AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level MATHEMATICS

Paper 1

Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 State the set of values of x which satisfies the inequality

$$(x - 3)(2x + 7) > 0$$

Tick (✓) **one** box.

[1 mark]

$\left\{x : -\frac{7}{2} < x < 3\right\}$ ☐

$\left\{x : x < -3 \text{ or } x > \frac{7}{2}\right\}$ ☐

$\left\{x : x < -\frac{7}{2} \text{ or } x > 3\right\}$ ✓

$\left\{x : -3 < x < \frac{7}{2}\right\}$ ☐

2 Given that $y = \ln(5x)$

find $\frac{dy}{dx}$

Circle your answer.

[1 mark]

(circled) $\frac{dy}{dx} = \frac{1}{x}$ $\quad \frac{dy}{dx} = \frac{1}{5x}$ $\quad \frac{dy}{dx} = \frac{5}{x}$ $\quad \frac{dy}{dx} = \ln 5$

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3 A geometric sequence has a sum to infinity of -3

A second sequence is formed by multiplying each term of the original sequence by -2

What is the sum to infinity of the new sequence?

Circle your answer.

[1 mark]

The sum to infinity does not exist | -6 | -3 | (6)

4 Millie is attempting to use proof by contradiction to show that the result of multiplying an irrational number by a non-zero rational number is always an irrational number.

Select the assumption she should make to start her proof.

Tick (✓) **one** box.

[1 mark]

- Every irrational multiplied by a non-zero rational is irrational. ☐
- Every irrational multiplied by a non-zero rational is rational. ☐
- There exists a non-zero rational and an irrational whose product is irrational. ☐
- There exists a non-zero rational and an irrational whose product is rational. ☑

Turn over for the next question

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5 The line L has equation

$$3y - 4x = 21$$

The point P has coordinates (15, 2)

5 (a) Find the equation of the line perpendicular to L which passes through P.

[2 marks]

L: $3y = 4x + 21 \Rightarrow y = \frac{4}{3}x + 7$, gradient $\frac{4}{3}$

Perpendicular gradient $= -\frac{3}{4}$

$y - 2 = -\frac{3}{4}(x - 15)$

$4y - 8 = -3x + 45$

$4y + 3x = 53$

5 (b) Hence, find the shortest distance from P to L.

[4 marks]

Solve $3y - 4x = 21$ and $4y + 3x = 53$

$\times 3$: $9y - 12x = 63$

$\times 4$: $16y + 12x = 212$

Add: $25y = 275 \Rightarrow y = 11$

$3(11) - 4x = 21 \Rightarrow 4x = 12 \Rightarrow x = 3$

Foot of perpendicular from P to L is (3, 11)

Distance$^2 = (3 - 15)^2 + (11 - 2)^2 = 144 + 81 = 225$

Shortest distance $= \sqrt{225} = 15$

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6 (a) The ninth term of an arithmetic series is 3

The sum of the first n terms of the series is S_n and $S_{21} = 42$

Find the first term and common difference of the series.

[4 marks]

$u_9 = a + 8d = 3$

$S_{21} = \frac{21}{2}(2a + 20d) = 42$

$2a + 20d = 4 \quad \Rightarrow \quad a + 10d = 2$

Subtract: $(a + 10d) - (a + 8d) = 2 - 3$

$2d = -1 \quad \Rightarrow \quad d = -0.5$

$a = 3 - 8(-0.5) = 3 + 4 = 7$

First term $a = 7$, common difference $d = -0.5$

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6 (b) A second arithmetic series has first term -18 and common difference $\frac{3}{4}$

The sum of the first n terms of this series is T_n

Find the value of n such that $T_n = S_n$

[3 marks]

$S_n = \frac{n}{2}(2(7) + (n-1)(-0.5)) = \frac{n}{2}(14 - 0.5(n-1))$

$T_n = \frac{n}{2}(2(-18) + (n-1)(0.75)) = \frac{n}{2}(-36 + 0.75(n-1))$

$T_n = S_n$: $\frac{n}{2}(-36 + 0.75(n-1)) = \frac{n}{2}(14 - 0.5(n-1))$

$n \neq 0$: $-36 + 0.75n - 0.75 = 14 - 0.5n + 0.5$

$1.25n = 51.25$

$n = 41$

Turn over for the next question

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7 The equation $x^2 = x^3 + x - 3$ has a single solution, $x = \alpha$

7 (a) By considering a suitable change of sign, show that α lies between 1.5 and 1.6

[2 marks]

$x^2 = x^3 + x - 3 \Rightarrow x^3 - x^2 + x - 3 = 0$

Let $f(x) = x^3 - x^2 + x - 3$, which is continuous

$f(1.5) = 3.375 - 2.25 + 1.5 - 3 = -0.375 < 0$

$f(1.6) = 4.096 - 2.56 + 1.6 - 3 = 0.136 > 0$

Change of sign, so α lies between 1.5 and 1.6

7 (b) Show that the equation $x^2 = x^3 + x - 3$ can be rearranged into the form

$$x^2 = x - 1 + \frac{3}{x}$$

[2 marks]

$x^2 = x^3 + x - 3$

$x^3 = x^2 - x + 3$

Divide by x $(x \neq 0)$: $x^2 = x - 1 + \frac{3}{x}$

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7 (c) Use the iterative formula

$$x_{n+1} = \sqrt{x_n - 1 + \frac{3}{x_n}}$$

with $x_1 = 1.5$, to find x_2, x_3 and x_4, giving your answers to four decimal places.

[2 marks]

$x_2 = \sqrt{(1.5 - 1 + \frac{3}{1.5})} = \sqrt{2.5} = 1.58113... = 1.5811$

$x_3 = \sqrt{(1.58113... - 1 + \frac{3}{1.58113...})} = 1.57432... = 1.5743$

$x_4 = \sqrt{(1.57432... - 1 + \frac{3}{1.57432...})} = 1.57477... = 1.5748$

7 (d) Hence, deduce an interval of width 0.001 in which α lies.

[1 mark]

$1.574 \leq \alpha \leq 1.575$

Turn over for the next question

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8 (a) Given that

$$9\sin^2\theta + \sin 2\theta = 8$$

show that

$$8\cot^2\theta - 2\cot\theta - 1 = 0$$

[4 marks]

$9\sin^2\theta + \sin 2\theta = 8$

$9\sin^2\theta + 2\sin\theta\cos\theta = 8$

Divide by $\sin^2\theta$: $9 + \frac{2\cos\theta}{\sin\theta} = \frac{8}{\sin^2\theta}$

$9 + 2\cot\theta = 8\operatorname{cosec}^2\theta$

Using $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$:

$9 + 2\cot\theta = 8(1 + \cot^2\theta)$

$9 + 2\cot\theta = 8 + 8\cot^2\theta$

$8\cot^2\theta - 2\cot\theta - 1 = 0$

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8 (b) Hence, solve

$$9\sin^2\theta + \sin 2\theta = 8$$

in the interval $0 < \theta < 2\pi$

Give your answers to two decimal places.

[3 marks]

$(4\cot\theta + 1)(2\cot\theta - 1) = 0$

$\cot\theta = -\frac{1}{4}$ or $\cot\theta = \frac{1}{2}$

$\tan\theta = -4$ or $\tan\theta = 2$

$\tan\theta = 2$: $\theta = 1.107...$ or $\theta = 1.107... + \pi = 4.248...$

$\tan\theta = -4$: $\theta = \pi - 1.3258... = 1.8157...$ or $\theta = 1.8157... + \pi = 4.957...$

$\theta = 1.11,\ 1.82,\ 4.25,\ 4.96$

8 (c) Solve

$$9\sin^2\left(2x - \frac{\pi}{4}\right) + \sin\left(4x - \frac{\pi}{2}\right) = 8$$

in the interval $0 < x < \frac{\pi}{2}$

Give your answers to one decimal place.

[2 marks]

Let $\theta = 2x - \frac{\pi}{4}$, then $4x - \frac{\pi}{2} = 2\theta$, same equation as (b)

$0 < x < \frac{\pi}{2} \Rightarrow -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$, so $\theta = 1.1071...$ or $1.8157...$

$2x = 1.1071... + \frac{\pi}{4} \Rightarrow x = 0.9462...$

$2x = 1.8157... + \frac{\pi}{4} \Rightarrow x = 1.3005...$

$x = 0.9,\ 1.3$

Turn over ▶

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9 The table below shows the annual global production of plastics, P, measured in millions of tonnes per year, for six selected years.

Year	1980	1985	1990	1995	2000	2005
$\boldsymbol{P}$	75	94	120	156	206	260

It is thought that P can be modelled by

$$P = A \times 10^{kt}$$

where t is the number of years after 1980 and A and k are constants.

9 (a) Show algebraically that the graph of $\log_{10} P$ against t should be linear.

[3 marks]

$P = A \times 10^{kt}$

$\log_{10} P = \log_{10}(A \times 10^{kt})$

$\log_{10} P = \log_{10} A + \log_{10} 10^{kt}$

$\log_{10} P = \log_{10} A + kt$

This is of the form $Y = mt + c$ with $Y = \log_{10} P$, gradient k and intercept $\log_{10} A$ both constant, so the graph of $\log_{10} P$ against t is linear.

9 (b) (i) Complete the table below.

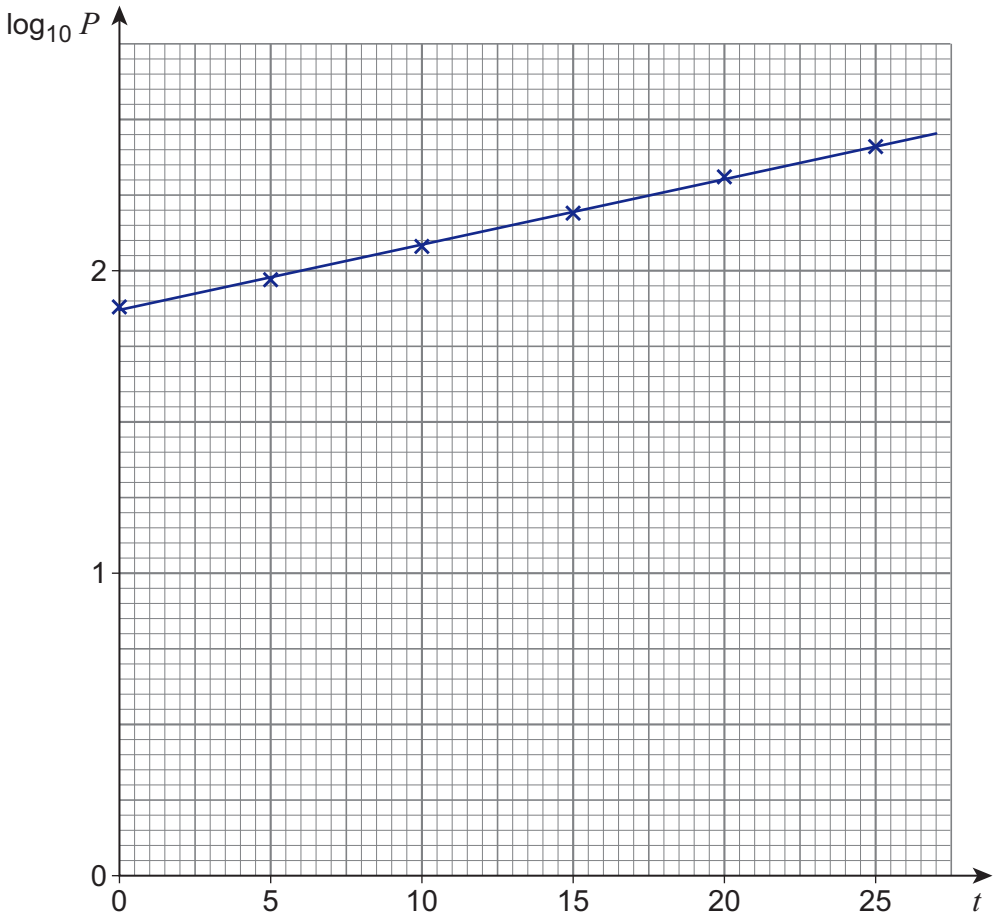
$\boldsymbol{t}$	0	5	10	15	20	25
$\boldsymbol{\log_{10} P}$	1.88	1.97	2.08	2.19	2.31	2.41

[1 mark]

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9 (b) (ii) Plot $\log_{10} P$ against t, and draw a line of best fit for the data.

[2 marks]



9 (c) (i) Hence, show that k is approximately 0.02

[2 marks]

Gradient of line of best fit = $\frac{2.41 - 1.88}{25 - 0} = \frac{0.53}{25} = 0.0212$

k = gradient ≈ 0.02

9 (c) (ii) Find the value of A.

[1 mark]

When $t = 0$, $P = A \times 10^0 = A$, so $A = 75$

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9 (d) Using the model with $k = 0.02$ predict the number of tonnes of annual global production of plastics in 2030.

[2 marks]

2030: $t = 50$

$P = 75 \times 10^{0.02 \times 50} = 75 \times 10^{1} = 750$

750 million tonnes

9 (e) Using the model with $k = 0.02$ predict the year in which P first exceeds 8000

[3 marks]

$75 \times 10^{0.02t} > 8000$

$10^{0.02t} > \frac{8000}{75} = 106.66...$

$0.02t > \log_{10} 106.66... = 2.0280...$

$t > 101.40...$

1980 + 101.4 = 2081.4, so P first exceeds 8000 in 2082

9 (f) Give a reason why it may be inappropriate to use the model to make predictions about future annual global production of plastics.

[1 mark]

Global plastic production may decrease in the future (e.g. for environmental reasons), so extrapolating the model far beyond the data is unreliable.

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10 (a) Given that

$$y = \tan x$$

use the quotient rule to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 x$$

[3 marks]

$y = \tan x = \frac{\sin x}{\cos x}$

$\frac{dy}{dx} = \frac{\cos x \times \cos x - \sin x \times (-\sin x)}{\cos^2 x}$

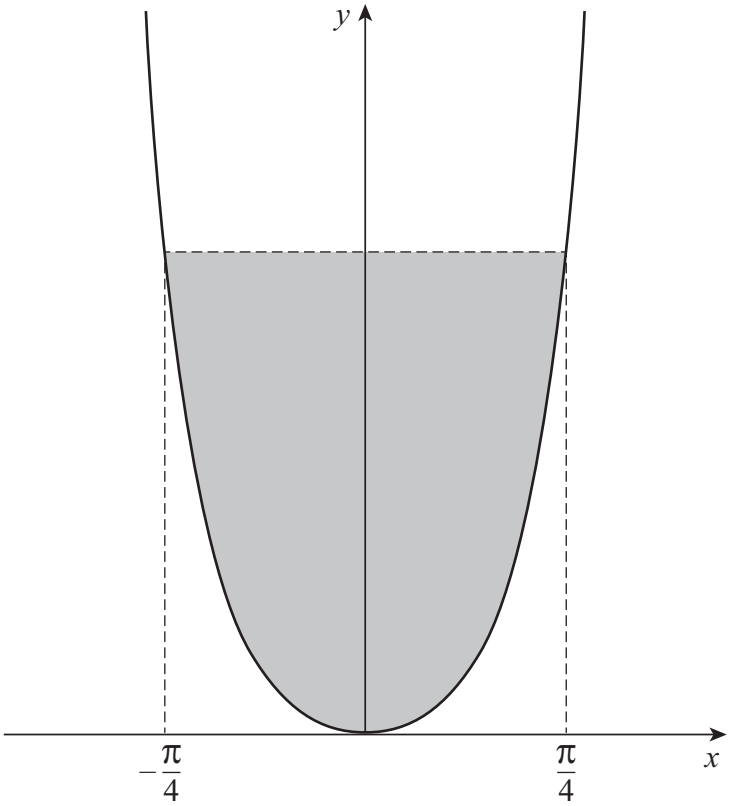
$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$ since $\sin^2 x + \cos^2 x = 1$

$= \sec^2 x$

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10 (b) The region enclosed by the curve $y = \tan^2 x$ and the horizontal line, which intersects the curve at $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$, is shaded in the diagram below.



Show that the area of the shaded region is

$$\pi - 2$$

Fully justify your answer.

[5 marks]

Horizontal line: $y = \tan^2(\frac{\pi}{4}) = 1$

Rectangle area $= 1 \times (\frac{\pi}{4} - (-\frac{\pi}{4})) = \frac{\pi}{2}$

Area under curve $= \int_{-\pi/4}^{\pi/4} \tan^2 x \, dx = \int_{-\pi/4}^{\pi/4} (\sec^2 x - 1) \, dx$

$= [\tan x - x]_{-\pi/4}^{\pi/4}$

$= (\tan \frac{\pi}{4} - \frac{\pi}{4}) - (\tan(-\frac{\pi}{4}) - (-\frac{\pi}{4}))$

$= (1 - \frac{\pi}{4}) - (-1 + \frac{\pi}{4})$

$= 2 - \frac{\pi}{2}$

Shaded area = rectangle − area under curve $= \frac{\pi}{2} - (2 - \frac{\pi}{2})$

$= \pi - 2$

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11 A curve, C, passes through the point with coordinates (1, 6)

The gradient of C is given by

$$\frac{dy}{dx} = \frac{1}{6}(xy)^2$$

Show that C intersects the coordinate axes at exactly one point and state the coordinates of this point.

Fully justify your answer.

[8 marks]

$\frac{dy}{dx} = \frac{1}{6}x^2y^2$

$\int \frac{1}{y^2}\,dy = \int \frac{1}{6}x^2\,dx$

$-\frac{1}{y} = \frac{x^3}{18} + c$

At (1, 6): $-\frac{1}{6} = \frac{1}{18} + c \Rightarrow c = -\frac{4}{18} = -\frac{2}{9}$

$-\frac{1}{y} = \frac{x^3}{18} - \frac{2}{9}$

x-axis ($y = 0$): $-\frac{1}{y}$ is undefined when $y = 0$, so there is no value of x for which $y = 0$. $\therefore$ C does not intersect the x-axis.

y-axis ($x = 0$): $-\frac{1}{y} = 0 - \frac{2}{9} = -\frac{2}{9} \Rightarrow y = \frac{9}{2} = 4.5$

Only one value of y when $x = 0$, so C crosses the y-axis only at (0, 4.5).

Hence C intersects the coordinate axes at exactly one point, (0, 4.5)

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Turn over for the next question

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12 The equation of a curve is

$$(x+y)^2 = 4y + 2x + 8$$

The curve intersects the positive x-axis at the point P.

12 (a) Show that the gradient of the curve at P is $-\frac{3}{2}$

[6 marks]

At P, $y = 0$: $x^2 = 2x + 8$

$x^2 - 2x - 8 = 0 \Rightarrow (x - 4)(x + 2) = 0$

$x = 4$ (positive x-axis), so $P(4, 0)$

Differentiate implicitly:

$2(x + y)(1 + \frac{dy}{dx}) = 4\frac{dy}{dx} + 2$

At $(4, 0)$: $2(4)(1 + \frac{dy}{dx}) = 4\frac{dy}{dx} + 2$

$8 + 8\frac{dy}{dx} = 4\frac{dy}{dx} + 2$

$4\frac{dy}{dx} = -6$

$\frac{dy}{dx} = -\frac{3}{2}$

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12 (b) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by = c$, where a, b and c are integers.

[2 marks]

Gradient of normal = $\frac{2}{3}$

$y - 0 = \frac{2}{3}(x - 4)$

$3y = 2x - 8$

$2x - 3y = 8$

Turn over for the next question

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13 (a) Given that

$$P(x) = 125x^3 + 150x^2 + 55x + 6$$

use the factor theorem to prove that $(5x + 1)$ is a factor of $P(x)$.

[2 marks]

$P(-\frac{1}{5}) = 125(-\frac{1}{5})^3 + 150(-\frac{1}{5})^2 + 55(-\frac{1}{5}) + 6$

$= -1 + 6 - 11 + 6 = 0$

Since $P(-\frac{1}{5}) = 0$, $(5x + 1)$ is a factor of $P(x)$ by the factor theorem

13 (b) Factorise $P(x)$ completely.

[3 marks]

$P(x) = (5x + 1)(25x^2 + 25x + 6)$

$25x^2 + 25x + 6 = (5x + 2)(5x + 3)$

$P(x) = (5x + 1)(5x + 2)(5x + 3)$

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13 (c) Hence, prove that $250n^3 + 300n^2 + 110n + 12$ is a multiple of 12 when n is a positive whole number.

[3 marks]

$250n^3 + 300n^2 + 110n + 12 = 2(125n^3 + 150n^2 + 55n + 6) = 2P(n)$

$= 2(5n + 1)(5n + 2)(5n + 3)$

For positive whole n, $5n + 1$, $5n + 2$, $5n + 3$ are three consecutive whole numbers. Among any three consecutive whole numbers at least one is a multiple of 2 and exactly one is a multiple of 3, so

$(5n + 1)(5n + 2)(5n + 3)$ is a multiple of $2 \times 3 = 6$

$\therefore 2(5n + 1)(5n + 2)(5n + 3)$ is a multiple of $2 \times 2 \times 3 = 12$

Hence $250n^3 + 300n^2 + 110n + 12$ is a multiple of 12

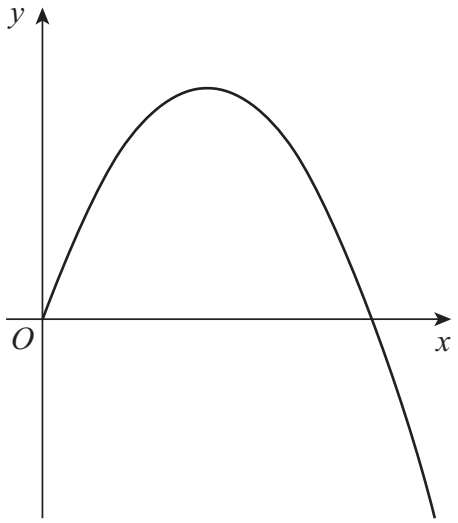
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14 The curve C is defined for $t \geq 0$ by the parametric equations

$$x = t^2 + t \quad \text{and} \quad y = 4t^2 - t^3$$

C is shown in the diagram below.



14 (a) Find the gradient of C at the point where it intersects the positive x-axis.

[5 marks]

Crosses x-axis when $y = 0$: $4t^2 - t^3 = 0 \Rightarrow t^2(4 - t) = 0$

$t = 0$ (origin) or $t = 4$, so the positive x-axis is at $t = 4$

$\frac{dy}{dt} = 8t - 3t^2$ $\quad$ $\frac{dx}{dt} = 2t + 1$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{8t - 3t^2}{2t + 1}$

$t = 4$: $\frac{dy}{dx} = \frac{32 - 48}{8 + 1} = -\frac{16}{9}$

Gradient $= -\frac{16}{9}$

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14 (b) (i) The area A enclosed between C and the x-axis is given by

$$A = \int_0^b y \, \mathrm{d}x$$

Find the value of b.

[1 mark]

At $t = 4$: $x = 4^2 + 4 = 20$, so $b = 20$

14 (b) (ii) Use the substitution $y = 4t^2 - t^3$ to show that

$$A = \int_0^4 (4t^2 + 7t^3 - 2t^4) \, \mathrm{d}t$$

[3 marks]

$x = t^2 + t \Rightarrow \frac{dx}{dt} = 2t + 1 \Rightarrow dx = (2t + 1)\,dt$

Limits: $x = 0 \Rightarrow t = 0$ $(t \geq 0)$; $x = 20 \Rightarrow t = 4$

$A = \int_0^{20} y\,dx = \int_0^4 (4t^2 - t^3)(2t + 1)\,dt$

$= \int_0^4 (8t^3 + 4t^2 - 2t^4 - t^3)\,dt$

$= \int_0^4 (4t^2 + 7t^3 - 2t^4)\,dt$

14 (b) (iii) Find the value of A.

[1 mark]

$A = \left[\frac{4t^3}{3} + \frac{7t^4}{4} - \frac{2t^5}{5} \right]_0^4 = \frac{256}{3} + 448 - \frac{2048}{5}$

$= \frac{1856}{15}$ $(= 123.7)$

Turn over ▶

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15 (a) Show that

$$\sin x - \sin x \cos 2x \approx 2x^3$$

for small values of x.

[3 marks]

For small x: $\sin x \approx x$, $\cos 2x \approx 1 - \frac{(2x)^2}{2} = 1 - 2x^2$

$\sin x - \sin x \cos 2x \approx x - x(1 - 2x^2)$

$\approx x - x + 2x^3$

$\approx 2x^3$

15 (b) Hence, show that the area between the graph with equation

$$y = \sqrt{8(\sin x - \sin x \cos 2x)}$$

the positive x-axis and the line $x = 0.25$ can be approximated by

$$\text{Area} \approx 2^m \times 5^n$$

where m and n are integers to be found.

[4 marks]

Area $\approx \int_0^{0.25} \sqrt{(8 \times 2x^3)}\, dx = \int_0^{0.25} \sqrt{(16x^3)}\, dx = \int_0^{0.25} 4x^{3/2}\, dx$

$= \left[\frac{8}{5}x^{5/2}\right]_0^{0.25}$

$= \frac{8}{5} \times (0.25)^{5/2} - 0 = \frac{8}{5} \times (\frac{1}{2})^5 = \frac{8}{5} \times \frac{1}{32} = \frac{1}{20}$

$= \frac{1}{2^2 \times 5} = 2^{-2} \times 5^{-1}$

$m = -2, \quad n = -1$

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15 (c) (i) Explain why

$$\int_{6.3}^{6.4} 2x^3 \, \mathrm{d}x$$

is **not** a suitable approximation for

$$\int_{6.3}^{6.4} (\sin x - \sin x \cos 2x) \, \mathrm{d}x$$

[1 mark]

The approximation sin x − sin x cos 2x ≈ 2x³ is only valid for small values of x, and 6.3 and 6.4 are not small.

Question 15 continues on the next page

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15 (c) (ii) Explain how

$$\int_{6.3}^{6.4} (\sin x - \sin x \cos 2x)\,\mathrm{d}x$$

may be approximated by

$$\int_{a}^{b} 2x^3\,\mathrm{d}x$$

for suitable values of a and b.

[2 marks]

sin x − sin x cos 2x is periodic with period 2π, so the integral over [6.3, 6.4] equals the integral over [6.3 − 2π, 6.4 − 2π].

These limits are small, so the approximation $2x^3$ is valid there:

a = 6.3 − 2π = 0.0168... ≈ 0.017

b = 6.4 − 2π = 0.1168... ≈ 0.117

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