AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level MATHEMATICS

Paper 2

Thursday 12 June 2025 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
TOTAL	

Do not write outside the box

Section A

Answer **all** questions in the spaces provided.

1 Describe the single transformation which maps the curve with the equation

$$y = \ln x$$

onto the curve with the equation

$$y = 2\ln x$$

Tick (✓) **one** box.

[1 mark]

Stretch, scale factor 2, parallel to the y-axis ☑

Stretch, scale factor $\frac{1}{2}$, parallel to the x-axis ☐

Translation $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ ☐

Translation $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ ☐

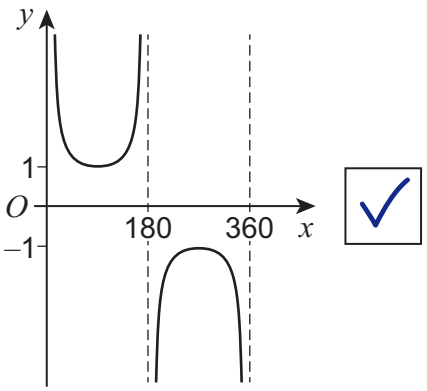
Do not write outside the box

2 One of the diagrams below shows the graph of $y = \text{cosec}\, x°$ for $0 \leq x \leq 360$

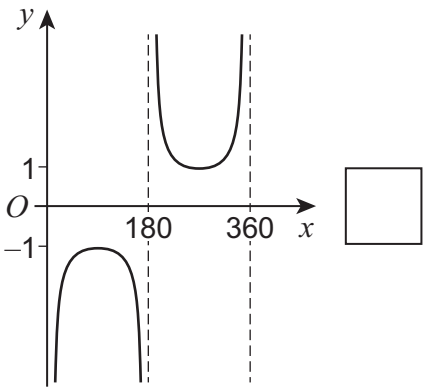
Identify the correct graph.

Tick (✓) **one** box.

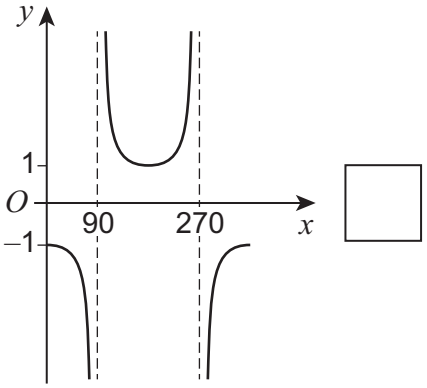
[1 mark]



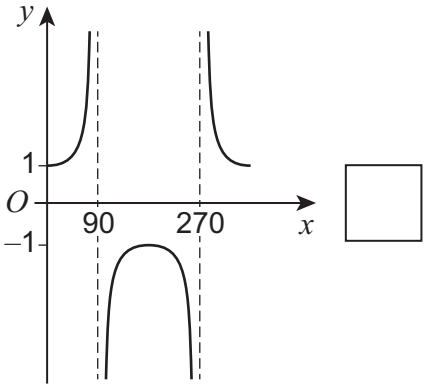
☑



☐



☐

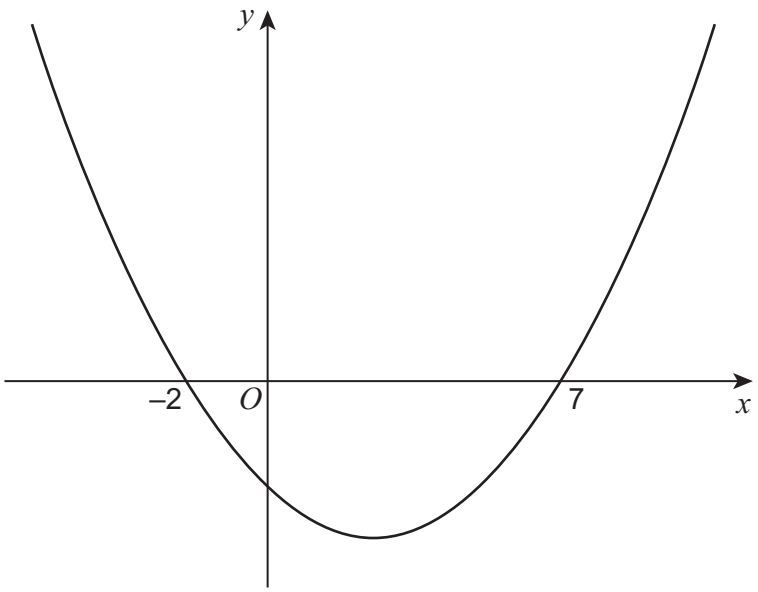


☐

Turn over for the next question

Do not write outside the box

3 The diagram shows the graph with equation $y = (x + 2)(x - 7)$



Solve the inequality.

$$(x + 2)(x - 7) > 0$$

Tick (✓) **one** box.

[1 mark]

$x \in (-\infty, -2) \cap (7, \infty)$ ☐

$x \in (-\infty, -2) \cup (7, \infty)$ ✓

$x \in (-\infty, -2] \cap [7, \infty)$ ☐

$x \in (-\infty, -2] \cup [7, \infty)$ ☐

Do not write outside the box

4 Solve the equation

$$5^{x-1} = 20$$

Give your answer in an exact form.

[2 marks]

$\log_5(5^{x-1}) = \log_5 20$

$x - 1 = \log_5 20$

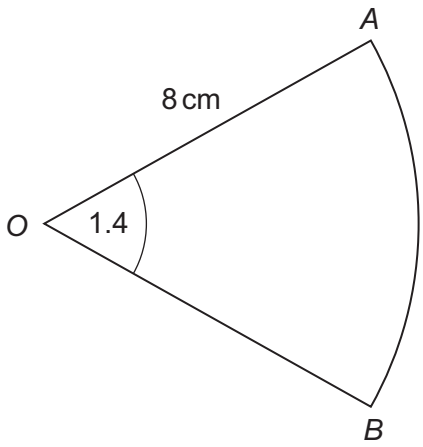
$x = 1 + \log_5 20$

$\left(= 1 + \frac{\ln 20}{\ln 5} \approx 2.86\right)$

Turn over for the next question

Do not write outside the box

5 The diagram shows a sector AOB of a circle with centre O and radius 8 cm.



The angle AOB is 1.4 radians.

Find the area of the sector.

[2 marks]

$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 8^2 \times 1.4$

$= 44.8 \text{ cm}^2$

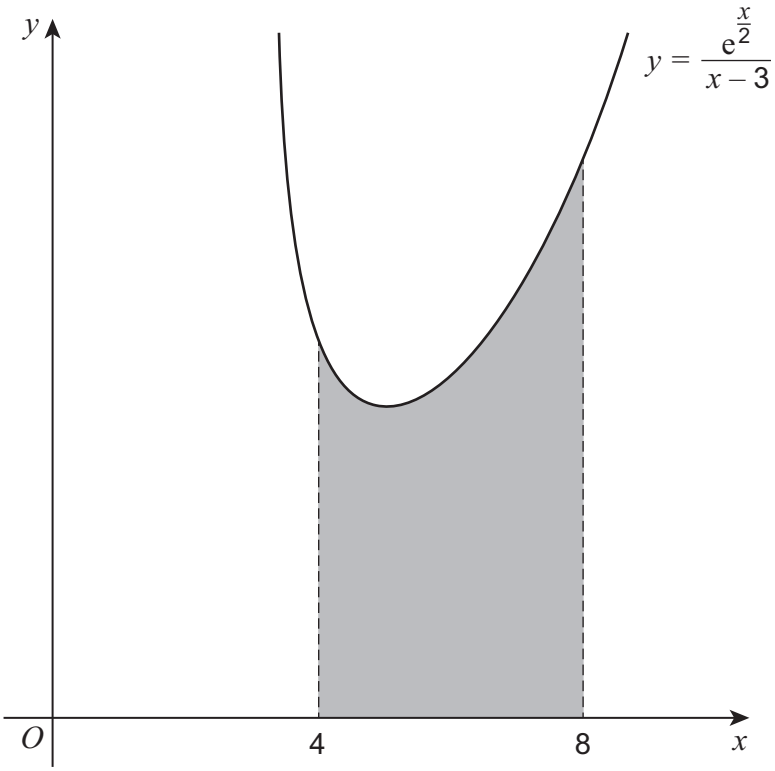
Do not write outside the box

Turn over for the next question

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

6 The curve with equation $y = \dfrac{e^{\frac{x}{2}}}{x-3}$ is shown in the diagram.



The region shaded is bounded by the curve, the x-axis, and the lines $x = 4$ and $x = 8$

The trapezium rule with six ordinates (5 strips) is to be used to find an approximate value for the area of the shaded region.

Some of the values required to obtain this approximation are shown in the table below.

x	4	4.8	5.6	6.4	7.2	8
y	7.3891	6.1240	6.3249	7.2155	8.7139	10.9196

6 (a) (i) Find the y-value that is missing from the table.

[1 mark]

$y = \dfrac{e^{3.2}}{6.4 - 3} = \dfrac{24.5325...}{3.4} = 7.21545...$

$y = 7.2155$ (4 d.p.)

Do not write outside the box

6 (a) (ii) Use the trapezium rule with six ordinates (5 strips) to find an approximate value for the area of the shaded region.

Give your answer to **five** significant figures.

[3 marks]

$h = \frac{8-4}{5} = 0.8$

$A \approx \frac{0.8}{2}[7.3891 + 10.9196 + 2(6.1240 + 6.3249 + 7.2155 + 8.7139)]$

$= 0.4 \times [18.3087 + 2 \times 28.3783]$

$= 0.4 \times 75.0653$

$= 30.02612$

$A \approx 30.026$ (5 s.f.)

6 (b) A student finds an improved approximation for the area of the shaded region by using the trapezium rule with 11 ordinates.

The student correctly obtains 29.759 as their improved approximation.

The student claims that the exact area must be greater than 29.759

Without further calculation, explain whether or not the student is correct.

[2 marks]

The curve is convex (concave upwards) for $4 \leq x \leq 8$,

so the trapezium rule gives an over-estimate of the area.

So the exact area is less than 29.759, therefore

the student is incorrect.

Do not write outside the box

7 The point A lies on the curve with equation

$$y = x^3 + px^2 + qx + 12$$

7 (a) Given that A has coordinates $(-5, 37)$, show that

$$5p - q = 30$$

[2 marks]

Sub $x = -5$, $y = 37$:

$37 = (-5)^3 + p(-5)^2 + q(-5) + 12$

$37 = -125 + 25p - 5q + 12$

$150 = 25p - 5q$

$5p - q = 30$ $(\div 5)$ as required

7 (b) Given that A is a stationary point, show that

$$10p - q = 75$$

[3 marks]

$\frac{dy}{dx} = 3x^2 + 2px + q$

A is stationary, so $\frac{dy}{dx} = 0$ when $x = -5$:

$3(-5)^2 + 2p(-5) + q = 0$

$75 - 10p + q = 0$

$10p - q = 75$ as required

Do not write outside the box

7 (c) Hence find the value of p and the value of q

[1 mark]

$(10p - q) - (5p - q) = 75 - 30 \Rightarrow 5p = 45$

$p = 9, \quad q = 5(9) - 30 = 15$

7 (d) The curve with equation

$$y = x^3 + px^2 + qx + 12$$

has a second stationary point *B*

Find the coordinates of *B*

Fully justify your answer.

[3 marks]

$y = x^3 + 9x^2 + 15x + 12$

Stationary points occur when $\frac{dy}{dx} = 0$:

$\frac{dy}{dx} = 3x^2 + 18x + 15 = 0$

$x^2 + 6x + 5 = 0$

$(x + 5)(x + 1) = 0$

$x = -5$ (this is A) or $x = -1$

At B, $x = -1$: $y = (-1)^3 + 9(-1)^2 + 15(-1) + 12$

$= -1 + 9 - 15 + 12 = 5$

B is $(-1, 5)$

Do not write outside the box

8 (a) Given that $\cos x \neq 0$ state the value of $\sec^2 x - \tan^2 x$

[1 mark]

$\sec^2 x - \tan^2 x = 1$

8 (b) Show that

$$(3\sec x + 5\tan x)(5\sec x - 3\tan x) - \frac{16\tan x}{\cos x} = N$$

where N is an integer to be found.

[4 marks]

$(3 \sec x + 5 \tan x)(5 \sec x - 3 \tan x)$

$= 15 \sec^2 x - 9 \sec x \tan x + 25 \sec x \tan x - 15 \tan^2 x$

$= 15 \sec^2 x + 16 \sec x \tan x - 15 \tan^2 x$

$\frac{16 \tan x}{\cos x} = 16 \times \frac{1}{\cos x} \times \tan x = 16 \sec x \tan x$

So expression $= 15 \sec^2 x + 16 \sec x \tan x - 15 \tan^2 x - 16 \sec x \tan x$

$= 15 \sec^2 x - 15 \tan^2 x$

$= 15(\sec^2 x - \tan^2 x)$

$= 15 \times 1 = 15$

$\therefore N = 15$

Do not write outside the box

8 (c) State the **two** values of x between 0° and 360° for which the value of N in part **(b)** is **not** valid.

[1 mark]

$x = 90°$ and $x = 270°$ $(\cos x = 0)$

Turn over for the next question

Do not write outside the box

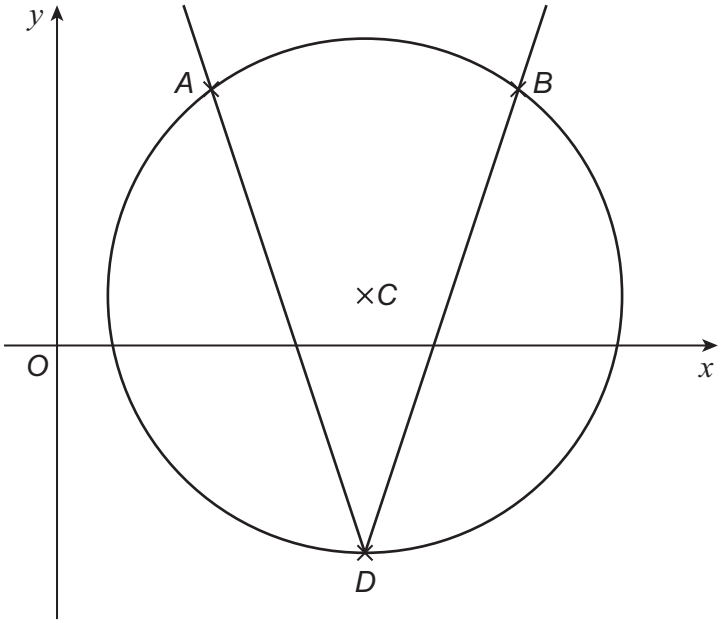
9 A circle with centre C has equation

$$(x - 12)^2 + (y - 2)^2 = 100$$

The graph with equation

$$y = |3x - 36| - 8$$

intersects the circle at the points A, B and D as shown in the diagram.



Point D is vertically below point C

9 (a) State the coordinates of C

[1 mark]

C(12, 2)

9 (b) State the coordinates of D

[1 mark]

D(12, −8)

Do not write outside the box

9 (c) The coordinates of A are $(a,10)$

Find the value of a

Fully justify your answer.

[3 marks]

A lies on the circle with y = 10:

$(a - 12)^2 + (10 - 2)^2 = 100$

$(a - 12)^2 = 100 - 64 = 36$

$a - 12 = \pm 6 \Rightarrow a = 6$ or $a = 18$

A is to the left of C ($a < 12$), so $a = 6$

(check: $|3(6) - 36| - 8 = 18 - 8 = 10$ ✓)

9 (d) (i) Find, in radians, the angle ADB

Give your answer to **three** significant figures.

[2 marks]

A(6, 10), D(12, −8), DC vertical: from D to A is 6 across, 18 up

$\tan(\frac{1}{2}ADB) = \frac{6}{18} \Rightarrow \frac{1}{2}ADB = 0.32175...$

$ADB = 0.6435... = 0.644$ rad (3 s.f.)

9 (d) (ii) Hence or otherwise find the length of the minor arc AB

Give your answer to **three** significant figures.

[2 marks]

Angle ACB = 2 × angle ADB = 1.2870... rad (angle at centre)

Arc $AB = r\theta = 10 \times 1.2870...$

$= 12.870... = 12.9$ (3 s.f.)

Turn over ▶

Do not write outside the box

10 A curve C has equation

$$y = x^k \ln x \quad \text{for} \quad x > 0$$

where k is a positive integer.

10 (a) Show that

$$\frac{dy}{dx} = x^{k-1}\left[A + k \ln x\right]$$

where A is a constant to be found.

[4 marks]

Product rule: $u = x^k$, $v = \ln x$

$\frac{dy}{dx} = kx^{k-1} \ln x + x^k \times \frac{1}{x}$

$= kx^{k-1} \ln x + \frac{x^k}{x}$

$= kx^{k-1} \ln x + x^{k-1}$

$= x^{k-1}[1 + k \ln x]$

$A = 1$

10 (b) Hence show that the y-coordinate of the stationary point of C can be written as $-\frac{1}{ke}$

Fully justify your answer.

[5 marks]

At a stationary point $\frac{dy}{dx} = 0$: $x^{k-1}(1 + k \ln x) = 0$

$x > 0$ so $x^{k-1} \neq 0$ ($x = 0$ is rejected, $\ln 0$ is undefined)

$\therefore\ 1 + k \ln x = 0$

$\ln x = -\frac{1}{k} \quad \Rightarrow \quad x = e^{-1/k}$

$y = (e^{-1/k})^k \times \ln(e^{-1/k})$

$= e^{-1} \times (-\frac{1}{k})$

Do not write outside the box

$= \frac{1}{e} \times (-\frac{1}{k})$

$= -\frac{1}{ke}$ as required

10 (c) Given that the stationary point of C has coordinates $\left(\frac{1}{e}, -\frac{1}{e}\right)$ state the value of k

[1 mark]

$k = 1$

10 (d) Prove that C does **not** have a point of inflection.

[3 marks]

$k = 1$: $y = x \ln x$

$\frac{dy}{dx} = 1 + \ln x$

$\frac{d^2y}{dx^2} = \frac{1}{x}$

For a point of inflection $\frac{d^2y}{dx^2} = 0$ (and changes sign)

But $x > 0$, so $\frac{1}{x} > 0$ for all x, so $\frac{d^2y}{dx^2}$ is never 0

$\therefore$ C does not have a point of inflection.

TURN OVER FOR SECTION B

Do not write outside the box

Section B

Answer **all** questions in the spaces provided.

11 An object is pulled along a horizontal surface. The normal reaction between the surface and the object is 125 newtons.

The object experiences a constant frictional force of 40 newtons.

Find the value of the coefficient of friction.

Circle your answer.

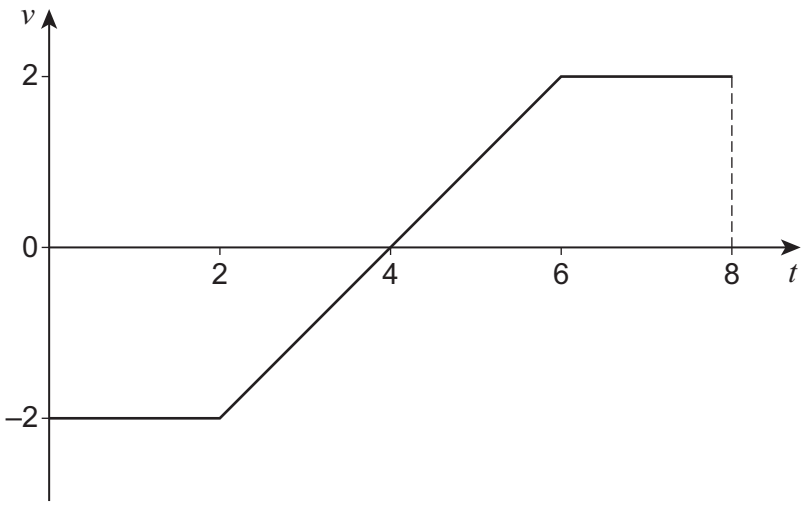
[1 mark]

0.245 | 0.32 | 0.5 | 3.125

Do not write outside the box

12 A particle moves in a straight line for 8 seconds.

The velocity v, in m s^{-1}, of the particle at time t seconds is shown in the diagram.



Find the displacement, in metres, of the particle after 8 seconds.

Circle your answer.

[1 mark]

–12 | 0 | 6 | 12

Turn over for the next question

Do not write outside the box

13 A car moves, in a straight line on a horizontal road, from a point A to a point B

The distance AB is 225 metres.

At the point B, the velocity of the car is 30 m s^{-1}

The car is modelled as having a constant acceleration of 2 m s^{-2}

13 (a) Show that the car starts from rest at point A

[2 marks]

$u = ?,\ v = 30,\ a = 2,\ s = 225$

$v^2 = u^2 + 2as$

$30^2 = u^2 + 2 \times 2 \times 225$

$900 = u^2 + 900 \Rightarrow u^2 = 0 \Rightarrow u = 0$

So the car starts from rest at A.

13 (b) The car continues to move past the point B in the same straight line.

Explain why the model would eventually become invalid.

[1 mark]

The acceleration is unlikely to remain constant (the car would reach its maximum speed), so the speed cannot keep increasing.

Do not write outside the box

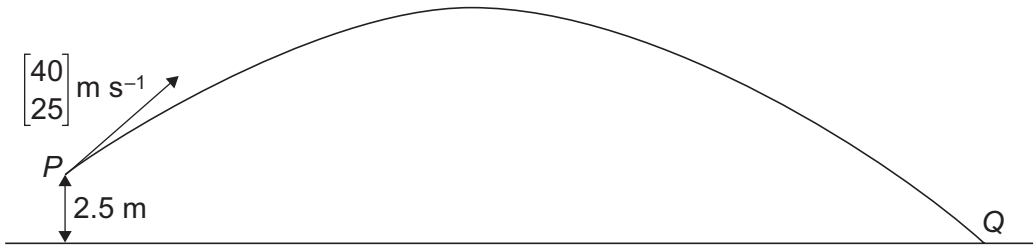
14 **In this question use $g = 9.8$ m s^{-2}**

An arrow is projected from a point P which is at a height of 2.5 metres above the horizontal ground.

The arrow has an initial velocity of $\begin{bmatrix} 40 \\ 25 \end{bmatrix}$ m s^{-1}

The arrow lands on the horizontal ground at a point Q

The path of the arrow is shown in the diagram.



Find the **speed** of the arrow at point Q

[4 marks]

Vertically (up +ve): $u = 25$, $a = -9.8$, $s = -2.5$

$v^2 = u^2 + 2as = 25^2 + 2(-9.8)(-2.5)$

$v^2 = 625 + 49 = 674$

$v = -\sqrt{674} = -25.96...$ m s^{-1} (downwards)

Horizontal velocity stays 40 m s^{-1}

Speed $= \sqrt{(40^2 + 674)} = \sqrt{2274}$

$= 47.686... = 47.7$ m s^{-1}

Turn over ▶

Do not write outside the box

15 A particle moves under the actions of two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$

$\mathbf{F}_1$ has magnitude 17 newtons and acts due East.

$\mathbf{F}_2$ has magnitude 26 newtons and acts at a bearing of 310°

The **resultant** of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$

15 (a) Show that the magnitude of $\mathbf{R}$ is 17.0 newtons, correct to **three** significant figures.

[4 marks]

i east, j north. $F_1 = 17i$

$F_2 = 26 \sin 310° \, i + 26 \cos 310° \, j = -19.917...i + 16.712...j$

$R = (17 - 19.917...)i + 16.712...j$

$= -2.917...i + 16.712...j$

$|R| = \sqrt{((-2.917...)^2 + 16.712...^2)}$

$= \sqrt{287.82...}$

$= 16.965...$

$= 17.0$ N (3 s.f.) as required

15 (b) Find the angle that $\mathbf{R}$ makes with $\mathbf{F}_1$

Give your answer to the **nearest** degree.

[2 marks]

$\tan \alpha = \frac{16.712...}{2.917...} \Rightarrow \alpha = 80.098...°$

Angle between R and $F_1 = 180° - 80.098...°$

$= 99.9...° = 100°$ (nearest degree)

Do not write outside the box

15 (c) A third force, $\mathbf{F}_3$, acts upon the particle so that the particle is in equilibrium.

15 (c) (i) State the magnitude of $\mathbf{F}_3$

[1 mark]

17.0 N

15 (c) (ii) State the bearing on which $\mathbf{F}_3$ acts.

[1 mark]

R acts on bearing 360° − 9.9° = 350°, F_3 is opposite to R:

bearing 350° − 180° = 170°

Turn over for the next question

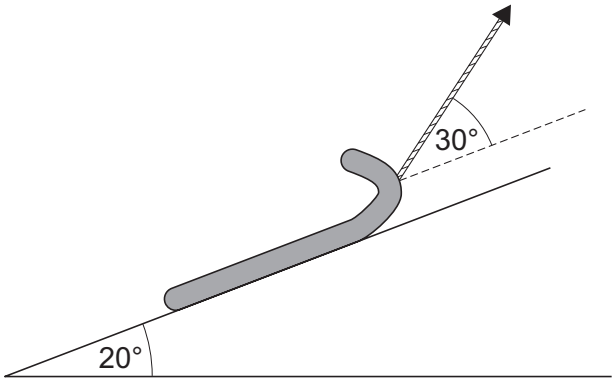
Do not write outside the box

16 **In this question use $g = 9.8$ m s^{-2}**

A sledge is pulled in a straight line up a rough path, as shown in the diagram.

The path is inclined at an angle of 20° to the horizontal.

The sledge is pulled by a light, inextensible rope inclined at an angle of 30° to the path.



The mass of the sledge is 10 kilograms.

16 (a) In one model, the sledge moves at a constant speed and experiences a combined resistance force of 15 newtons.

Find the tension in the rope for this model.

[4 marks]

Constant speed so resultant force = 0.

Resolving parallel to the path:

$T \cos 30° = 10g \sin 20° + 15$

$T \cos 30° = 33.518... + 15 = 48.518...$

$T = \frac{48.518...}{\cos 30°} = 56.02...$

$T = 56$ N

Do not write outside the box

16 (b) In a **different** model, the sledge experiences no air resistance.

The tension in the rope for this model is 54 N

The coefficient of friction between the sledge and the path is 0.2

Find the acceleration of the sledge for this model.

[7 marks]

Resolving perpendicular to the path:

$R + 54 \sin 30° = 10g \cos 20°$

$R = 92.089... - 27 = 65.089...$ N

$F = \mu R = 0.2 \times 65.089... = 13.017...$ N

Resolving parallel to the path ($F = ma$):

$54 \cos 30° - 10g \sin 20° - F = 10a$

$46.765... - 33.518... - 13.017... = 10a$

$10a = 0.2294...$

$a = 0.02294... = 0.023$ m s^{-2}

16 (c) State an assumption you have used to answer both parts **(a)** and **(b)**.

[1 mark]

The sledge is modelled as a particle.

Turn over ▶

Do not write outside the box

17 A triangle has vertices A, B and C with position vectors given by

$$\overrightarrow{OA} = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}, \quad \overrightarrow{OB} = \begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix} \text{ and } \overrightarrow{OC} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

17 (a) (i) Find $\overrightarrow{AB}$

[1 mark]

$AB = OB - OA = \begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix} - \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \\ -3 \end{bmatrix}$

17 (a) (ii) Find the magnitude of $\overrightarrow{AB}$

[1 mark]

$|AB| = \sqrt{(8^2 + (-1)^2 + (-3)^2)} = \sqrt{74} \quad (\approx 8.60)$

17 (b) Hence show that triangle ABC is a scalene triangle.

[5 marks]

$AC = OC - OA = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ -1 \end{bmatrix}$

$BC = OC - OB = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} -4 \\ -5 \\ 2 \end{bmatrix}$

$|AC| = \sqrt{(4^2 + (-6)^2 + (-1)^2)} = \sqrt{(16 + 36 + 1)} = \sqrt{53}$

$|BC| = \sqrt{((-4)^2 + (-5)^2 + 2^2)} = \sqrt{(16 + 25 + 4)} = \sqrt{45}$

Do not write outside the box

$AB = \sqrt{74}$, $AC = \sqrt{53}$, $BC = \sqrt{45}$

All three sides have different lengths,

therefore triangle ABC is a scalene triangle.

Turn over for the next question

Do not write outside the box

18 **In this question use $g = 9.81$ m s^{-2}**

A uniform platform AD has length 2.5 metres and weight 400 newtons.

The platform is attached to a chain at A. The other end of the chain is fixed to the floor.

The platform rests on a support at a point B which is 0.6 metres from A

The support exerts an upwards reaction force on the platform.

A child of mass 30 kilograms stands at a point C

The point C is 0.2 metres from D, as shown in the diagram.



The system is in equilibrium with the platform resting horizontally.

18 (a) By taking moments about A, find the reaction force at B

[4 marks]

Weight of platform acts at 1.25 m from A, child at 2.5 − 0.2 = 2.3 m

Moments about A (clockwise = anticlockwise):

0.6R = 400 × 1.25 + 30 × 9.81 × 2.3

0.6R = 500 + 676.89 = 1176.89

R = 1961.48...

R = 1960 N

Do not write outside the box

18 (b) (i) The child moves from *C* to *B*

State what happens to the reaction force at *B* during this movement.

[1 mark]

The reaction force at B decreases.

18 (b) (ii) Having reached *B*, the child steps off the platform.

State what happens to the tension in the chain during this movement.

[1 mark]

The tension in the chain does not change.

Turn over for the next question

Do not write outside the box

19 The displacement, **s** metres, of a particle P, at time t seconds, is given by

$$\mathbf{s} = (2t^3)\mathbf{i} + (2t^2 + qt)\mathbf{j}$$

19 (a) Find an expression for the velocity of particle P after t seconds.

[2 marks]

$v = \frac{ds}{dt}$

$v = 6t^2\, i + (4t + q)\, j$

19 (b) The acceleration, **a** m s^{-2}, of a particle Q, at time t seconds, is given by

$$\mathbf{a} = 6t\mathbf{i} + 7\mathbf{j}$$

Particle Q has an initial velocity of $4\mathbf{j}$ m s^{-1}

Particles P and Q are moving parallel to each other when $t = 2$

Find the value of q

[6 marks]

$v_Q = \int a\, dt = \int (6t\, i + 7\, j)\, dt = 3t^2\, i + 7t\, j + c$

$t = 0$: $v_Q = 4j \Rightarrow c = 4j$

$v_Q = 3t^2\, i + (7t + 4)\, j$

At $t = 2$: $v_P = 24\, i + (8 + q)\, j$

$v_Q = 12\, i + 18\, j$

Parallel: $v_P = k\, v_Q$

$24 = 12k \Rightarrow k = 2$

$8 + q = 2 \times 18 = 36$

$q = 28$

Do not write outside the box

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Copyright information

For confidentiality purposes, all acknowledgements of third-party copyright material are published in a separate booklet. This booklet is published after each live examination series and is available for free download from www.aqa.org.uk.

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team.

Copyright © 2025 AQA and its licensors. All rights reserved.