AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level MATHEMATICS

Paper 3

Friday 12 June 2020 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Given that

$$\int_0^{10} \mathrm{f}(x)\,\mathrm{d}x = 7$$

deduce the value of

$$\int_0^{10} \left(\mathrm{f}(x) + 1\right)\mathrm{d}x$$

Circle your answer.

[1 mark]

-3 7 8 (17)

2 Given that

$$6\cos\theta + 8\sin\theta \equiv R\cos(\theta + \alpha)$$

find the value of R.

Circle your answer.

[1 mark]

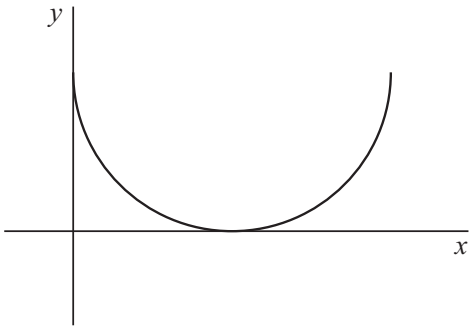
6 8 (10) 14

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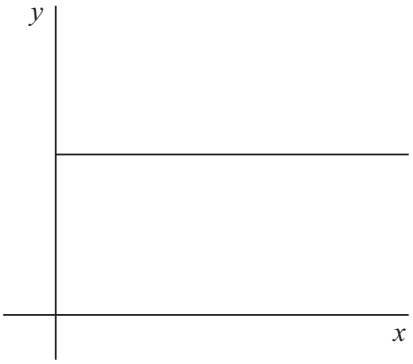
3 Determine which one of these graphs does **not** represent y as a function of x.

Tick (✓) **one** box.

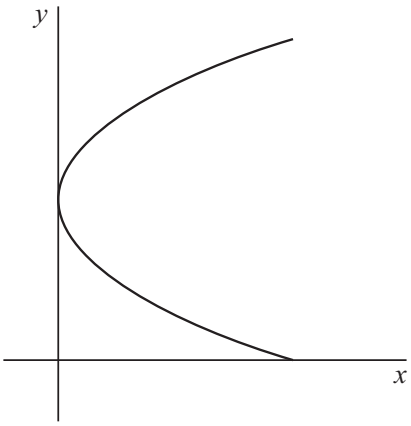
[1 mark]



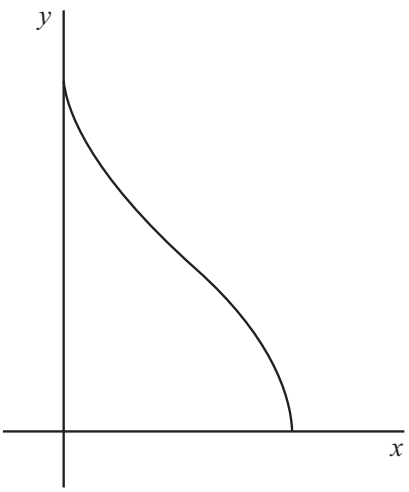
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4 $\mathrm{p}(x) = 4x^3 - 15x^2 - 48x - 36$

4 (a) Use the factor theorem to prove that $x - 6$ is a factor of $\mathrm{p}(x)$.

[2 marks]

$p(6) = 4(6)^3 - 15(6)^2 - 48(6) - 36$

$= 864 - 540 - 288 - 36 = 0$

$p(6) = 0$, so by the factor theorem $(x - 6)$ is a factor of $p(x)$.

4 (b) (i) Prove that the graph of $y = \mathrm{p}(x)$ intersects the x-axis at exactly one point.

[4 marks]

$4x^3 - 15x^2 - 48x - 36 = (x - 6)(4x^2 + 9x + 6)$

For $4x^2 + 9x + 6 = 0$:

$b^2 - 4ac = 9^2 - 4 \times 4 \times 6 = 81 - 96 = -15$

$-15 < 0$, so $4x^2 + 9x + 6 = 0$ has no real roots.

Hence $p(x) = 0$ has exactly one real root, $x = 6$,

so the graph of $y = p(x)$ intersects the x-axis at

exactly one point.

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4 (b) (ii) State the coordinates of this point of intersection.

[1 mark]

(6, 0)

Turn over for the next question

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5 The number of radioactive atoms, N, in a sample of a sodium isotope after time t hours can be modelled by

$$N = N_0 e^{-kt}$$

where N_0 is the initial number of radioactive atoms in the sample and k is a positive constant.

The model remains valid for large numbers of atoms.

5 (a) It takes 15.9 hours for half of the sodium atoms to decay.

Determine the number of days required for at least 90% of the number of atoms in the original sample to decay.

[5 marks]

$t = 15.9$, $N = \frac{1}{2}N_0$: $\frac{1}{2}N_0 = N_0 e^{-15.9k}$

$e^{-15.9k} = 0.5 \Rightarrow k = \frac{\ln 2}{15.9} = 0.043594...$

At least 90% decayed ⇒ at most 10% remain: $N = 0.1N_0$

$0.1N_0 = N_0 e^{-0.043594... t}$

$-0.043594... t = \ln 0.1$

$t = \ln 10 \div 0.043594... = 52.8186...$ hours

$52.8186... \div 24 = 2.2007...$ days

Number of days required = 2.2 days

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5 (b) Find the percentage of the atoms remaining after the first week.

Give your answer to two significant figures.

[2 marks]

1 week = 7 × 24 = 168 hours

$N = N_0 e^{-0.043594... \times 168} = N_0 \times 0.00065963...$

Percentage remaining = 0.065963...%

= 0.066% (2 s.f.)

5 (c) Explain why the model can only provide an estimate for the number of remaining atoms.

[1 mark]

The model is continuous, but the number of atoms is discrete (a whole number), so the model can only give an estimate of the number of remaining atoms.

5 (d) Explain why the model is invalid in the long run.

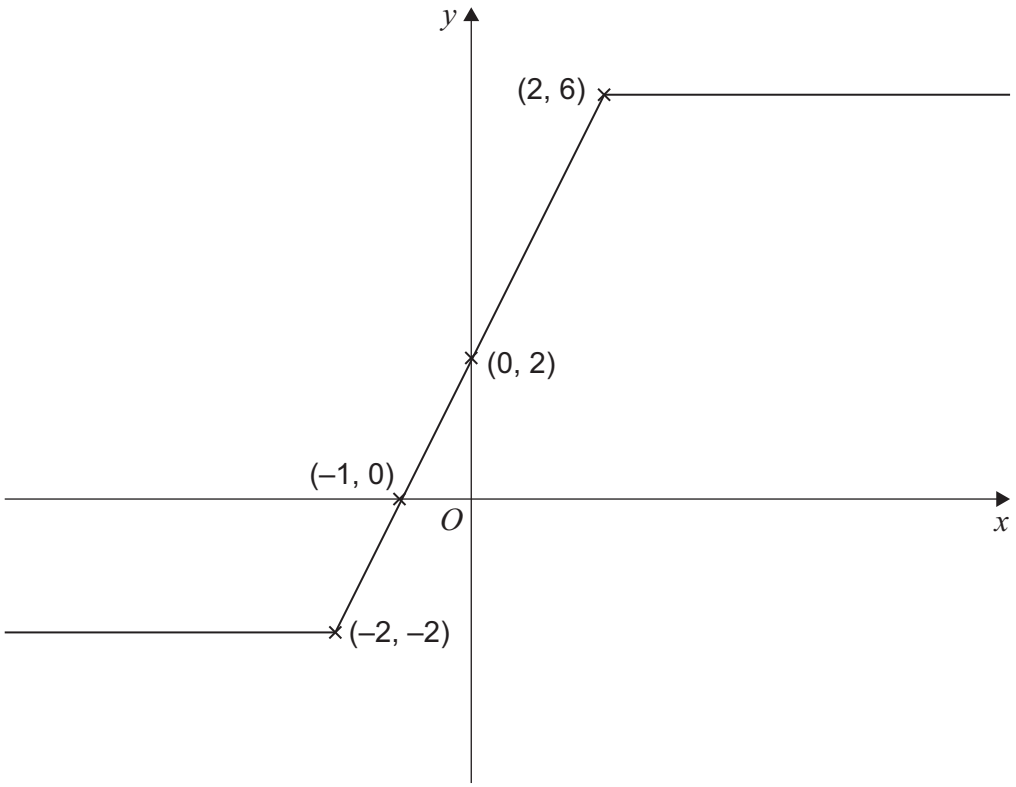
[1 mark]

As t gets large, the number of atoms predicted by the model becomes very small, and it eventually predicts less than one atom (but never zero atoms), so the model is no longer valid.

Turn over for the next question

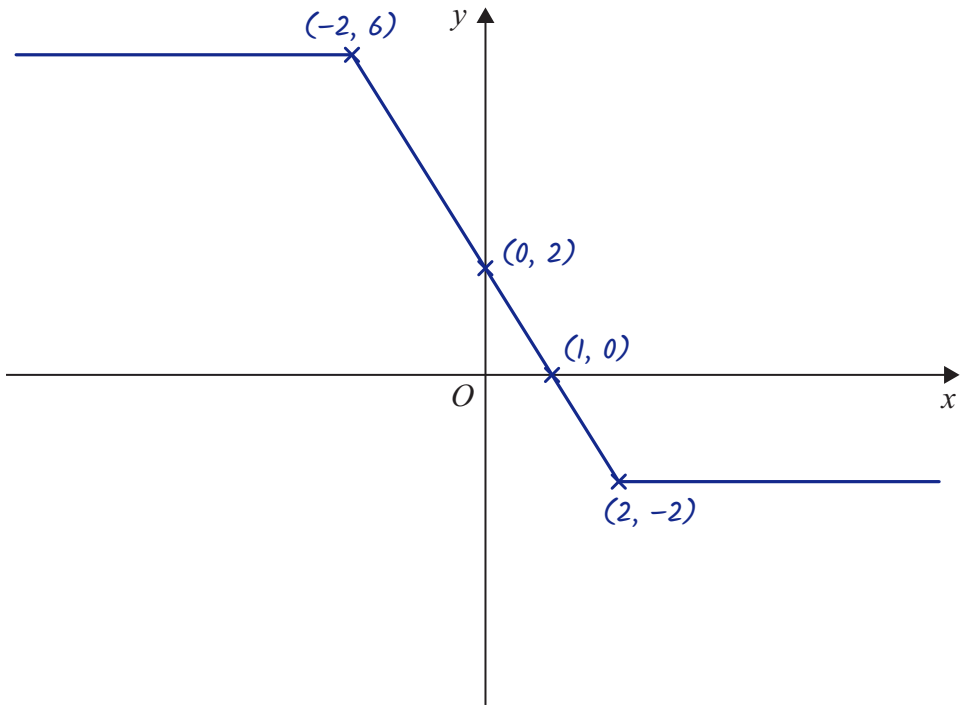
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6 The graph of $y = f(x)$ is shown below.



6 (a) Sketch the graph of $y = f(-x)$

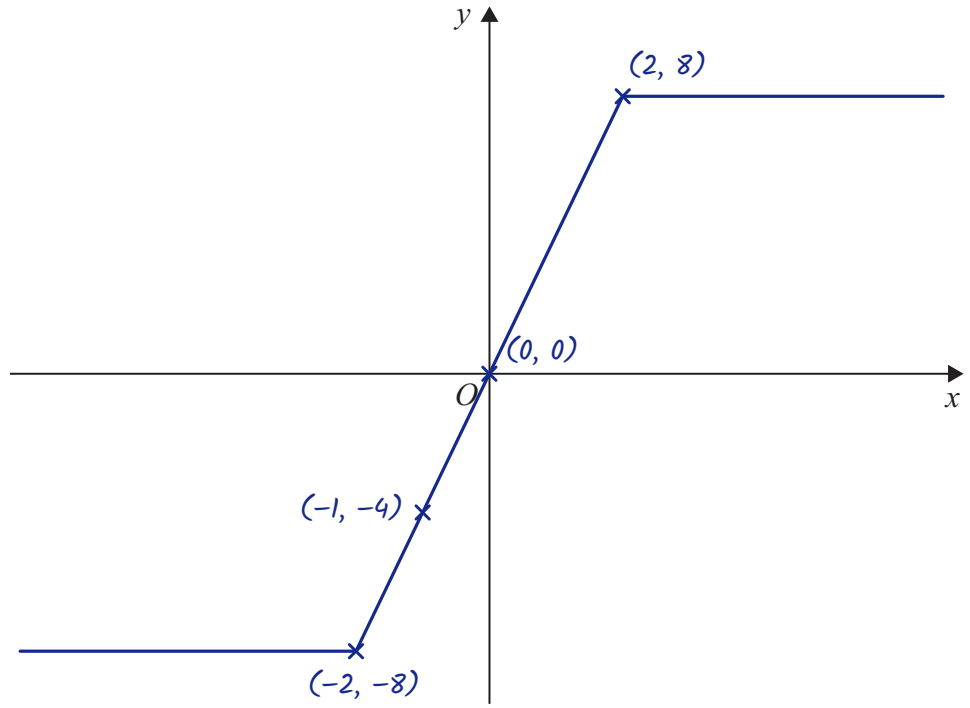
[2 marks]



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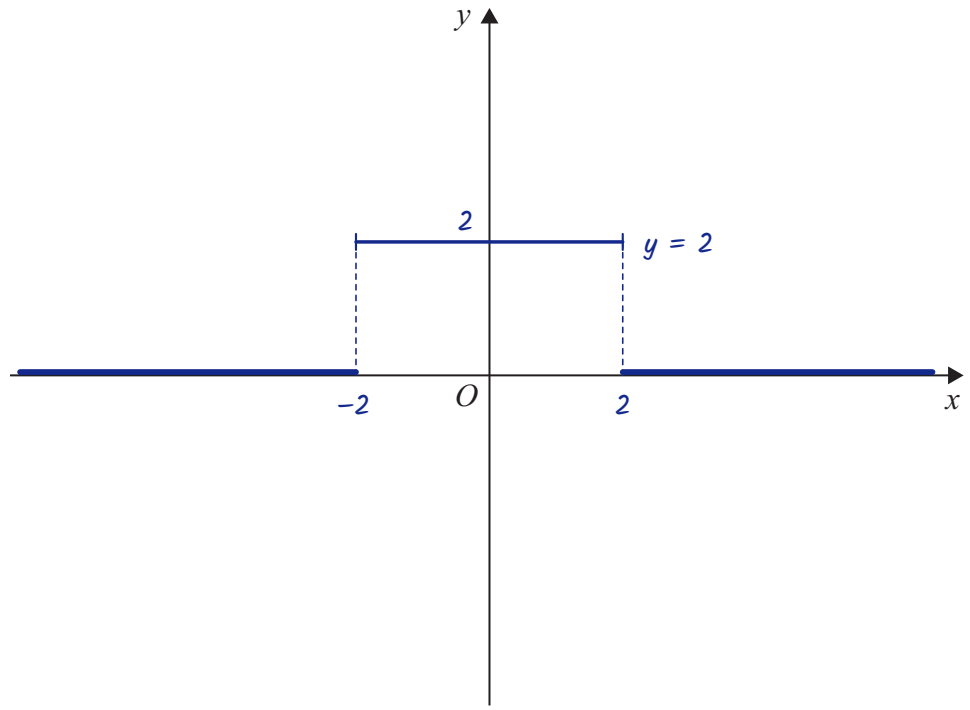
6 (b) Sketch the graph of $y = 2f(x) - 4$

[2 marks]



6 (c) Sketch the graph of $y = f'(x)$

[3 marks]



Turn over for the next question

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7 (a) Using ${}^nC_r = \dfrac{n!}{r!(n-r)!}$ show that ${}^nC_2 = \dfrac{n(n-1)}{2}$

[2 marks]

${}^nC_2 = \dfrac{n!}{2!(n-2)!}$

$= \dfrac{n \times (n-1) \times (n-2)!}{2 \times 1 \times (n-2)!}$

$= \dfrac{n(n-1)}{2}$ as required

7 (b) (i) Show that the equation

$$2 \times {}^nC_4 = 51 \times {}^nC_2$$

simplifies to

$$n^2 - 5n - 300 = 0$$

[3 marks]

$2 \times \dfrac{n!}{4!(n-4)!} = 51 \times \dfrac{n!}{2!(n-2)!}$

$2 \times \dfrac{n(n-1)(n-2)(n-3)}{24} = 51 \times \dfrac{n(n-1)}{2}$

Divide both sides by $n(n-1)/2$ $(n \geq 4$ so $n(n-1) \neq 0)$:

$\dfrac{(n-2)(n-3)}{6} = 51$

$(n-2)(n-3) = 306$

$n^2 - 5n + 6 = 306$

$n^2 - 5n - 300 = 0$ as required

Do not write outside the box

7 (b) (ii) Hence, solve the equation

$$2 \times {}^{n}C_4 = 51 \times {}^{n}C_2$$

[2 marks]

$n^2 - 5n - 300 = 0$

$(n - 20)(n + 15) = 0$

$n = 20$ or $n = -15$

n must be a positive integer, so $n = 20$

Turn over for the next question

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8 The sum to infinity of a geometric series is 96

The first term of the series is less than 30

The second term of the series is 18

8 (a) Find the first term and common ratio of the series.

[5 marks]

Second term: $ar = 18$

Sum to infinity: $\frac{a}{1-r} = 96$

$a = \frac{18}{r} \Rightarrow \frac{18}{r(1-r)} = 96$

$96r(1-r) = 18 \Rightarrow 96r^2 - 96r + 18 = 0$

$16r^2 - 16r + 3 = 0 \Rightarrow (4r-1)(4r-3) = 0$

$r = \frac{1}{4}$ or $r = \frac{3}{4}$

$r = \frac{1}{4} \Rightarrow a = 72$; $r = \frac{3}{4} \Rightarrow a = 24$

The first term is less than 30, so $a = 24$

First term $a = 24$, common ratio $r = \frac{3}{4}$

8 (b) (i) Show that the nth term of the series, u_n, can be written as

$$u_n = \frac{3^n}{2^{2n-5}}$$

[4 marks]

$u_n = ar^{n-1} = 24 \times (\frac{3}{4})^{n-1}$

$24 = 2^3 \times 3$ and $\frac{3}{4} = \frac{3}{2^2}$

Do not write outside the box

$u_n = 2^3 \times 3 \times \frac{3^{n-1}}{2^{2(n-1)}}$

$= \frac{3 \times 3^{n-1}}{2^{-3} \times 2^{2n-2}}$

$= \frac{3^n}{2^{2n-2-3}}$

$= \frac{3^n}{2^{2n-5}}$ as required

8 (b) (ii) Hence show that

$$\log_3 u_n = n(1 - 2\log_3 2) + 5\log_3 2$$

[3 marks]

$\log_3 u_n = \log_3 \frac{3^n}{2^{2n-5}}$

$= \log_3 3^n - \log_3 2^{2n-5}$

$= n \log_3 3 - (2n - 5) \log_3 2$

$= n - 2n \log_3 2 + 5 \log_3 2$

$= n(1 - 2 \log_3 2) + 5 \log_3 2$ as required

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9 (a) For $\cos\theta \neq 0$, prove that

$$\operatorname{cosec} 2\theta + \cot 2\theta = \cot\theta$$

[4 marks]

LHS = cosec 2θ + cot 2θ = $\frac{1}{\sin 2\theta} + \frac{\cos 2\theta}{\sin 2\theta}$

= $\frac{1 + \cos 2\theta}{\sin 2\theta}$

= $\frac{1 + (2\cos^2\theta - 1)}{2 \sin\theta \cos\theta}$

= $\frac{2\cos^2\theta}{2 \sin\theta \cos\theta}$

= $\frac{\cos\theta}{\sin\theta}$ (dividing by cos θ, since cos θ ≠ 0)

= cot θ = RHS as required

9 (b) Explain why

$$\cot\theta \neq \operatorname{cosec} 2\theta + \cot 2\theta$$

when $\cos\theta = 0$

[1 mark]

When cos θ = 0: cot θ = cos θ ÷ sin θ = 0, so the LHS is defined.

But sin 2θ = 2 sin θ cos θ = 0, so cosec 2θ = 1 ÷ sin 2θ and

cot 2θ = cos 2θ ÷ sin 2θ are undefined, so the RHS is undefined.

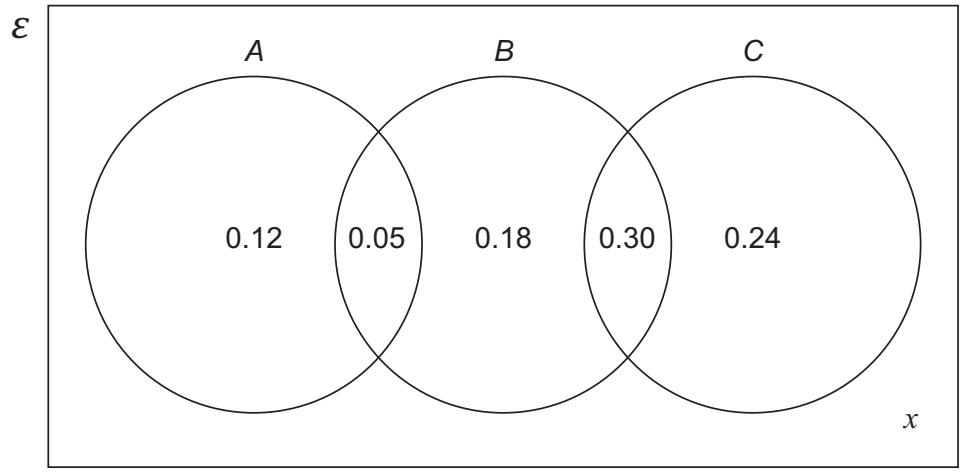
Hence cot θ ≠ cosec 2θ + cot 2θ when cos θ = 0.

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Section B

Answer **all** questions in the spaces provided.

10 The probabilities of events A, B and C are related, as shown in the Venn diagram below.



Find the value of x.

Circle your answer.

[1 mark]

0.11 0.46 0.54 0.89

11 The table below shows the temperature on Mount Everest on the first day of each month.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temperature (°C)	−17	−16	−14	−9	−2	2	6	5	−3	−4	−11	−18

Calculate the standard deviation of these temperatures.

Circle your answer.

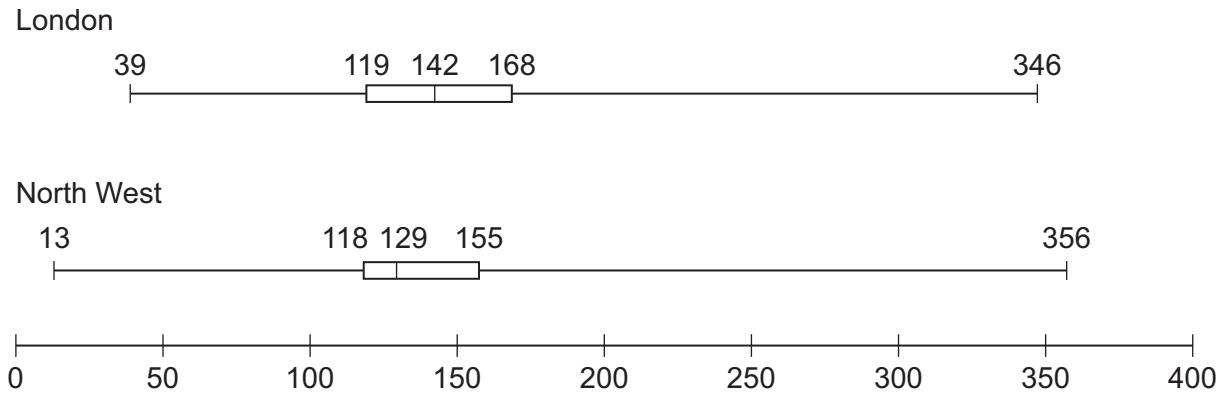
[1 mark]

−6.75 5.82 8.24 67.85

Turn over ▶

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12 The box plot below summarises the CO_2 emissions, in g/km, for cars in the Large Data Set from the London and North West regions.



12 (a) Using the box plot, give **one** comparison of central tendency and **one** comparison of spread for the two regions.

[2 marks]

Comparison of central tendency

The median for London (142 g/km) is greater than the median for the North West (129 g/km).

Comparison of spread

The range for London (346 – 39 = 307 g/km) is less than the range for the North West (356 – 13 = 343 g/km).

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12 (b) Jaspal, an environmental researcher, used all of the data in the Large Data Set to produce a statistical comparison of the CO_2 and CO emissions in regions of England.

Using your knowledge of the Large Data Set, give **two** reasons why his conclusions may be invalid.

[2 marks]

1. Not all makes of car are included in the Large Data Set, so the data may not be representative of all cars.

2. Not all regions of England are included in the Large Data Set, so he cannot compare the emissions in all regions of England.

Turn over for the next question

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13 Diedre is a head teacher in a school which provides primary, secondary and sixth-form education.

There are 200 teachers in her school.

The number of teachers in each level of education along with their gender is shown in the table below.

	Primary	**Secondary**	**Sixth-form**
Male	9	24	23
Female	35	85	24

13 (a) A teacher is selected at random. Find the probability that:

13 (a) (i) the teacher is female

[1 mark]

$P(\text{female}) = \frac{35 + 85 + 24}{200} = \frac{144}{200} = \frac{18}{25}$

13 (a) (ii) the teacher is **not** a sixth-form teacher.

[1 mark]

$P(\text{not sixth-form}) = 1 - \frac{23 + 24}{200} = \frac{153}{200} = 0.765$

13 (b) Given that a randomly chosen teacher is male, find the probability that this teacher is **not** a primary teacher.

[2 marks]

Number of male teachers $= 9 + 24 + 23 = 56$

$P(\text{not primary} \mid \text{male}) = \frac{24 + 23}{56} = \frac{47}{56}$

$= 0.839$ (3 s.f.)

Do not write outside the box

13 (c) Diedre wants to select three different teachers at random to be part of a school project.

Calculate the probability that all three chosen are secondary teachers.

[2 marks]

Number of secondary teachers = 24 + 85 = 109

$P(\text{all three secondary}) = \frac{109}{200} \times \frac{108}{199} \times \frac{107}{198}$

$= 0.159840...$

$= 0.160$ (3 s.f.)

Turn over for the next question

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14 It is known that a hospital has a mean waiting time of 4 hours for its Accident and Emergency (A&E) patients.

After some new initiatives were introduced, a random sample of 12 patients from the hospital's A&E Department had the following waiting times, in hours.

4.25	3.90	4.15	3.95	4.20	4.15
5.00	3.85	4.25	4.05	3.80	3.95

Carry out a hypothesis test at the 10% significance level to investigate whether the mean waiting time at this hospital's A&E department has changed.

You may assume that the waiting times are normally distributed with standard deviation 0.8 hours.

[7 marks]

μ = population mean waiting time (hours) at this A&E

H_0: $\mu = 4$ H_1: $\mu \neq 4$ (two-tailed test)

$\bar{x} = \frac{49.5}{12} = 4.125$

Under H_0: $\bar{x} \sim N(4, \frac{0.8^2}{12})$

Test statistic $z = \frac{4.125 - 4}{0.8 \div \sqrt{12}} = 0.54126...$

10% significance, two-tailed: critical values $z = \pm 1.6449$

$-1.6449 < 0.541 < 1.6449$, so z is not in the critical region

Do not reject H_0 (accept H_0).

There is insufficient evidence at the 10% level to suggest that the mean waiting time at this hospital's A&E department has changed.

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Turn over for the next question

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15 A political party is holding an election to choose a new leader.

A statistician within the party decides to sample 70 party members to find their opinions of the leadership candidates.

There are 4735 members under 30 years old and 8565 members 30 years old and over.

The statistician wants to use a sample of 70 party members in the survey.

He decides to use a random stratified sample.

15 (a) Calculate how many of each age group should be included in his sample.

[2 marks]

Total members = 4735 + 8565 = 13 300

Under 30: $\frac{4735}{13\,300} \times 70 = 24.92... \approx 25$

30 and over: $\frac{8565}{13\,300} \times 70 = 45.07... \approx 45$

25 members under 30 and 45 members aged 30 and over

15 (b) Explain how he could collect the random sample of members under 30 years old.

[3 marks]

Obtain a list of all 4735 party members under 30 years old and give each member a number from 1 to 4735.

Use a calculator (random number generator) to generate random four-digit integers between 1 and 4735.

Select the members whose numbers are generated, ignoring repeats and numbers outside 1 to 4735, until 25 different members have been chosen.

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16 An educational expert found that the correlation coefficient between the hours of revision and the scores achieved by 25 students in their A-level exams was 0.379

Her data came from a bivariate normal distribution.

Carry out a hypothesis test at the 1% significance level to determine if there is a positive correlation between the hours of revision and the scores achieved by students in their A-level exams.

The critical value of the correlation coefficient is 0.4622

[4 marks]

ρ = population correlation coefficient

H_0: $\rho = 0$ H_1: $\rho > 0$ (one-tailed test)

$r = 0.379$, critical value = 0.4622 (1% level, $n = 25$)

$0.379 < 0.4622$, so r is not in the critical region

Do not reject H_0 (accept H_0).

There is insufficient evidence at the 1% level to suggest that there is a positive correlation between the hours of revision and the scores achieved by students in their A-level exams.

Turn over for the next question

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17 The lifetime of Zaple smartphone batteries, X hours, is normally distributed with mean 8 hours and standard deviation 1.5 hours.

17 (a) (i) Find $P(X \neq 8)$

[1 mark]

X is continuous, so $P(X = 8) = 0$

$P(X \neq 8) = 1 - 0 = 1$

17 (a) (ii) Find $P(6 < X < 10)$

[1 mark]

$X \sim N(8, 1.5^2)$

$P(6 < X < 10) = 0.81757...$

$= 0.818$ (3 s.f.)

17 (b) Determine the lifetime exceeded by 90% of Zaple smartphone batteries.

[2 marks]

$P(X > x) = 0.9 \quad \Rightarrow \quad P(X < x) = 0.1$

$\frac{x - 8}{1.5} = -1.28155...$

$x = 8 - 1.5 \times 1.28155... = 6.0777...$

Lifetime = 6.08 hours (3 s.f.)

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17 (c) A different smartphone, Kaphone, has its battery's lifetime, Y hours, modelled by a normal distribution with mean 7 hours and standard deviation σ.

25% of randomly selected Kaphone batteries last less than 5 hours.

Find the value of σ, correct to three significant figures.

[4 marks]

$Y \sim N(7, \sigma^2)$, $P(Y < 5) = 0.25$

$P(Z < z) = 0.25 \Rightarrow z = -0.67449...$

$\frac{5 - 7}{\sigma} = -0.67449...$

$\sigma = \frac{-2}{-0.67449...} = 2.96520...$

$\sigma = 2.97$ (3 s.f.)

Turn over for the next question

Do not write outside the box

18 Tiana is a quality controller in a clothes factory. She checks for four possible types of defects in shirts.

Of the shirts with defects, the proportion of each type of defect is as shown in the table below.

Type of defect	Colour	Fabric	Sewing	Sizing
Probability	0.25	0.30	0.40	0.05

Shirts with defects are packed in boxes of 30 at random.

18 (a) Find the probability that:

18 (a) (i) a box contains exactly 5 shirts with a colour defect

[2 marks]

C = number of shirts with a colour defect, $C \sim B(30, 0.25)$

$P(C = 5) = {}^{30}C_5 \times 0.25^5 \times 0.75^{25}$

$= 0.104728...$

$= 0.105$ (3 s.f.)

18 (a) (ii) a box contains fewer than 15 shirts with a sewing defect

[2 marks]

S = number of shirts with a sewing defect, $S \sim B(30, 0.40)$

$P(S < 15) = P(S \leq 14)$

$= 0.824630...$

$= 0.825$ (3 s.f.)

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18 (a) (iii) a box contains at least 20 shirts which do **not** have a fabric defect.

[3 marks]

Y = number of shirts without a fabric defect, $Y \sim B(30, 0.70)$

$P(Y \geq 20) = 1 - P(Y \leq 19)$

$= 1 - 0.269629...$

$= 0.730370...$

$= 0.730$ (3 s.f.)

Question 18 continues on the next page

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18 (b) Tiana wants to investigate the proportion, p, of defective shirts with a fabric defect.

She wishes to test the hypotheses

$$H_0 : p = 0.3$$

$$H_1 : p < 0.3$$

She takes a random sample of 60 shirts with a defect and finds that x of them have a fabric defect.

18 (b) (i) Using a 5% level of significance, find the critical region for x.

[5 marks]

Under H_0: $X \sim B(60, 0.3)$

Lower-tailed test at 5%: need $P(X \leq c) \leq 0.05$

$P(X \leq 11) = 0.029475\ldots < 0.05$

$P(X \leq 12) = 0.056770\ldots > 0.05$

Critical region is $x \leq 11$

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18 (b) (ii) In her sample she finds 13 shirts with a fabric defect.

Complete the test stating her conclusion in context.

[2 marks]

13 > 11, so 13 is not in the critical region

Do not reject H_0 (accept H_0).

There is insufficient evidence at the 5% level to suggest that the proportion of defective shirts with a fabric defect has decreased.

END OF QUESTIONS

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