also awarded to those who used $r \tan 8^\circ$ or the arc length for an angle of 8° . The next step was to calculate the loss in gravitational potential energy corresponding to this distance; about 56% of students were able to do this. In the use of $T = mgh/\theta$ for the last step, it was not uncommon to find students converting 172° to radians, despite 3.00 rad being given in the stem of the question.
- (c) Students usually cope well with the angular versions of the equations of motion and this question was no exception. Common errors were not squaring $\omega 1$ in $\omega 2^2 = \omega 1^2 - 2\alpha\theta$ or not converting 573 rev to radian correctly (or not even attempting to).

Students were allowed an error carried forward in their calculation of the moment of inertia for the last mark. Pleasingly, 57.2% of students gained maximum marks here.

Q3.

- (a) About half of all students scored 2 marks here. The rest mainly wrote 'inertia' instead of 'moment of inertia' as being analogous to mass.
- (b) The majority of students were successful here. Those who did not get the mark either calculated only the moment of inertia of the trolley and load, or failed to square the 35 m.
- (c) If this question had been set in a linear context, with a velocity-time graph in place of an angular velocity against time graph, it is likely to have been answered well. Only about 60% of students scored any marks at all. A common error was to oversimplify the situation and use average angular speed = total angular displacement / total time.
- (d) The majority of students scored the first mark for stating that the moment of inertia increases as the trolley moves away from the axis, but that is as far as they went. There were good answers using $T = I\alpha$ with T constant and I increasing, but many tried to use conservation of either angular momentum or kinetic energy.

Q4.

- (a) This was a straightforward calculation of torque which was answered well.
- (b) This question, however, proved much more demanding, with only just over one quarter of students gaining any marks. Gears are not on the specification, but the scene was set carefully, and all relevant information was given. Students who realised that power ($T\omega$) or work ($T\theta$) is transferred from gear B to gear C, or that the force on the meshing teeth of gears B and C was the same, gained marks. Many students tried to use $T = I\alpha$ and got side-tracked into relative accelerations or moments of inertia of gears B and C.
- (c) The calculations for this question were done well.
- (d) Students were asked to refer to 'angular impulse', and an easy mark could be scored simply by defining angular impulse as change in angular momentum. There were some good answers, but some students got as far as saying there would be a high torque, then failed to go on to relate this high torque to a high force for the last mark.

Q9.

A little over half the students scored the full two marks for part (a). Common errors were not resolving the force in the horizontal direction, failing to multiply by three for the three nozzles, and writing $\sin 45^\circ$ instead of $\cos 45^\circ$. Although $\sin 45^\circ$ gave the same numerical answer, the maximum they could score was 1 mark. Part (b)(i) was answered fully by only about one quarter of the students. All too often students muddled linear and rotational motion with 'force' being used for 'torque', or 'force' being equated to 'torque' (e.g. the frictional force equals the applied torque produced by the jets). Many students only addressed one of the two parts of the question (why the spinner accelerates and why it reaches constant angular speed). The situation is analogous to a falling object reaching terminal velocity in linear motion.

Most students realised the need to use $P = T\omega$ in part **(b)(ii)**, but a surprising number were not able to convert rev min^{-1} to rad s^{-1} , a fundamental requirement for this option. In part **(c)(i)**, three quarters of students were able to select an equation of rotational motion and use it correctly. A carried error was given for those who had not been able to make the rad s^{-1} conversion in part **(b)(ii)**.

The expected route for part **(c)(ii)** was the use of either $W = T\theta$ or $W = \text{average power} / \text{time}$ and many used either of these methods. Parts **(c)(ii)** and **(c)(iii)** were marked together, so that those who had used $I = T/\alpha$ in **(c)(iii)** and had then gone back to **(c)(ii)** to use their value of I in $E_K = \frac{1}{2}I\omega^2$ could score the marks. Part **(c)(ii)** was a 'show that' type of question, so that those who couldn't get the 2 J answer could use 2 J in part **(c)(iii)** to find I .

Q10.

Over half the students were able to achieve three or more marks out of the six allocated, but only a small minority were able to give the detail and quality of writing needed to reach the highest tier. For this tier, all three bullet points in the question needed to be addressed, and there were some excellent answers. A pleasing proportion of students saw the need to put more mass at the circumference of the flywheel, to use a material of high density and to increase the angular velocity. Examiners were not expecting students to have a detailed knowledge of how gears work, but it was satisfying to see that a good number were able to make sensible suggestions concerning the gears or the toothed segment attached to the handle, or that reducing friction by, for example, lubrication, would help the flywheel rotate for longer. A few students failed to read the question carefully and concentrated on the aspects of generating electricity.

Misconceptions were that:

- *reducing* the moment of inertia of the flywheel would increase energy stored because it would accelerate more quickly to a higher speed
- angular momentum is conserved, so reducing I would increase ω and hence energy stored.
- because $I = \sum mr^2$ and r cannot be increased, the *only* way to increase I is to increase m .

Some students wrote "inertia" for "moment of inertia".

The examiners were often frustrated by having to decipher illegible handwriting.

Q11.

- (a) The majority of students gained full marks.
- (b)
 - (i) The calculation was generally well done but few could state the correct units kg m^2 . The alternative N ms^2 was acceptable but those who included rad as N mrad s^2 were penalised.
 - (ii) A common error was to use $\theta = \omega t$, thus giving an answer for constant speed. Many of those who used $\theta = \omega t + \frac{1}{2} \alpha t^2$ did not use a negative value for the deceleration.
- (c)
 - (i) Most students had no problem here, however, a few took the angular speed to be the linear speed.
 - (ii) Many students demonstrated a lack of understanding of centripetal force. Some thought that its origin was the axis of rotation and others thought it acted outwards. Those who knew that it was provided by friction needed to indicate where the friction was acting. Some showed in pointing outwards on

the diagram. Most students stated that the operator was leaning inwards to be nearer the axis and very few gave the correct reason.

Q12.

Over 90 per cent of students scored the full 2 marks for part (a)(i).

Nearly 70 per cent of students were able to give confident and fully correct answers to part (a)(ii). Of those who did not score full marks, some calculated the area under the graph in radians, but failed to convert to revolutions. Others calculated one or two areas in radians, the rest in revolutions and added them together. An alternative acceptable route was to use the information given that 126 rad s^{-1} was equal to 1200 rev s^{-1} and do all the calculation without bringing in radians at all. Very few students seemed to know how to calculate the area of a trapezium directly.

Parts (b)(i) and (b)(ii) were marked together. The mark for (b)(i) was given provided what students had written in (b)(ii) for the way the acceleration was changing matched their drawn curve on Figure 2. It was clear from answers to (b)(ii) that many students did not know the principle behind the process of spin drying. Credit was given for either the moment of inertia increasing because the clothes would be forced closer towards the drum, hence decreasing angular acceleration, or the moment of inertia decreasing because the clothes were losing the mass of water, hence increasing angular acceleration. The marks were also awarded to those answers in terms of frictional torque increasing as speed increases, reducing resultant torque and so reducing angular acceleration.

Q13.

The majority of students were able to see that part (a) was a question requiring the application of conservation of angular momentum, even though angular momentum was not mentioned in the text of the question. Some, however, thought the conservation of rotational kinetic energy applied. Nearly all students scored the 1 mark for realising that the plasticine increased the moment of inertia of the turntable.

In part (b) answers that failed to get the physics (and numbers) correct initially could only score the third mark, and then only if they wrote any answer to 3 significant figures. Those students who made an error in converting g to kg or mm to m (or both) lost one of the two marks for the calculation. A common error was to use mr rather than mr^2 for the moment of inertia of the plasticine.

Part (c)(i) was reasonably well-answered. Examiners used 'carried error' where incorrect values of moments of inertia for the turntable and plasticine from part (b) were used appropriately in the calculation.

Part (c)(ii) elicited some very weak answers. Some just referred to loss of energy because the turntable slows down, or 'energy after is less than energy before', or the extra mass makes it slow down, without realising that there would be energy loss (or work done in accelerating the plasticine) *during the collision*. If this had been set as a collision in a linear dynamics context, they would probably have done better.