AQA

Please write clearly in block capitals.

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Surname StemUp Model Solution

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I declare this is my own work.

A-level MATHEMATICS

Paper 1

Wednesday 4 June 2025 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Answer **all** questions in the spaces provided.

1 The equation of a curve is given by

$$y = 3e^{x}$$

Find an expression for $\frac{dy}{dx}$

Circle your answer.

[1 mark]

$3e^{-x}$ $3e^{1}$ $3e^{x}$ $3xe^{x-1}$

2 A sequence is defined by

$$x_{n+1} = -\frac{1}{4}x_n \quad \text{with} \quad x_1 = 32$$

The first four terms of the sequence are

$$32, \quad -8, \quad 2, \quad -0.5$$

Which **one** of the following can be used to describe this sequence?

Circle your answer.

[1 mark]

Convergent Decreasing Increasing Periodic

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3 Express

$$\log_3 2x - \log_3 x$$

as a single logarithm.

Circle your answer.

[1 mark]

$\log_3 \frac{1}{2}$ $\quad$ $\log_3 2$ $\quad$ $\log_3 x$ $\quad$ $\log_3 2x^2$

Turn over for the next question

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4 The first three terms, in ascending powers of x, of the binomial expansion of $(1-8x)^{\frac{1}{2}}$ are

$$1+nx-8x^2$$

where n is a constant.

4 (a) State the range of values of x for which the expansion is valid.

Circle your answer.

[1 mark]

$|x|>-8$ $\quad$ $|x|>-\frac{1}{8}$ $\quad$ (circled) $|x|<\frac{1}{8}$ $\quad$ $|x|<8$

4 (b) State the value of the constant n

Circle your answer.

[1 mark]

-16 $\quad$ (circled) -4 $\quad$ $\frac{1}{2}$ $\quad$ 4

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5 Find $\int \left(4x - 3 + x^{-\frac{1}{2}}\right) \mathrm{d}x$

[3 marks]

$\int (4x - 3 + x^{-½})\ dx$

$= \frac{4x^2}{2} - 3x + \frac{x^{½}}{½} + c$

$= 2x^2 - 3x + 2x^{½} + c$

6 An arithmetic series has first term 3 and common difference -0.5

Find the sum of the first 250 terms.

[2 marks]

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{250} = \frac{250}{2}(2 \times 3 + (250 - 1) \times (-0.5))$

$= 125 \times (6 - 124.5)$

$= 125 \times (-118.5)$

$= -14812.5$

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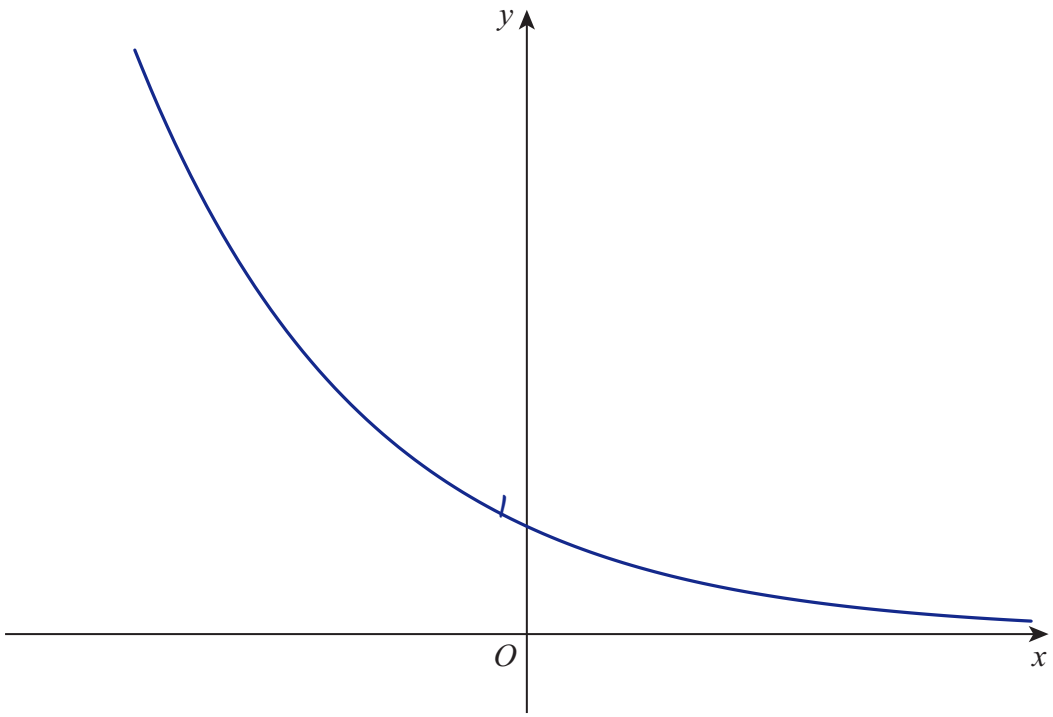
7 It is given that $0 < a < 1$

Sketch the graph with equation

$$y = a^x$$

on the axes below.

[2 marks]



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8 Show that, **for small values of x**, the graph with equation

$$y = \frac{4 + \sin 5x - 3\cos 2x}{1 + 2\tan x}$$

can be approximated by the straight line with equation of the form

$$y = ax + 1$$

where a is a constant to be found.

[5 marks]

For small values of x:

$\sin 5x \approx 5x, \quad \tan x \approx x$

$\cos 2x \approx 1 - \frac{(2x)^2}{2} = 1 - 2x^2$

$y \approx \frac{4 + 5x - 3(1 - \frac{1}{2}(2x)^2)}{1 + 2x}$

$= \frac{4 + 5x - 3 + 6x^2}{1 + 2x}$

$= \frac{6x^2 + 5x + 1}{1 + 2x}$

$= \frac{(2x + 1)(3x + 1)}{2x + 1}$

$= 3x + 1$

$\therefore$ for small x, $y \approx 3x + 1$, a straight line of the form

$y = ax + 1$ with $a = 3$

Turn over ▶

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9 (a) A geometric series, S, has **second** term 60

The common ratio of S is 0.2

Find the exact value of the sum of the first five terms of S

[4 marks]

Second term: $ar = 60$ with $r = 0.2$

$0.2a = 60 \Rightarrow a = \frac{60}{0.2} = 300$

$S_n = \frac{a(1 - r^n)}{1 - r}$

$S_5 = \frac{300(1 - 0.2^5)}{1 - 0.2}$

$= \frac{300 \times 0.99968}{0.8}$

$= 374.88 = \frac{9372}{25}$

9 (b) A different geometric series, T, has **second** term 60 and positive common ratio r

The sum to infinity of T is T_∞

9 (b) (i) Show that

$$T_\infty = \frac{60}{r - r^2}$$

[2 marks]

$ar = 60 \Rightarrow a = \frac{60}{r}$

$T_\infty = \frac{a}{1 - r}$ $(0 < r < 1)$

$= \frac{60}{r(1 - r)}$

$T_\infty = \frac{60}{r - r^2}$ as required

Do not write outside the box

9 (b) (ii) Find the maximum value of $r - r^2$

[2 marks]

Let $f(r) = r - r^2$, $\frac{df}{dr} = 1 - 2r = 0$

$\Rightarrow r = \frac{1}{2}$, $\frac{d^2f}{dr^2} = -2 < 0$ so a maximum

Maximum value of $r - r^2 = \frac{1}{2} - (\frac{1}{2})^2 = \frac{1}{4}$

9 (b) (iii) Hence find the range of possible values of T_∞

Fully justify your answer.

[2 marks]

For $0 < r < 1$, $r - r^2 > 0$ and $T_\infty = \frac{60}{r - r^2}$ is least

when $r - r^2$ is greatest

Min $T_\infty = \frac{60}{1/4} = 240$

Hence $T_\infty \geq 240$

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10 A researcher working for a frozen-food manufacturer uses the formula

$$\theta = 21 - Ae^{-kt}$$

to model the temperature of a dessert once it is taken out of a freezer.

In this model:

- θ is the temperature of the dessert in °C
- t is the time in hours since the dessert was removed from the freezer
- A and k are positive constants.

10 (a) Show how

$$\theta = 21 - Ae^{-kt}$$

can be rearranged to obtain

$$\ln(21 - \theta) = -kt + \ln A$$

[3 marks]

$\theta = 21 - Ae^{-kt}$

$Ae^{-kt} = 21 - \theta$

$\ln(Ae^{-kt}) = \ln(21 - \theta)$

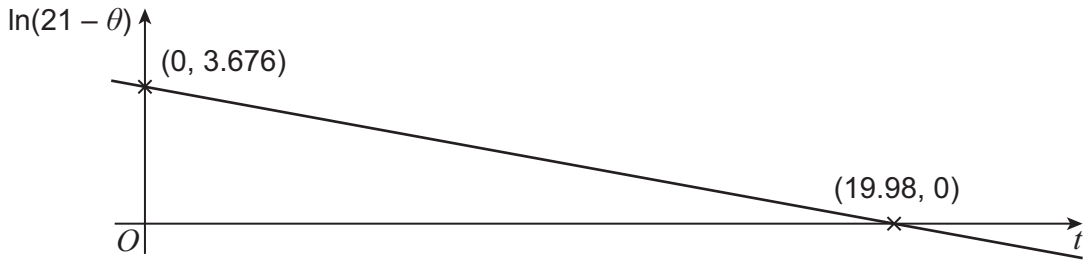
$\ln A + \ln e^{-kt} = \ln(21 - \theta)$

$\ln A - kt = \ln(21 - \theta)$

$\therefore \ln(21 - \theta) = -kt + \ln A$

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10 (b) The researcher uses measurements they have recorded to plot the graph of $\ln(21 - \theta)$ against t as shown in the diagram below.



10 (b) (i) Use the information on the graph to find the value of A

Give your answer to **three** significant figures.

[2 marks]

Intercept on vertical axis: $\ln A = 3.676$

$A = e^{3.676} = 39.488125...$

$A = 39.5$ (3 s.f.)

10 (b) (ii) Use the information on the graph to find the value of k

Give your answer to **three** significant figures.

[2 marks]

Gradient $= -k = \frac{0 - 3.676}{19.98 - 0}$

$-k = -0.183983...$

$k = 0.184$ (3 s.f.)

Turn over ▶

Do not write outside the box

10 (b) (iii) Find the temperature of the dessert when it is initially removed from the freezer.

Give your answer to **three** significant figures.

[2 marks]

At $t = 0$: $\theta = 21 - 39.488... \times e^0$

$\theta = 21 - 39.488... = -18.488...$

Initial temperature = −18.5 °C

10 (c) The dessert is ready to be eaten when its temperature reaches 4 °C

Use the model to determine the time, after being removed from the freezer, for the dessert to reach this temperature.

Give your answer to the nearest 10 minutes.

[3 marks]

$\theta = 4$: $\ln(21 - 4) = -kt + \ln A$

$\ln 17 = -0.183983...t + 3.676$

$t = \frac{3.676 - \ln 17}{0.183983...}$

$t = 4.580760...$ hours

$0.580760... \times 60 = 34.8$ minutes, so $t = 4$ h 34.8 min

Time = 4 hours 30 minutes (to the nearest 10 minutes)

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11 The equation of a curve is

$$x^2y + 4y^3 = 8x$$

The curve has two stationary points.

11 (a) Use implicit differentiation to show that at the stationary points $y = \frac{4}{x}$

[4 marks]

$x^2y + 4y^3 = 8x$

$2xy + x^2\frac{dy}{dx} + 12y^2\frac{dy}{dx} = 8$

At the stationary points $\frac{dy}{dx} = 0$:

$2xy + 0 + 0 = 8$

$2xy = 8$

$\Rightarrow y = \frac{4}{x}$

11 (b) Hence show that the x-coordinates of the stationary points can be written in the form $\pm\sqrt{n}$ where n is an integer to be found.

[3 marks]

$x^2 \times \frac{4}{x} + 4\left(\frac{4}{x}\right)^3 = 8x$

$4x + \frac{256}{x^3} = 8x$

$4x = \frac{256}{x^3} \quad \Rightarrow \quad x^4 = 64$

$x^2 = 8 \quad (x^2 = -8 \text{ has no real solutions})$

$x = \pm\sqrt{8}$, so $n = 8$

Turn over ▶

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12 Functions f and g are defined by

$$f(x) = x^2 + 5 \qquad x \in \mathbb{R}$$
$$g(x) = \sqrt{x} \qquad x \geq 0$$

12 (a) Using set notation, state the range of f

[2 marks]

$x^2 \geq 0$ for all x, so $f(x) = x^2 + 5 \geq 5$

Range of f: $\{y : y \geq 5\}$

12 (b) Determine whether f has an inverse.

Fully justify your answer.

[2 marks]

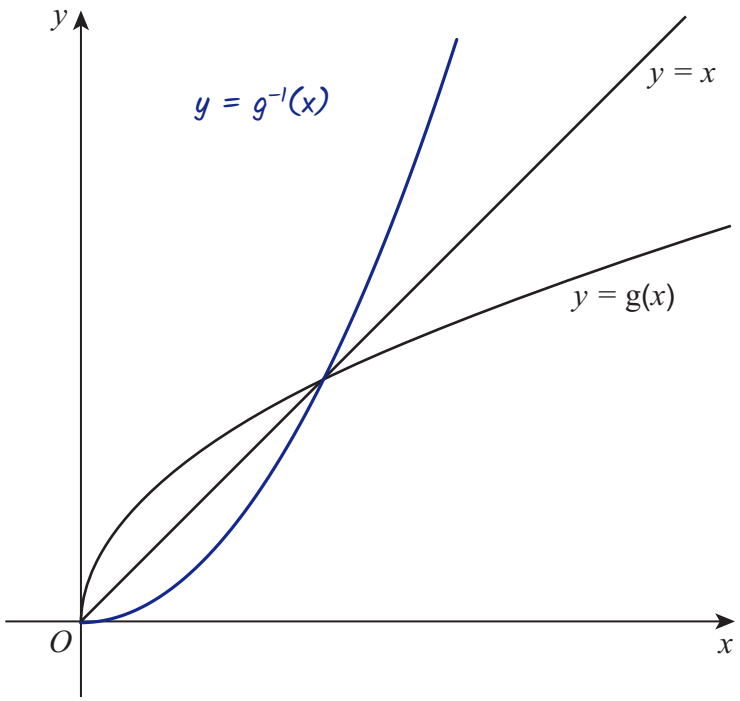
$f(-1) = 6$ and $f(1) = 6$

so f is many-to-one (it is not one-to-one),

therefore f does not have an inverse.

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12 (c) The graph of $y = g(x)$, and the line with equation $y = x$, are shown on the diagram below.



Sketch the graph of $y = g^{-1}(x)$ on the diagram.

[2 marks]

12 (d) The composite function gf is denoted by h

12 (d) (i) Write down an expression for $h(x)$

[1 mark]

$h(x) = g(f(x)) = \sqrt{(x^2 + 5)}$

12 (d) (ii) State the range of h

[1 mark]

$h(x) \geq \sqrt{5}$

Turn over ▶

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13 A curve C has parametric equations

$$x = 4(4t + 1)^2$$
$$y = e^{-4t}$$

for $-\frac{1}{4} \leq t \leq 0$

13 (a) Find $\frac{dy}{dx}$ in terms of t

[3 marks]

$\frac{dx}{dt} = 4 \times 2(4t + 1) \times 4 = 32(4t + 1)$

$\frac{dy}{dt} = -4e^{-4t}$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{-4e^{-4t}}{32(4t + 1)}$

$= \frac{-e^{-4t}}{8(4t + 1)}$

13 (b) Find an equation of the tangent to C at the point where $t = 0$

[3 marks]

$t = 0$: $x = 4(0 + 1)^2 = 4$, $y = e^0 = 1$

$\frac{dy}{dx} = \frac{-e^0}{8(0 + 1)} = -\frac{1}{8}$

$y - 1 = -\frac{1}{8}(x - 4)$

$y = -\frac{1}{8}x + \frac{3}{2}$

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13 (c) Find a Cartesian equation for C in the form $y = f(x)$

Fully justify your answer.

[3 marks]

$(4t + 1)^2 = \frac{x}{4} \quad \Rightarrow \quad 4t + 1 = \pm\frac{\sqrt{x}}{2}$

Since $-\frac{1}{4} \leq t \leq 0$, $4t + 1 \geq 0$, so take the positive root:

$4t + 1 = \frac{\sqrt{x}}{2} \quad \Rightarrow \quad -4t = 1 - \frac{\sqrt{x}}{2}$

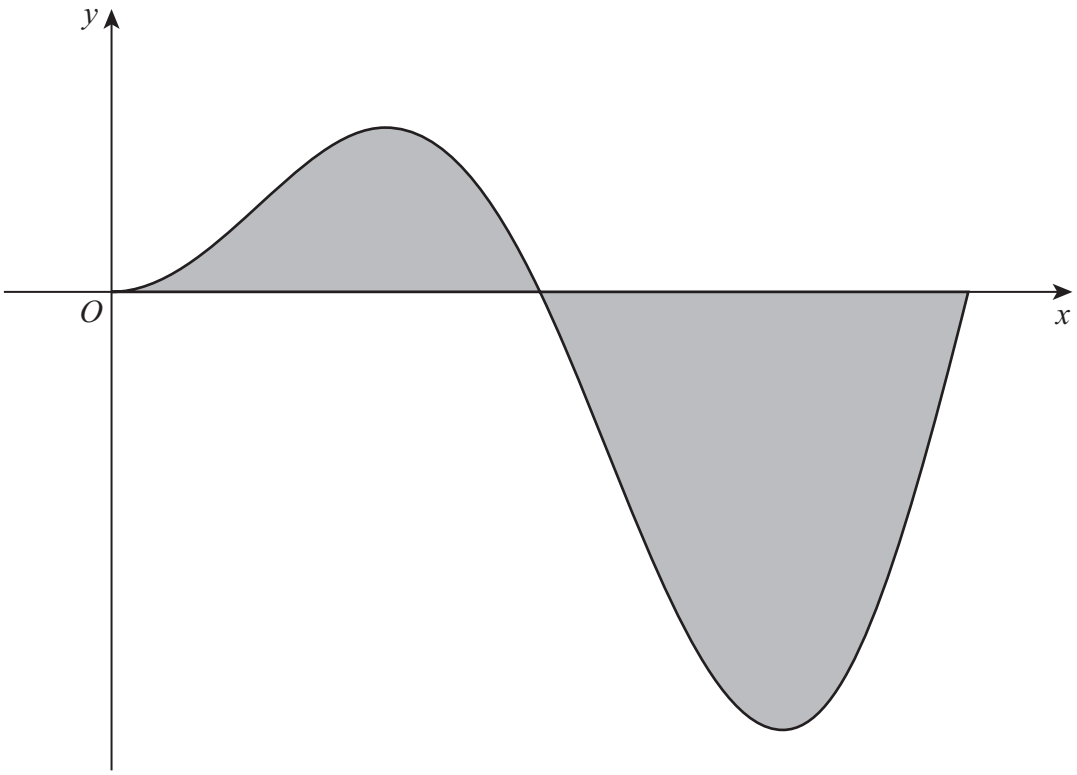
$y = e^{-4t}$

$\therefore y = e^{1 - \frac{\sqrt{x}}{2}}$

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14 The graph of $y = 4x\sin 2x$ for $0 \leq x \leq \pi$ is shown below.



Show that the shaded area enclosed by the x-axis and the curve is $k\pi$, where k is an integer to be found.

Fully justify your answer.

[8 marks]

Curve meets x-axis where $4x \sin 2x = 0$: $x = 0$ or $\sin 2x = 0$

$\Rightarrow 2x = \pi \Rightarrow x = \frac{\pi}{2}$ (for $0 < x < \pi$)

Integration by parts: $u = 4x$, $\frac{dv}{dx} = \sin 2x$

$\frac{du}{dx} = 4$, $v = -\frac{1}{2}\cos 2x$

$\int 4x \sin 2x \, dx = -\frac{4x}{2}\cos 2x - \int -\frac{4}{2}\cos 2x \, dx$

$= -2x \cos 2x + \int 2 \cos 2x \, dx$

$= -2x \cos 2x + \sin 2x + c$

Area above the axis ($0 \leq x \leq \frac{\pi}{2}$):

$[-2x \cos 2x + \sin 2x]_0^{\pi/2} = (-\pi \cos \pi + \sin \pi) - (0 + 0)$

$= \pi$

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Area below the axis $\left(\frac{\pi}{2} \leq x \leq \pi\right)$:

$[-2x \cos 2x + \sin 2x]_{\pi/2}^{\pi} = (-2\pi \cos 2\pi + \sin 2\pi) - (-\pi \cos \pi + \sin \pi)$

$= (-2\pi + 0) - (\pi + 0) = -3\pi$

The integral is negative as the region is below the axis, so its area is 3π

Total shaded area $= \pi + 3\pi = 4\pi$

$\therefore$ the area is $k\pi$ with $k = 4$

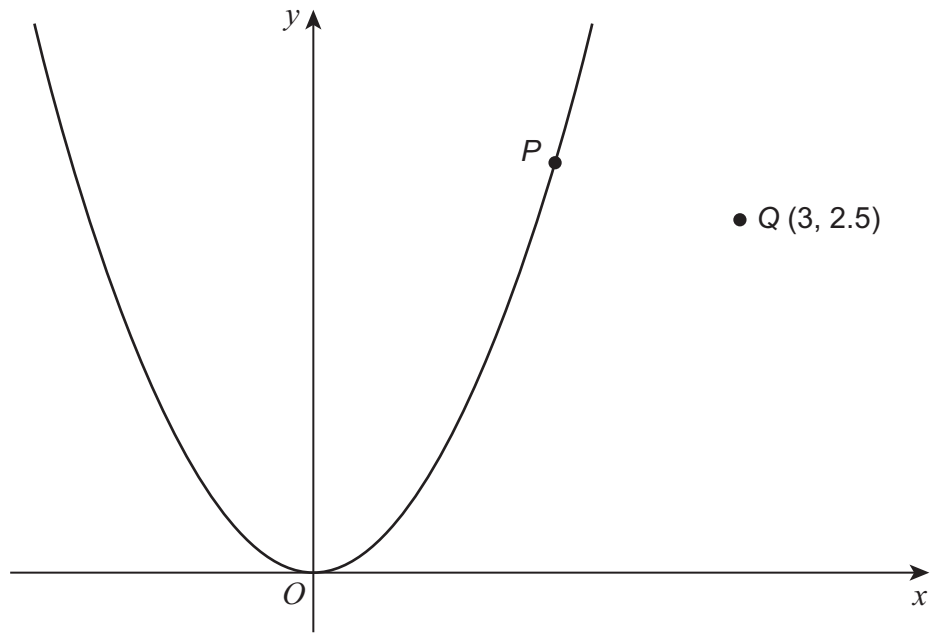
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15 A curve has equation

$$y = x^2$$

The point Q has coordinates (3, 2.5)

The point P on the curve which is closest to the point Q is shown on the diagram below.



15 (a) Show that the x-coordinate of P satisfies the equation

$$2x^3 - 4x - 3 = 0$$

[4 marks]

Let $P = (x, x^2)$. $\frac{dy}{dx} = 2x$, so gradient of normal at $P = -\frac{1}{2x}$

P is the closest point to Q, so PQ is along the normal at P:

$\frac{x^2 - 2.5}{x - 3} = -\frac{1}{2x}$

$2x(x^2 - 2.5) = -(x - 3)$

$2x^3 - 5x = -x + 3$

$2x^3 - 4x - 3 = 0$ as required

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Question 15 continues on the next page

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15 (b) The Newton–Raphson method is to be used to find an approximate solution to the equation

$$2x^3 - 4x - 3 = 0$$

Show that the Newton–Raphson method generates the iterative formula

$$x_{n+1} = \frac{4x_n^3 + 3}{6x_n^2 - 4}$$

[4 marks]

$f(x) = 2x^3 - 4x - 3, \quad f'(x) = 6x^2 - 4$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_{n+1} = x_n - \frac{2x_n^3 - 4x_n - 3}{6x_n^2 - 4}$

$= \frac{x_n(6x_n^2 - 4) - (2x_n^3 - 4x_n - 3)}{6x_n^2 - 4}$

$= \frac{6x_n^3 - 4x_n - 2x_n^3 + 4x_n + 3}{6x_n^2 - 4}$

$x_{n+1} = \frac{4x_n^3 + 3}{6x_n^2 - 4}$ as required

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15 (c) Starting with $x_0 = 3$, use the iterative formula given in part **(b)** to find the value of x_3

Give your answer to **three** decimal places.

[2 marks]

$x_1 = \frac{4(3)^3 + 3}{6(3)^2 - 4} = \frac{111}{50} = 2.22$

$x_2 = 1.828840...$

$x_3 = 1.709452...$

$x_3 = 1.709$ (3 d.p.)

15 (d) Hence find the distance PQ

Give your answer to **two** decimal places.

[2 marks]

$P \approx (1.709452..., 1.709452...^2) = (1.709452..., 2.922226...)$

$PQ = \sqrt{((1.709452... - 3)^2 + (2.922226... - 2.5)^2)}$

$= 1.357861...$

$PQ = 1.36$ (2 d.p.)

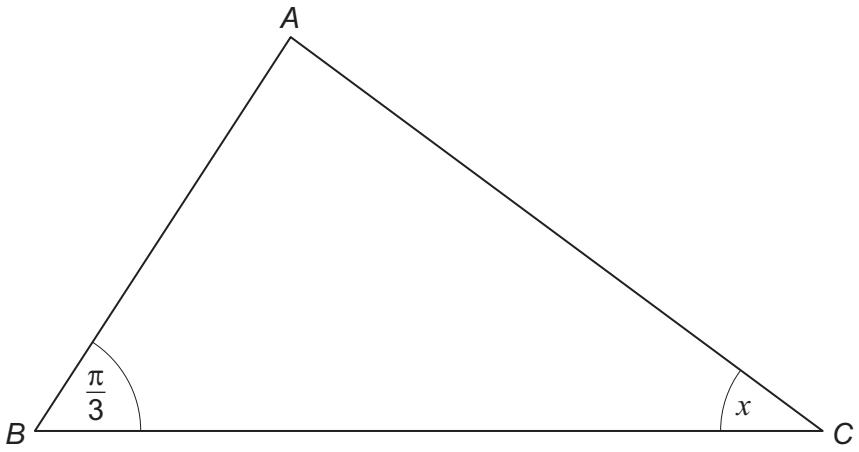
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16 The triangle *ABC* is shown in the diagram below.

The angle *ABC* is $\frac{\pi}{3}$ radians.

The angle *BCA* is x radians.



16 (a) Explain why angle $BAC = \frac{2\pi}{3} - x$

[1 mark]

The angles in a triangle add up to π radians, so

angle BAC = $\pi - \frac{\pi}{3} - x = \frac{2\pi}{3} - x$

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16 (b) (i) Use the sine rule to show that

$$\frac{BC}{AB} = \frac{\sqrt{3}\cot x + p}{q}$$

where p and q are integers.

[5 marks]

Sine rule: $\frac{BC}{\sin BAC} = \frac{AB}{\sin BCA}$

$\frac{BC}{\sin(2\pi/3 - x)} = \frac{AB}{\sin x}$

$\frac{BC}{AB} = \frac{\sin(2\pi/3 - x)}{\sin x}$

$\sin(\frac{2\pi}{3} - x) = \sin\frac{2\pi}{3}\cos x - \cos\frac{2\pi}{3}\sin x$

$= \frac{\sqrt{3}}{2}\cos x - (-\frac{1}{2})\sin x = \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$

$\frac{BC}{AB} = \frac{\sqrt{3}\cos x + \sin x}{2\sin x}$

$= \frac{\sqrt{3}\cot x + 1}{2}$ so $p = 1$, $q = 2$

16 (b) (ii) Hence state the exact value of x when

$$\frac{BC}{AB} = \frac{\sqrt{3}+1}{2}$$

[1 mark]

$\sqrt{3}\cot x + 1 = \sqrt{3} + 1 \Rightarrow \cot x = 1$

$x = \frac{\pi}{4}$

Turn over ▶

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17 (a) Use the substitution $u = e^x + 1$ to show that

$$\int \frac{e^{2x}}{e^x + 1}\,dx = e^x - \ln(e^x + 1) + k$$

[5 marks]

$u = e^x + 1 \Rightarrow \frac{du}{dx} = e^x$, so $dx = \frac{du}{e^x}$, $e^x = u - 1$

$\int \frac{e^{2x}}{e^x + 1}\,dx = \int \frac{e^{2x}}{u} \times \frac{1}{e^x}\,du$

$= \int \frac{e^x}{u}\,du$

$= \int \frac{u - 1}{u}\,du$

$= \int (1 - \frac{1}{u})\,du$

$= u - \ln u + c$

$= e^x + 1 - \ln(e^x + 1) + c$

$= e^x - \ln(e^x + 1) + k$ where $k = c + 1$

Do not write outside the box

17 (b) Solve the differential equation

$$\left(\frac{e^x + 1}{e^{2x}}\right)\frac{dy}{dx} = \cos^2 y$$

given that $y = \pi$ when $x = 0$

Write your answer in the form

$$\tan y = e^x + \ln\left(\frac{A}{e^x + 1}\right) + B$$

where A and B are constants to be found.

[5 marks]

$\left(\frac{e^x + 1}{e^{2x}}\right)\frac{dy}{dx} = \cos^2 y$

$\frac{1}{\cos^2 y}\frac{dy}{dx} = \frac{e^{2x}}{e^x + 1}$

$\int \sec^2 y \, dy = \int \frac{e^{2x}}{e^x + 1} dx$

$\tan y = e^x - \ln(e^x + 1) + k$ (using part (a))

$y = \pi, x = 0$: $\tan \pi = e^0 - \ln(e^0 + 1) + k$

$0 = 1 - \ln 2 + k \Rightarrow k = \ln 2 - 1$

$\tan y = e^x - \ln(e^x + 1) + \ln 2 - 1$

$\tan y = e^x + \ln\left(\frac{2}{e^x + 1}\right) - 1$

$A = 2, \quad B = -1$

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