AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level MATHEMATICS

Paper 3

Thursday 19 June 2025 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do not write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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Section A

Answer **all** questions in the spaces provided.

1 A student states that the product of two irrational numbers is always irrational.

One of the options is a counter example that shows the student's statement is incorrect.

Identify the counter example.

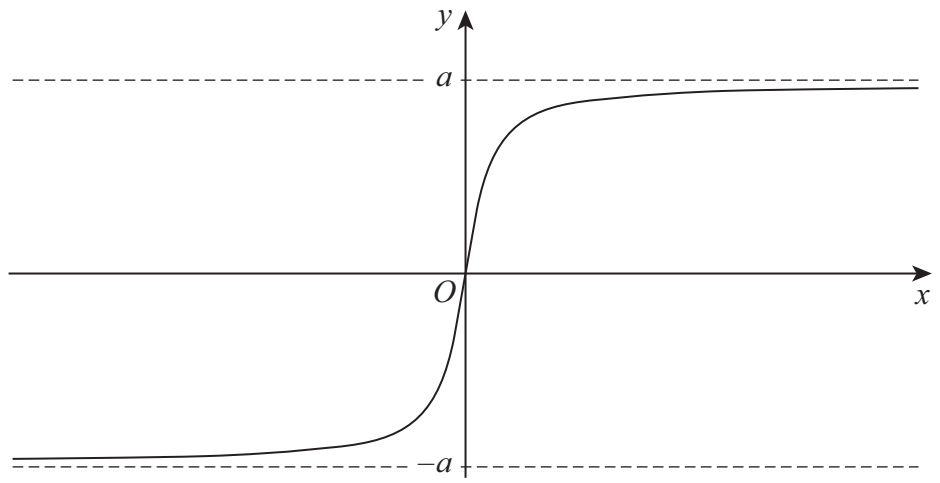
Circle your answer.

[1 mark]

$e \times 0 = 0$ $\quad \frac{1}{\pi} \times \pi = 1$ $\quad \frac{1}{\sqrt{2}} \times \sqrt{6} = \sqrt{3}$ $\quad 2 \times \sqrt{3} = \sqrt{6}$

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2 The diagram shows the graph of $y = \arctan x$



The graph has horizontal asymptotes at $y = -a$ and $y = a$

State the value of a

Circle your answer.

[1 mark]

$\frac{\pi}{4}$ 1 $\frac{\pi}{2}$ π

Turn over for the next question

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3 The function f is defined by

$$f(x) = e^x \quad \text{for} \quad x \in \mathbb{R}$$

Identify which one of the following statements describes the function f

Tick (✓) **one** box.

[1 mark]

Decreasing and concave ☐

Decreasing and convex ☐

Increasing and concave ☐

Increasing and convex ☑

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4 (a) State the value of

$$\log_a a^2$$

where $a > 0$

[1 mark]

$\log_a a^2 = 2 \log_a a = 2$

4 (b) Find the value of

$$\log_a \sqrt{a} - \log_a \frac{1}{a^2}$$

where $a > 0$

[2 marks]

$\log_a \sqrt{a} - \log_a \frac{1}{a^2} = \log_a a^{1/2} - \log_a a^{-2}$

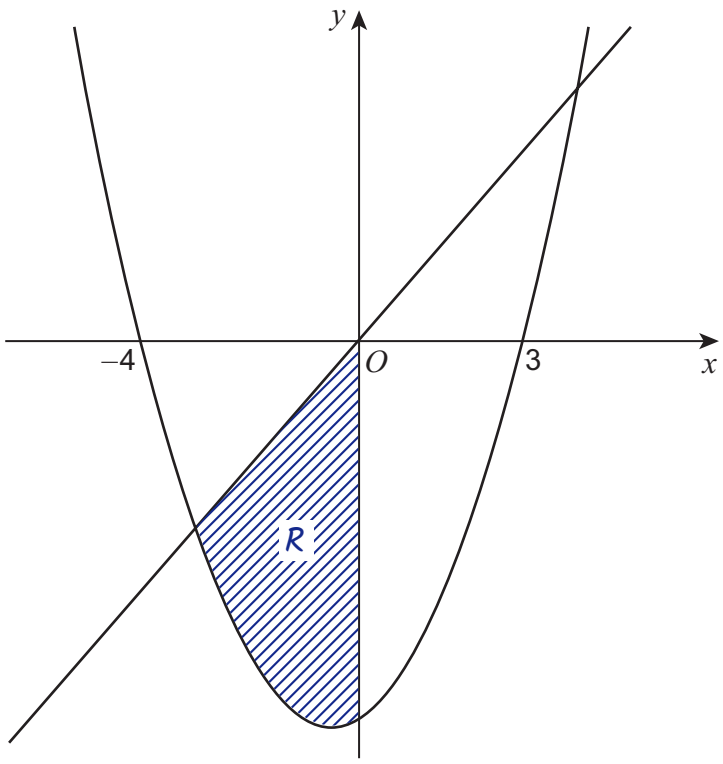
$= \frac{1}{2} - (-2)$

$= \frac{5}{2} \quad (= 2.5)$

Turn over for the next question

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5 The diagram shows the graphs of $y = 2x$ and $y = (x - 3)(x + 4)$.



The region R is defined by the inequalities

$$(x - 3)(x + 4) \leq y \leq 2x \quad \textbf{and} \quad x \leq 0$$

Shade the region R on the diagram.

[2 marks]

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6 The first four terms in ascending powers of x of the binomial expansion of

$$(2 - 3x)^5$$

are given by

$$32 + px + qx^2 - 1080x^3$$

where p and q are constants.

6 (a) Find the value of p and the value of q

[2 marks]

$(2 - 3x)^5 = 2^5 + {}^5C_1(2^4)(-3x) + {}^5C_2(2^3)(-3x)^2 + \ldots$

$p = {}^5C_1 \times 2^4 \times (-3) = 5 \times 16 \times (-3) = -240$

$q = {}^5C_2 \times 2^3 \times (-3)^2 = 10 \times 8 \times 9 = 720$

$p = -240, \quad q = 720$

6 (b) Hence find an approximation for 1.94^5

Give your answer to five decimal places.

[2 marks]

$2 - 3x = 1.94 \quad \Rightarrow \quad x = 0.02$

$1.94^5 \approx 32 - 240(0.02) + 720(0.02)^2 - 1080(0.02)^3$

$= 32 - 4.8 + 0.288 - 0.00864$

$= 27.47936$

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7 Solve the equation

$$\sum_{r=1}^{3} (ar + 5) = 57$$

to find the value of a

[3 marks]

$\Sigma (ar + 5)$ for $r = 1$ to $3 = (a + 5) + (2a + 5) + (3a + 5)$

$= 6a + 15$

$6a + 15 = 57$

$6a = 42$

$a = 7$

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Turn over for the next question

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8 (a) The expression

$$\frac{x}{2x^2 + 3x + 1}$$

can be written in the form

$$\frac{A}{x+1} + \frac{B}{2x+1}$$

Find the value of A and the value of B

[3 marks]

$2x^2 + 3x + 1 = (x + 1)(2x + 1)$

$\frac{x}{(x+1)(2x+1)} = \frac{A}{x+1} + \frac{B}{2x+1} \Rightarrow x = A(2x + 1) + B(x + 1)$

$x = -1$: $-1 = A(-1) \Rightarrow A = 1$

$x = -\frac{1}{2}$: $-\frac{1}{2} = B(\frac{1}{2}) \Rightarrow B = -1$

$A = 1, \quad B = -1$

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8 (b) Use your answer to part **(a)** to show that

$$\int_0^4 \frac{x}{2x^2 + 3x + 1}\,\mathrm{d}x = \ln q$$

where q is a rational number to be found.

[4 marks]

$\int_0^4 \frac{x}{2x^2 + 3x + 1}\,dx = \int_0^4 \left(\frac{1}{x + 1} - \frac{1}{2x + 1} \right) dx$

$= \left[\ln(x + 1) - \frac{1}{2} \ln(2x + 1) \right]_0^4$

$= \left(\ln 5 - \frac{1}{2} \ln 9 \right) - \left(\ln 1 - \frac{1}{2} \ln 1 \right)$

$= \ln 5 - \ln 9^{1/2} - 0$

$= \ln 5 - \ln 3$

$= \ln \frac{5}{3}$ so $q = \frac{5}{3}$

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9 (a) Describe a sequence of **two** transformations which maps the graph with equation

$$y = \frac{1}{x}$$

onto the graph with equation

$$y = \frac{3}{x-4}$$

[2 marks]

Stretch parallel to the y-axis, scale factor 3 $\left(y = \frac{3}{x}\right)$

followed by a translation by the vector $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

9 (b) State the equation of the vertical asymptote of the graph with equation

$$y = \frac{3}{x-4}$$

[1 mark]

$x = 4$

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9 (c) A student is attempting to use a change of sign to determine if the equation

$$\frac{3}{x-4} = x$$

has a solution between 3 and 5

The student **correctly** writes

$$\frac{3}{x-4} = x \Leftrightarrow \frac{3}{x-4} - x = 0$$

$$\text{Let } f(x) = \frac{3}{x-4} - x$$

$$f(3) = -6 < 0 \text{ and } f(5) = -2 < 0$$

The student then **incorrectly** states:

"Since there is no change of sign, there is no solution between 3 and 5"

Give two reasons why the student's argument is invalid.

[2 marks]

Reason 1: *There could be more than one root (e.g. two roots) between 3 and 5, so the signs can be the same even when roots exist.*

Reason 2: *f(x) is not continuous between 3 and 5; there is a vertical asymptote at x = 4 (the points are either side of it).*

Turn over for the next question

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10 (a) Express

$$2\sin x + 5\cos x$$

in the form

$$R\sin(x + \alpha)$$

where $R > 0$ and $0° \leq \alpha \leq 90°$

[3 marks]

$2 \sin x + 5 \cos x \equiv R \sin x \cos \alpha + R \cos x \sin \alpha$

$R \cos \alpha = 2, \quad R \sin \alpha = 5$

$R = \sqrt{(2^2 + 5^2)} = \sqrt{29} \quad (= 5.385...)$

$\tan \alpha = \frac{5}{2} \quad \Rightarrow \quad \alpha = 68.198...° = 68.2°$

$2 \sin x + 5 \cos x = \sqrt{29} \sin (x + 68.2°)$

10 (b) In 2010, the water temperature at a beach could be modelled by the formula

$$T = 14.2 - (2\sin d° + 5\cos d°)$$

where:

- T is the temperature of the water in °C
- d is the number of days after 1 January 2010

10 (b) (i) Find the value of d when the temperature of the water reached its lowest value in 2010

[1 mark]

T lowest when $\sin (d + 68.2) = 1$: $d + 68.2 = 90$

$d = 90 - 68.2 = 21.8$

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10 (b) (ii) Find the maximum temperature of the water predicted by the model in 2010

Give your answer to one decimal place.

[1 mark]

$T_{max} = 14.2 + \sqrt{29} = 19.585... = 19.6\ °C$

10 (b) (iii) Find the number of **weeks** in 2010 for which the temperature of the water was higher than 15 °C

Give your answer to the nearest whole number.

[5 marks]

$T = 14.2 - \sqrt{29} \sin(d + 68.2) = 15$

$\sin(d + 68.2) = \frac{14.2 - 15}{\sqrt{29}} = -0.14855...$

$d + 68.2 = 188.543...$ or $351.456...$

$(\text{as } 68.2 \leq d + 68.2 \leq 433.2)$

$d = 120.34...$ or $d = 283.25...$

$T > 15$ for $120.34 < d < 283.25$

Number of days $= 283.25 - 120.34 = 162.91$ days

Number of weeks $= 162.91 \div 7 = 23.27...$

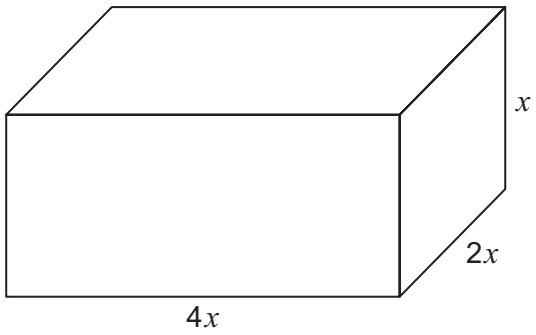
23 weeks

Turn over ▶

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11 A block of ice is melting.

At time t minutes, the block is in the shape of a cuboid with dimensions of $4x$, $2x$ and x, as shown in the diagram.



All measurements are in centimetres.

11 (a) The volume, V cm^3, of the block of ice **decreases** at a rate which is proportional to its surface area.

When $x = 4$ the volume of the block of ice is decreasing at a rate of 7 cm^3 per minute.

Show that

$$\frac{dV}{dt} = -0.4375x^2$$

[4 marks]

Surface area $S = 2(4x \times 2x + 4x \times x + 2x \times x)$

$= 2(8x^2 + 4x^2 + 2x^2) = 28x^2$

$\frac{dV}{dt} = -kS = -28kx^2 \quad (k > 0)$

When $x = 4$, $\frac{dV}{dt} = -7$: $\quad -7 = -28k \times 4^2$

$-7 = -448k \Rightarrow k = \frac{7}{448} = \frac{1}{64}$

$\frac{dV}{dt} = -28 \times \frac{1}{64} x^2 = -\frac{28}{64} x^2 = -0.4375x^2$ as required

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11 (b) Find $\frac{dV}{dx}$ in terms of x

[2 marks]

$V = 4x \times 2x \times x = 8x^3$

$\frac{dV}{dx} = 24x^2$

11 (c) (i) Using the results from parts **(a)** and **(b)**, find $\frac{dx}{dt}$

[2 marks]

$\frac{dx}{dt} = \frac{dV}{dt} \times \frac{dx}{dV} = -0.4375x^2 \times \frac{1}{24x^2}$

$\frac{dx}{dt} = -\frac{0.4375}{24} = -\frac{7}{384}$ $(= -0.0182...)$

11 (c) (ii) Interpret, in context, your answer to part **(c)(i)**.

[2 marks]

The length x (the height of the block) is decreasing at a constant rate of 0.0182 cm per minute (7/384 cm per minute). The rate does not depend on x, i.e. on the size of the block.

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12 Given that the real numbers a, b, and c are such that

$$a \times b = c$$

Use proof by contradiction to show that if c is irrational then at least one of a or b is irrational.

[3 marks]

Assume that c is irrational and that a and b are both rational.

Then $a = m/n$ and $b = p/q$ where m, n, p, q are integers, $n, q \neq 0$

$c = a \times b = \frac{m}{n} \times \frac{p}{q} = \frac{mp}{nq}$

mp and nq are integers ($nq \neq 0$), so c is rational.

This contradicts the fact that c is irrational.

So the assumption is false: if c is irrational, then at least one of a or b is irrational.

END OF SECTION A

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Section B

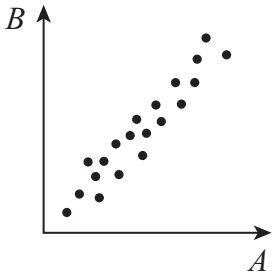
Answer **all** questions in the spaces provided.

13 The variables A and B have a product moment correlation coefficient of 0.47

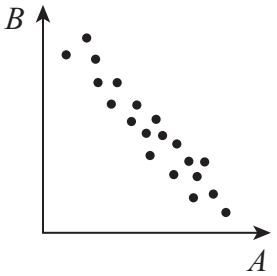
Identify which one of the scatter diagrams shows the relationship between A and B.

Tick (✓) **one** box.

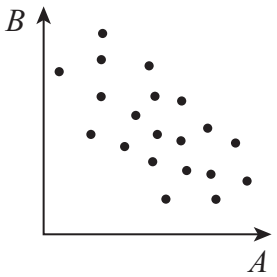
[1 mark]



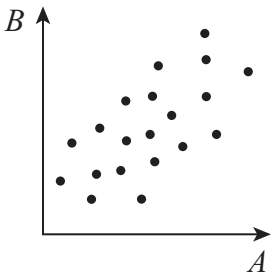
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14 It is given that $X \sim N(9, 1.5^2)$ and $P(a \leq X \leq b) \approx 95\%$

Identify the correct pair of values of a and b.

Tick (✓) **one** box.

[1 mark]

$a = 4.5$ and $b = 13.5$ ☐

$a = 6$ and $b = 12$ ☑

$a = 7$ and $b = 11$ ☐

$a = 7.5$ and $b = 10.5$ ☐

Turn over for the next question

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15 The proportion of diners at a restaurant who are vegan is 20%

The number of diners at the restaurant who are vegan can be modelled by a binomial distribution.

15 (a) In a random sample of 25 diners:

15 (a) (i) find the probability that no diner is a vegan.

[1 mark]

$X \sim B(25, 0.2)$

$P(X = 0) = 0.8^{25} = 0.003777... = 0.00378$

15 (a) (ii) find the probability that fewer than 6 diners are vegan.

[1 mark]

$P(X < 6) = P(X \leq 5) = 0.61668... = 0.617$

15 (a) (iii) find the probability that at least 9 diners are vegan.

[2 marks]

$P(X \geq 9) = 1 - P(X \leq 8)$

$= 1 - 0.95322...$

$= 0.04677... = 0.0468$

15 (b) State one assumption necessary for the binomial distribution to be valid in the context of this question.

[1 mark]

Whether a diner is vegan is independent from one diner to the next (the probability of a diner being vegan is always 20%).

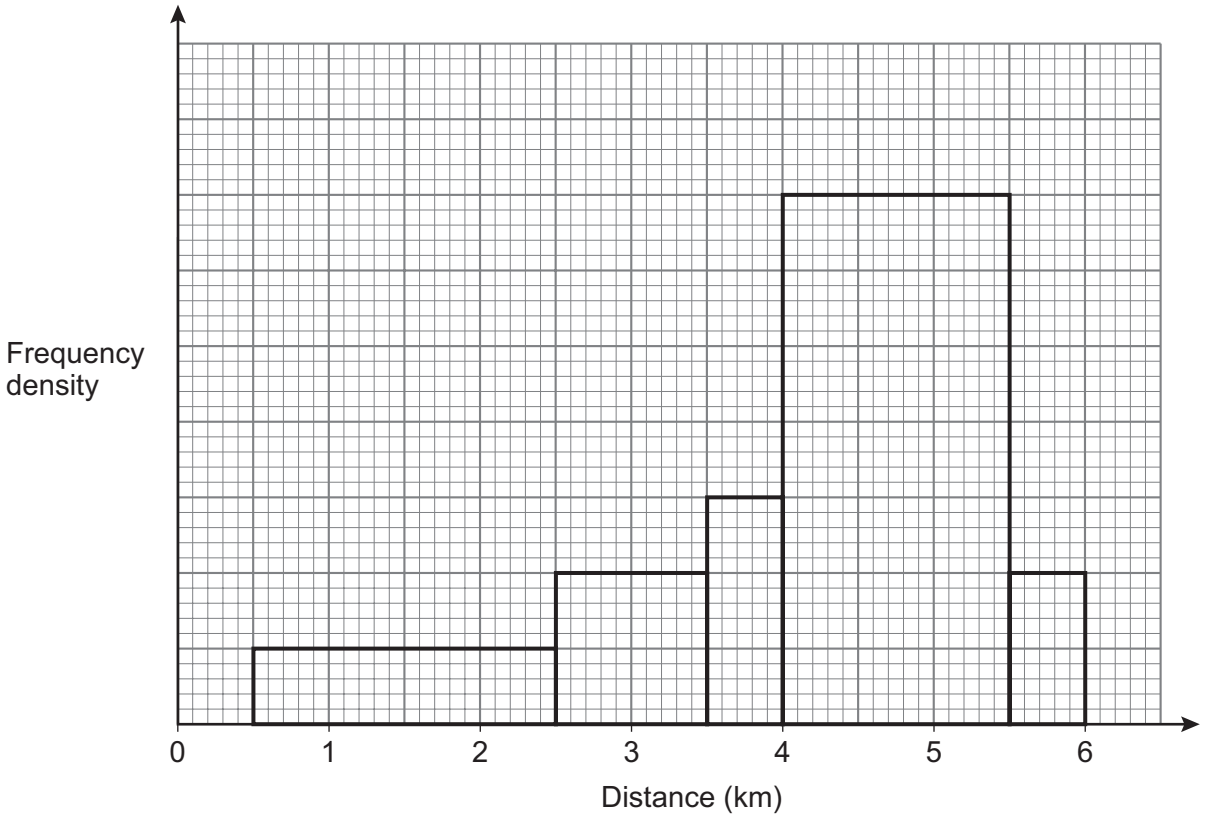
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16 The distance, in kilometres, travelled from home to work by 340 individuals is shown in the histogram.



16 (a) (i) Estimate the number of individuals who travel between 3 km and 5 km from home to work.

Fully justify your answer.

[4 marks]

Taking 1 cm wide by 1 cm high as one unit of area:

Total area = 2 × 1 + 1 × 2 + 0.5 × 3 + 1.5 × 7 + 0.5 × 2

= 2 + 2 + 1.5 + 10.5 + 1 = 17 units

340 individuals ⇔ 17 units, so 1 unit = 340 ÷ 17 = 20 individuals

Area between 3 km and 5 km:

3 to 3.5: 0.5 × 2 = 1; 3.5 to 4: 0.5 × 3 = 1.5; 4 to 5: 1 × 7 = 7

Total area = 1 + 1.5 + 7 = 9.5 units

Number = 20 × 9.5 = 190

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Estimate: 190 individuals

16 (a) (ii) Give one reason why your answer to part **(a)(i)** is only an estimate.

[1 mark]

The actual distance travelled by each person is unknown: the distances are grouped, so we assume they are evenly spread.

16 (b) The histogram shows that the data is skewed.

State the type of skewness.

[1 mark]

Negative skew

16 (c) Explain why frequency density is used in this histogram.

[1 mark]

Because the class widths are different (unequal), so the area of each bar represents the frequency.

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17 A maths teacher holds a revision session every Wednesday.

In a random sample of eight Wednesdays, the number of students who attended is listed below.

2 4 3 1 4 5 2 3

17 (a) Find the mean of these eight values.

[1 mark]

Mean = $(2 + 4 + 3 + 1 + 4 + 5 + 2 + 3) \div 8 = 24 \div 8 = 3$

17 (b) Find the variance of these eight values.

[1 mark]

$\Sigma x^2 = 4 + 16 + 9 + 1 + 16 + 25 + 4 + 9 = 84$

Variance $= 84 \div 8 - 3^2 = 10.5 - 9 = 1.5$

17 (c) The teacher believes that the number of students who attended a revision session each Wednesday can be modelled by the binomial distribution B(30, 0.1).

Comment on whether the mean and variance found in parts **(a)** and **(b)** support the teacher's belief.

Fully justify your answer.

[3 marks]

For B(30, 0.1):

Mean $= np = 30 \times 0.1 = 3$

Variance $= np(1 - p) = 30 \times 0.1 \times 0.9 = 2.7$

The sample mean (3) is the same as the model mean,

but the sample variance (1.5) is very different from 2.7.

So the mean and variance do not support the teacher's belief:

B(30, 0.1) is not a suitable model.

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17 (d) The teacher wanted to decide if a Saturday morning revision session was worth doing.

The teacher asked the first 10 students who entered the classroom if they would attend a Saturday session.

17 (d) (i) Name this method of sampling.

[1 mark]

Opportunity sampling

17 (d) (ii) Describe **one advantage** of this method of sampling.

[1 mark]

It is quick and easy to carry out.

Turn over for the next question

Do not write outside the box

18 The cholesterol level, X, of an adult can be modelled by a normal distribution with mean 5.7 mmol/l and standard deviation 1.2 mmol/l.

18 (a) The lower quartile of X is a and the upper quartile is b

It is given that the $P(X \leq a) = 0.25$ and $P(X \geq b) = 0.25$

Find the interquartile range of X

[4 marks]

$X \sim N(5.7, 1.2^2)$

Standard normal: lower 25% point is $z = -0.67448...$

$a = 5.7 - 0.67448 \times 1.2 = 4.8906...$ (4.89)

$b = 5.7 + 0.67448 \times 1.2 = 6.5093...$ (6.51)

$IQR = b - a = 6.5093... - 4.8906...$

$= 1.6187... = 1.62$ mmol/l

Do not write outside the box

18 (b) A group of scientists believe that a new dietary supplement can reduce the mean cholesterol level.

A random sample of 160 adults were given the new dietary supplement for a trial period of one year.

After the trial, their cholesterol level was found to have a mean of 5.6 mmol/l.

Carry out a hypothesis test at the 5% significance level to investigate whether the mean cholesterol level has **reduced** after taking the dietary supplement.

It can be assumed that the cholesterol level after taking the dietary supplement follows a normal distribution with an unchanged variance.

[6 marks]

H_0: $\mu = 5.7$ H_1: $\mu < 5.7$

where μ is the mean cholesterol level (mmol/l) after the supplement

Under H_0: $\bar{X} \sim N(5.7, \frac{1.2^2}{160})$ i.e. $\bar{X} \sim N(5.7, 0.009)$

Standard deviation of $\bar{X} = 1.2 \div \sqrt{160} = 0.094868...$

$P(\bar{X} \leq 5.6) = 0.14592... = 0.146$

(test statistic $z = \frac{5.6 - 5.7}{1.2 \div \sqrt{160}} = -1.054...$)

$0.146 > 0.05$ (equivalently $-1.054 > -1.645$)

The result is not significant, so do not reject H_0

There is insufficient evidence at the 5% level of significance to suggest that the mean cholesterol level has reduced after taking the dietary supplement.

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19 Three events X, Y and Z are such that

$$P(X) = P(Y) = 0.38$$

$$P(X \cup Z) = 0.5$$

$$P[(X \cup Y)'] = 0.35$$

X and Z are mutually exclusive.

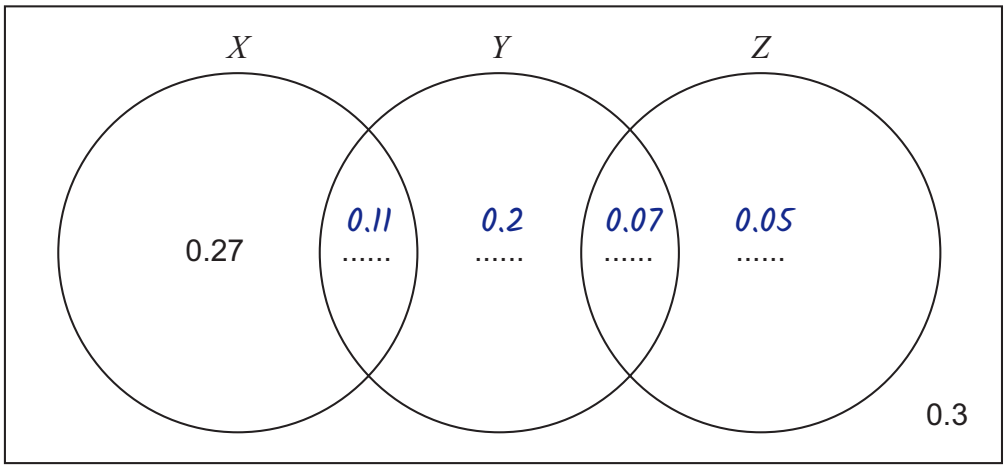
19 (a) Find $P(Z)$

[1 mark]

$P(Z) = P(X \cup Z) - P(X) = 0.5 - 0.38 = 0.12$

19 (b) Complete the Venn diagram.

[3 marks]



Do not write outside the box

19 (c) Find $P[(X \cap Y) \cup (Y \cap Z)]$

[2 marks]

$P[(X \cap Y) \cup (Y \cap Z)] = P(X \cap Y) + P(Y \cap Z)$

$= 0.11 + 0.07$

$= 0.18$

19 (d) Find $P(Y' \mid Z')$

[2 marks]

$P(Y' \cap Z') = 0.27 + 0.3 = 0.57, \quad P(Z') = 1 - 0.12 = 0.88$

$P(Y' \mid Z') = \frac{P(Y' \cap Z')}{P(Z')} = \frac{0.57}{0.88} = \frac{57}{88} \quad (= 0.6477...)$

19 (e) Show that the events Y and Z are **not** independent of each other.

[2 marks]

$P(Y) \times P(Z) = 0.38 \times 0.12 = 0.0456$

$P(Y \cap Z) = 0.07$

$0.0456 \neq 0.07$, so $P(Y \cap Z) \neq P(Y) \times P(Z)$

Hence Y and Z are not independent.

Do not write outside the box

20 The oxides of nitrogen emissions, F g/km, of a car registered in 2000 can be modelled by a normal distribution with mean 0.41 and standard deviation 0.07

20 (a) Find $P(F < 0.39)$

[1 mark]

$F \sim N(0.41, 0.07^2)$

$P(F < 0.39) = 0.38754... = 0.3875$

20 (b) Find $P(0.3 < F < 0.5)$

[1 mark]

$P(0.3 < F < 0.5) = 0.84268... = 0.8427$

20 (c) (i) Find $P(F > 0.6)$

[1 mark]

$P(F > 0.6) = 0.003320... = 0.00332$

20 (c) (ii) Explain why $P(F \geq 0.6) = P(F > 0.6)$ in this model.

[2 marks]

F is a continuous random variable (normal distribution),

so the probability of any single exact value is zero:

$P(F = 0.6) = 0$

$P(F \geq 0.6) = P(F = 0.6) + P(F > 0.6) = P(F > 0.6)$

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20 (d) The oxides of nitrogen emissions, F g/km, of a car in 2010 can be modelled by a normal distribution with mean 0.36 and standard deviation 0.09

Compare the oxides of nitrogen emissions in 2000 with those in 2010.

[2 marks]

On average, the oxides of nitrogen emissions in 2000 were higher than in 2010 (mean 0.41 > 0.36 g/km). The emissions in 2000 were less varied (more consistent) than in 2010 (standard deviation 0.07 < 0.09 g/km).

20 (e) A researcher collected data on the oxides of nitrogen emissions from the Large Data Set for cars registered in 2002 and 2016.

The researcher cleaned the data by removing the details for some cars.

Using your knowledge of the Large Data Set, give one reason why the researcher cleaned the data.

[1 mark]

Some cars in the Large Data Set have blank (missing) values for their oxides of nitrogen emissions.

20 (f) The researcher wished to make a comparison between the oxides of nitrogen emissions for a sample of Nissan cars in their local town from 2016 with the data from all cars registered in the same year in the Large Data Set.

Using your knowledge of the Large Data Set, give one reason why a meaningful comparison could **not** be made.

[1 mark]

There are no Nissan cars in the Large Data Set (it only has BMW, Ford, Toyota, Vauxhall and Volkswagen cars).

END OF QUESTIONS

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