AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level MATHEMATICS

Paper 3

Thursday 20 June 2024 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do not write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Each of the series below shows the first four terms of a geometric series.

Identify the only one of these geometric series that is convergent.

[1 mark]

Tick (✓) **one** box.

$0.1 + 0.2 + 0.4 + 0.8 + \ldots$ ☐

$1 - 1 + 1 - 1 + \ldots$ ☐

$128 - 64 + 32 - 16 + \ldots$ ☑

$1 + 2 + 4 + 8 + \ldots$ ☐

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2 The quadratic equation

$$4x^2 + bx + 9 = 0$$

has one repeated real root.

Find b

Circle your answer.

[1 mark]

$b = 0$ $\quad$ $b = \pm 12$ $\quad$ $b = \pm 13$ $\quad$ $b = \pm 36$

$b^2 - 4(4)(9) = 0 \Rightarrow b^2 = 144 \Rightarrow b = \pm 12$

Turn over for the next question

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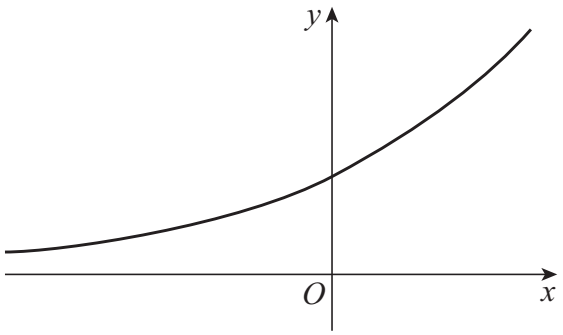
3 One of the graphs shown below **cannot** have an equation of the form

$$y = a^x \qquad \text{where } a > 0$$

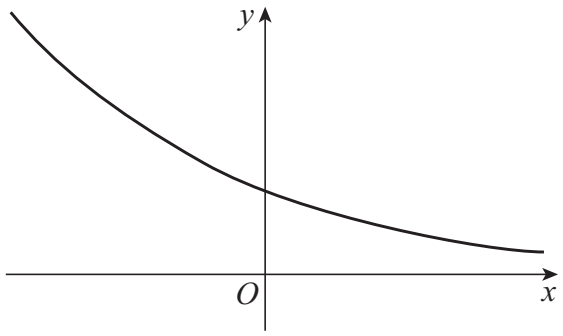
Identify this graph.

Tick (✓) **one** box.

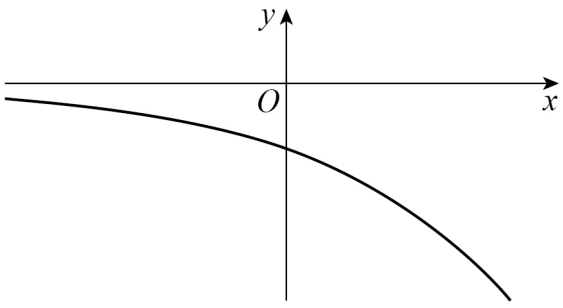
[1 mark]



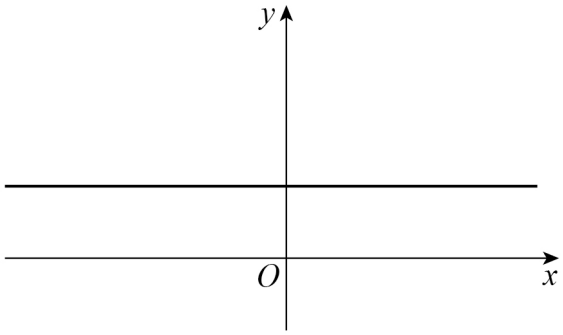
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4 A curve has equation $y = x^4 + 2^x$

Find an expression for $\frac{dy}{dx}$

[2 marks]

$y = x^4 + 2^x$

$\frac{dy}{dx} = 4x^3 + 2^x \ln 2$

Turn over for the next question

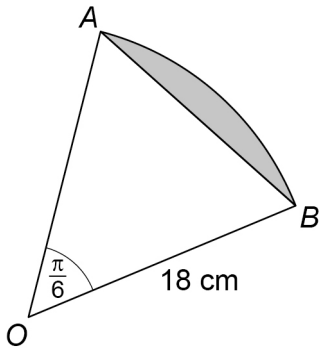
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5 The diagram below shows a sector of a circle OAB.

The chord AB divides the sector into a triangle and a shaded segment.

Angle AOB is $\frac{\pi}{6}$ radians.

The radius of the sector is 18 cm.



Show that the area of the shaded segment is

$$k(\pi - 3)\ \text{cm}^2$$

where k is an integer to be found.

[3 marks]

Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 18^2 \times \frac{\pi}{6} = 27\pi$

Area of triangle $OAB = \frac{1}{2}r^2 \sin\theta = \frac{1}{2} \times 18^2 \times \sin\frac{\pi}{6}$

$= 162 \times \frac{1}{2} = 81$

Area of segment $= 27\pi - 81$

$= 27(\pi - 3)\ \text{cm}^2$

so $k = 27$

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Turn over for the next question

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6 (a) Find $\int \left(6x^2 - \frac{5}{\sqrt{x}}\right) \mathrm{d}x$

[3 marks]

$\frac{5}{\sqrt{x}} = 5x^{-1/2}$

$\int (6x^2 - 5x^{-1/2})\, dx = \frac{6x^3}{3} - \frac{5x^{1/2}}{1/2} + c$

$= 2x^3 - 10x^{1/2} + c$

Do not write outside the box

6 (b) The gradient of a curve is given by

$$\frac{dy}{dx} = 6x^2 - \frac{5}{\sqrt{x}}$$

The curve passes through the point (4, 90).

Find the equation of the curve.

[2 marks]

$y = 2x^3 - 10x^{1/2} + c$

At (4, 90): $90 = 2(4)^3 - 10(4)^{1/2} + c$

$90 = 128 - 20 + c \Rightarrow c = -18$

$y = 2x^3 - 10x^{1/2} - 18$

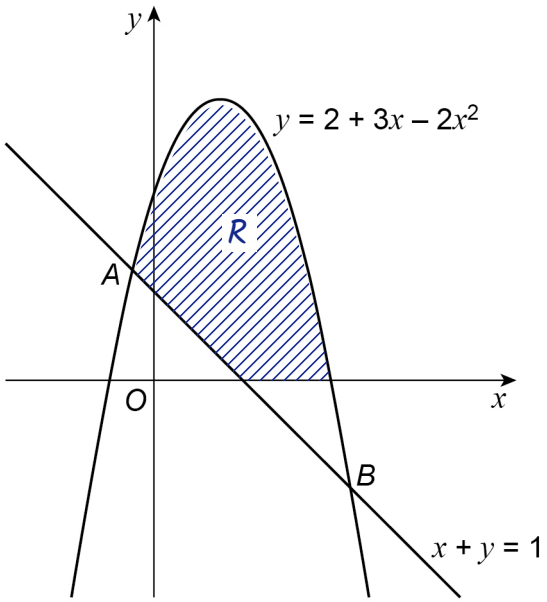
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7 The graphs with equations

$$y = 2 + 3x - 2x^2 \text{ and } x + y = 1$$

are shown in the diagram below.



The graphs intersect at the points *A* and *B*

7 (a) On the diagram above, shade and label the region, *R*, that is satisfied by the inequalities

$$0 \leq y \leq 2 + 3x - 2x^2$$

and

$$x + y \geq 1$$

[2 marks]

Do not write outside the box

7 (b) Find the exact coordinates of A

[3 marks]

$x + y = 1 \Rightarrow y = 1 - x$

$1 - x = 2 + 3x - 2x^2$

$2x^2 - 4x - 1 = 0$

$x = \frac{4 \pm \sqrt{(16 + 8)}}{4} = \frac{4 \pm 2\sqrt{6}}{4} = \frac{2 \pm \sqrt{6}}{2}$

At A, $x < 0$ so $x = \frac{2 - \sqrt{6}}{2}$

$y = 1 - \frac{2 - \sqrt{6}}{2} = \frac{\sqrt{6}}{2}$

$A\left(\frac{2 - \sqrt{6}}{2}, \frac{\sqrt{6}}{2}\right)$

Turn over for the next question

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8 The temperature θ °C of an oven t minutes after it is switched on can be modelled by the equation

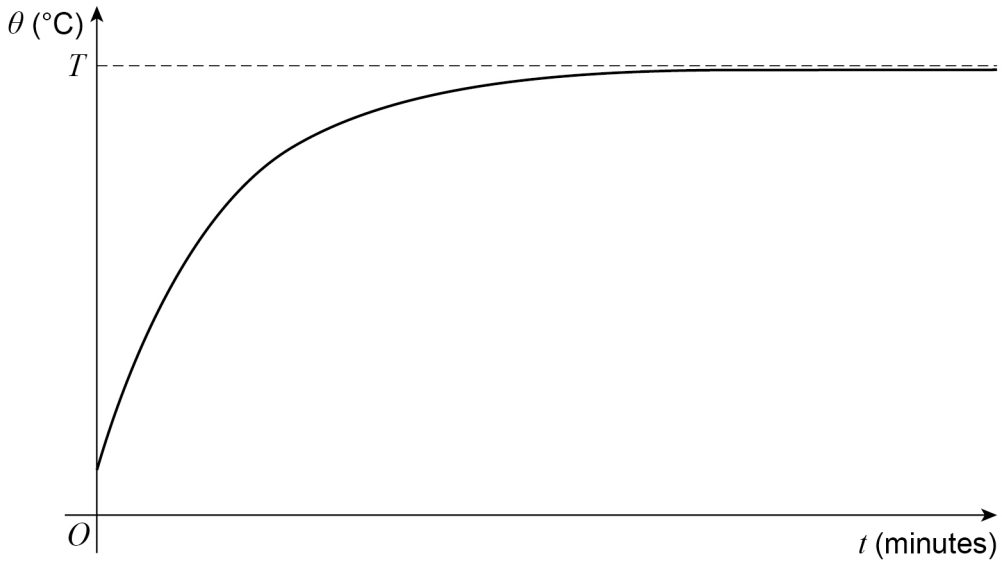
$$\theta = 20(11 - 10e^{-kt})$$

where k is a positive constant.

Initially the oven is at room temperature.

The maximum temperature of the oven is T °C

The temperature predicted by the model is shown in the graph below.



8 (a) Find the room temperature.

[2 marks]

Initially $t = 0$:

$\theta = 20(11 - 10e^{0}) = 20(11 - 10)$

$= 20$

Room temperature = 20 °C

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8 (b) Find the value of T

[2 marks]

As $t \to \infty$, $e^{-kt} \to 0$

$T = 20(11 - 10 \times 0) = 20 \times 11$

$T = 220$

Question 8 continues on the next page

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8 (c) The oven reaches a temperature of 86 °C one minute after it is switched on.

8 (c) (i) Find the value of k.

[2 marks]

$86 = 20(11 - 10e^{-k})$

$4.3 = 11 - 10e^{-k} \Rightarrow e^{-k} = 0.67$

$k = -\ln 0.67 = 0.400477...$

$k = 0.400$ (3 s.f.)

8 (c) (ii) Find the time it takes for the temperature of the oven to be within 1°C of its maximum.

[2 marks]

Within 1 °C of maximum: $\theta = 220 - 1 = 219$

$219 = 20(11 - 10e^{-0.400477t})$

$10.95 = 11 - 10e^{-0.400477t} \Rightarrow e^{-0.400477t} = 0.005$

$t = \frac{\ln 200}{0.400477} = 13.229...$

$t = 13.2$ minutes (3 s.f.)

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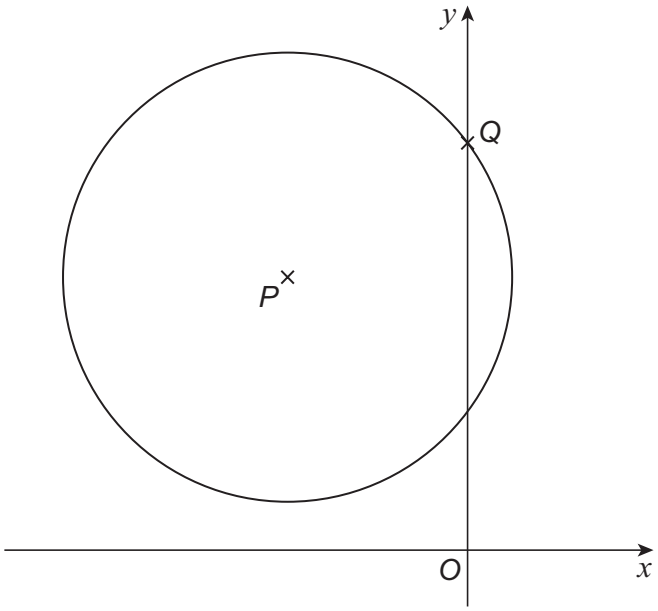
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9 **Figure 1** below shows a circle.

Figure 1



The centre of the circle is P and the circle intersects the y-axis at Q as shown in **Figure 1**.

The equation of the circle is

$$x^2 + y^2 = 12y - 8x - 27$$

9 (a) Express the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = k$$

where a, b and k are constants to be found.

[3 marks]

$x^2 + 8x + y^2 - 12y = -27$

$(x + 4)^2 - 16 + (y - 6)^2 - 36 = -27$

$(x + 4)^2 + (y - 6)^2 = 25$

$a = -4,\ b = 6,\ k = 25$

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9 (b) State the coordinates of P

[1 mark]

$P(-4, 6)$

9 (c) Find the y-coordinate of Q

[2 marks]

$x = 0$: $(0 + 4)^2 + (y - 6)^2 = 25$

$(y - 6)^2 = 9 \Rightarrow y = 6 \pm 3 \Rightarrow y = 3$ or $y = 9$

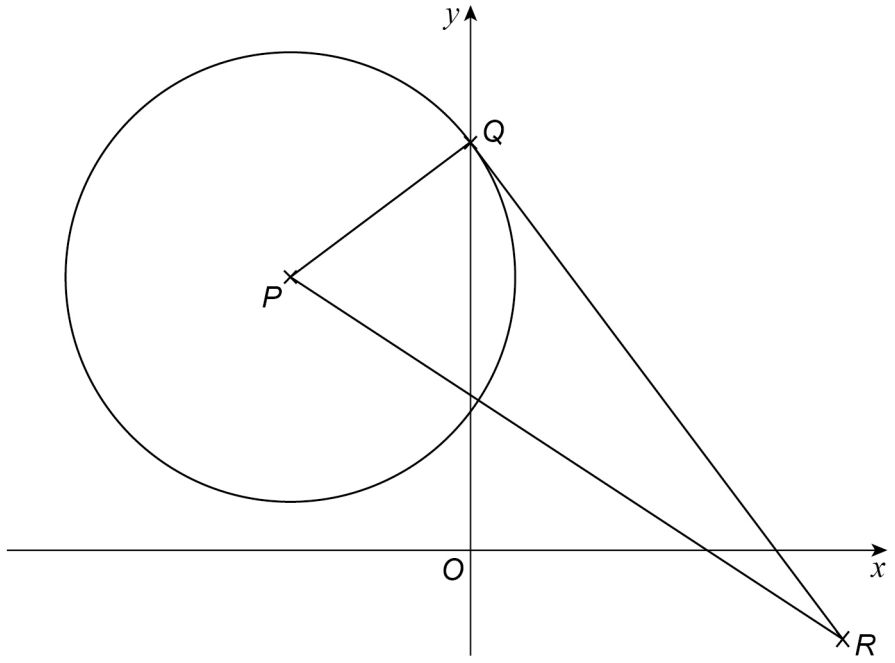
Q is the upper intersection, so y-coordinate of $Q = 9$

Question 9 continues on the next page

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9 (d) The line segment QR is a tangent to the circle as shown in **Figure 2** below.

Figure 2



The point R has coordinates $(9, -3)$.

Find the angle QPR

Give your answer in radians to three significant figures.

[3 marks]

$Q(0, 9)$, $P(-4, 6)$, $R(9, -3)$

PQ = radius = $\sqrt{25} = 5$

$PR = \sqrt{((9 + 4)^2 + (-3 - 6)^2)} = \sqrt{(169 + 81)} = \sqrt{250} = 5\sqrt{10}$

Angle $PQR = 90°$ (tangent is perpendicular to radius)

$\cos QPR = \frac{PQ}{PR} = \frac{5}{5\sqrt{10}} = \frac{1}{\sqrt{10}}$

$QPR = \cos^{-1}(1/\sqrt{10}) = 1.24904...$

Angle $QPR = 1.25$ radians (3 s.f.)

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10 It is given that

$$f(x) = 5x^3 + x$$

Use differentiation from first principles to prove that

$$f'(x) = 15x^2 + 1$$

[5 marks]

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x+h) - f(x) = 5(x+h)^3 + (x+h) - (5x^3 + x)$

$= 5(x^3 + 3x^2h + 3xh^2 + h^3) + x + h - 5x^3 - x$

$= 15x^2h + 15xh^2 + 5h^3 + h$

$\frac{f(x+h) - f(x)}{h} = \frac{15x^2h + 15xh^2 + 5h^3 + h}{h}$

$= 15x^2 + 15xh + 5h^2 + 1$

As $h \to 0$, $15xh \to 0$ and $5h^2 \to 0$

$f'(x) = \lim_{h \to 0} (15x^2 + 15xh + 5h^2 + 1)$

$f'(x) = 15x^2 + 1$ as required

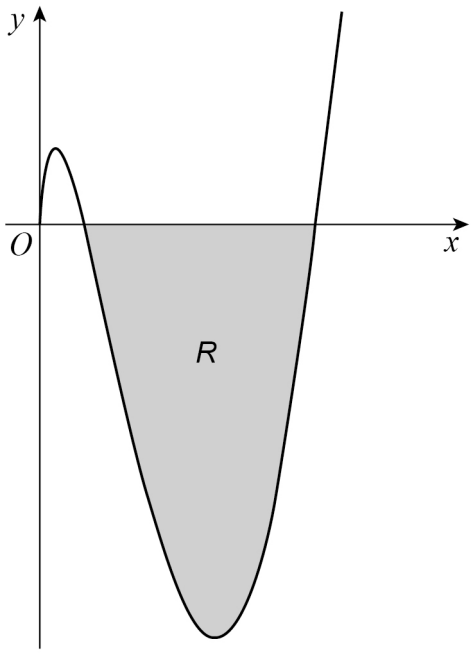
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11 The curve C with equation

$$y = (x^2 - 8x) \ln x$$

is defined for $x > 0$ and is shown in the diagram below.



The shaded region, R, lies below the x-axis and is bounded by C and the x-axis.

Show that the area of R can be written as

$$p + q \ln 2$$

where p and q are rational numbers to be found.

[10 marks]

Curve meets x-axis when $(x^2 - 8x) \ln x = 0$

$x(x - 8) = 0 \Rightarrow x = 0$ or $x = 8$; $\ln x = 0 \Rightarrow x = 1$

$x > 0$, so R lies between $x = 1$ and $x = 8$

Consider $\int_1^8 (x^2 - 8x) \ln x \, dx$

By parts: $u = \ln x \Rightarrow du/dx = 1/x$

$dv/dx = x^2 - 8x \Rightarrow v = (1/3)x^3 - 4x^2$

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$\int (x^2 - 8x) \ln x \, dx = ((1/3)x^3 - 4x^2) \ln x - \int ((1/3)x^3 - 4x^2) \times (1/x) \, dx$

$= ((1/3)x^3 - 4x^2) \ln x - \int ((1/3)x^2 - 4x) \, dx$

$= [\, ((1/3)x^3 - 4x^2) \ln x - (1/9)x^3 + 2x^2 \,]_1^8$

$x = 8$: $(\frac{512}{3} - 256) \ln 8 - \frac{512}{9} + 128$

$= -\frac{256}{3} \times 3 \ln 2 + \frac{640}{9} = -256 \ln 2 + \frac{640}{9}$

$x = 1$: $(\frac{1}{3} - 4) \ln 1 - \frac{1}{9} + 2 = \frac{17}{9}$

$\int_1^8 (x^2 - 8x) \ln x \, dx = (\frac{640}{9} - 256 \ln 2) - \frac{17}{9} = \frac{623}{9} - 256 \ln 2$

$= -108.22...$ (negative as R is below the x-axis)

Area of $R = 256 \ln 2 - \frac{623}{9}$

$= -\frac{623}{9} + 256 \ln 2$

so $p = -\frac{623}{9}$ and $q = 256$

END OF SECTION A

TURN OVER FOR SECTION B

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Section B

Answer **all** questions in the spaces provided.

12 A random sample of 84 students was asked how many revision websites they had visited in the past month.

The data is summarised in the table below.

Number of websites	Frequency
0	1
1	4
2	18
3	16
4	5
5	37
6	2
7	1

Find the interquartile range of the number of websites visited by these 84 students.

Circle your answer.

[1 mark]

(3) 4 19 42

Q_1 = 21.25th value = 2, Q_3 = 63.75th value = 5

IQR = 5 − 2 = 3

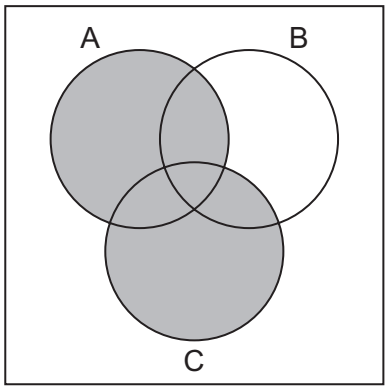
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13 The shaded region on one of the Venn diagrams below represents $(A \cup C) \cap B$

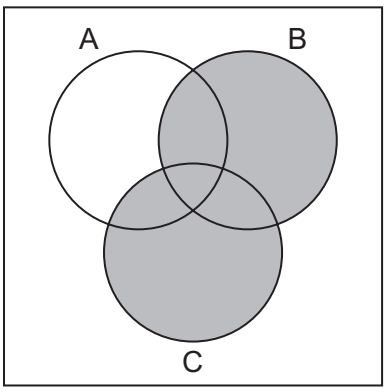
Identify this Venn diagram.

Tick (✓) **one** box.

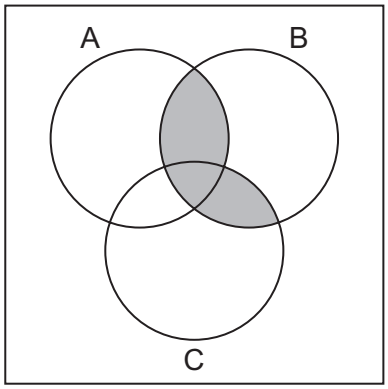
[1 mark]



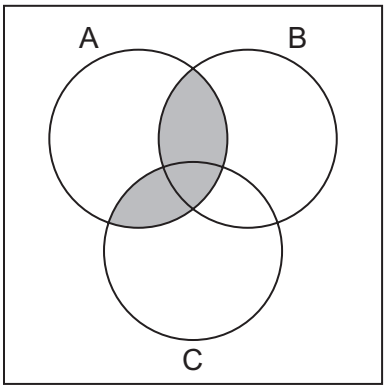
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Turn over for the next question

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14 The annual cost of energy in 2021 for each of the 350 households in Village A can be modelled by a random variable £X

It is given that

$$\sum x = 945\,000 \qquad \sum x^2 = 2\,607\,500\,000$$

14 (a) Calculate the mean of X.

[1 mark]

Mean $= \frac{945\,000}{350} = £2700$

14 (b) Calculate the standard deviation of X.

[2 marks]

s.d. $= \sqrt{\left(\frac{2\,607\,500\,000}{350} - 2700^2\right)}$

$= \sqrt{(7\,450\,000 - 7\,290\,000)} = \sqrt{160\,000}$

$= £400$

14 (c) For households in Village B the annual cost of energy in 2021 has mean £3100 and standard deviation £325

Compare the annual cost of energy in 2021 for households in Village A and Village B.

[2 marks]

On average, the annual cost of energy for households in Village A was lower than in Village B (£2700 < £3100). The cost of energy in Village A was more spread out (varied more) than in Village B (s.d. £400 > £325).

Do not write outside the box

15 It is given that

$$X \sim \text{B}(48, 0.175)$$

15 (a) Find the mean of X

[1 mark]

$E(X) = np = 48 \times 0.175 = 8.4$

15 (b) Show that the variance of X is 6.93

[1 mark]

$Var(X) = np(1 - p) = 48 \times 0.175 \times 0.825 = 6.93$

15 (c) Find $\text{P}(X < 10)$

[1 mark]

$P(X < 10) = P(X \leq 9) = 0.6742$ (4 d.p.)

15 (d) Find $\text{P}(X \geq 6)$

[2 marks]

$P(X \geq 6) = 1 - P(X \leq 5)$

$= 1 - 0.13248...$

$= 0.8675$ (4 d.p.)

Do not write outside the box

15 (e) Find $P(9 \leq X \leq 15)$

[2 marks]

$P(9 \leq X \leq 15) = P(X \leq 15) - P(X \leq 8)$

$= 0.99402... - 0.53172...$

$= 0.4623$ (4 d.p.)

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15 (f) The aeroplanes used on a particular route have 48 seats.

The proportion of passengers who use this route to travel for business is known to be 17.5%

Make **two** comments on whether it would be appropriate to use X to model the number of passengers on an aeroplane who are travelling for business using this route.

[2 marks]

Appropriate: the 48 seats and the 17.5% of passengers travelling for business match $n = 48$ and $p = 0.175$ in the model.

However, the plane may not be full, so the number of passengers n may not be 48. Also passengers may book in groups (e.g. work colleagues), so passengers travelling for business may not be independent of each other.

Turn over for the next question

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16 A medical student believes that, in adults, there is a **negative correlation** between the amount of nicotine in their blood stream and their energy level.

The student collected data from a random sample of 50 adults.

The correlation coefficient between the amount of nicotine in their blood stream and their energy level was -0.45

Carry out a hypothesis test at the 2.5% significance level to determine if this sample provides evidence to support the student's belief.

For $n = 50$, the critical value for a one-tailed test at the 2.5% level for the population correlation coefficient is 0.2787

[4 marks]

ρ = population correlation coefficient

H_0: $\rho = 0$ H_1: $\rho < 0$

$r = -0.45$, critical value $= -0.2787$ (one-tailed, 2.5%)

$-0.45 < -0.2787$, so r lies in the critical region

Reject H_0

There is sufficient evidence at the 2.5% level to suggest that, in adults, there is negative correlation between the amount of nicotine in their blood stream and their energy level, supporting the student's belief.

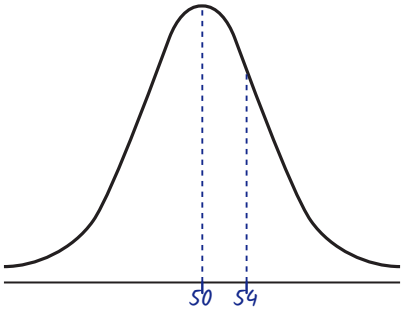
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17 In 2019, the lengths of new-born babies at a clinic can be modelled by a normal distribution with mean 50 cm and standard deviation 4 cm.

17 (a) This normal distribution is represented in the diagram below.

Label the values 50 and 54 on the horizontal axis.

[2 marks]



17 (b) State the probability that the length of a new-born baby is less than 50 cm.

[1 mark]

$P(X < 50) = 0.5$

17 (c) Find the probability that the length of a new-born baby is more than 56 cm.

[1 mark]

$X \sim N(50, 4^2)$: $P(X > 56) = 0.0668$ (3 s.f.)

17 (d) Find the probability that the length of a new-born baby is more than 40 cm but less than 60 cm.

[1 mark]

$P(40 < X < 60) = 0.9876$ (4 d.p.)

Turn over ▶

Do not write outside the box

17 (e) Determine the length exceeded by 95% of all new-born babies at the clinic.

[2 marks]

$P(X > x) = 0.95 \Rightarrow P(X < x) = 0.05$

$\frac{x - 50}{4} = -1.6449$

$x = 50 - 4 \times 1.6449 = 43.420...$

Length = 43.4 cm (3 s.f.)

17 (f) In 2020, the lengths of 40 new-born babies at the clinic were selected at random.

The total length of the 40 new-born babies was 2060 cm.

Carry out a hypothesis test at the 10% significance level to investigate whether the mean length of a new-born baby at the clinic in 2020 has **increased** compared to 2019.

You may assume that the length of a new-born baby is still normally distributed with standard deviation 4 cm.

[7 marks]

μ = mean length of new-born babies at the clinic in 2020 (cm)

H_0: $\mu = 50$ H_1: $\mu > 50$

$\bar{x} = 2060 \div 40 = 51.5$

Under H_0, $\bar{x} \sim N(50, \frac{4^2}{40})$ i.e. $N(50, 0.4)$

$P(\bar{x} \geq 51.5) = 0.008853... = 0.00885$

$0.00885 < 0.10$, so the result is significant at the 10% level

Reject H_0

There is sufficient evidence at the 10% level to suggest that the mean length of a new-born baby at the clinic in 2020 has increased compared to 2019.

Do not write outside the box

Turn over for the next question

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18 The Human Resources director in a company is investigating the graduate status and salaries of its employees.

Event G is defined as the employee is a graduate.

Event H is defined as the employee earns at least £40 000 a year.

The director summarised the findings in the table of probabilities below.

	H	H'
G	0.21	0.18
G'	0.07	0.54

18 (a) An employee is selected at random.

18 (a) (i) Find $P(G)$

[1 mark]

$P(G) = 0.21 + 0.18 = 0.39$

18 (a) (ii) Find $P[(G \cap H)']$

[2 marks]

$P[(G \cap H)'] = 1 - P(G \cap H)$

$= 1 - 0.21 = 0.79$

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18 (a) (iii) Find $P(H \mid G')$

[2 marks]

$P(H \mid G') = \frac{P(H \cap G')}{P(G')}$

$= \frac{0.07}{0.07 + 0.54} = \frac{0.07}{0.61}$

$= \frac{7}{61}$ (= 0.115 to 3 s.f.)

18 (b) Determine whether the events G and H are independent.

Fully justify your answer.

[2 marks]

$P(H) = 0.21 + 0.07 = 0.28$

$P(G) \times P(H) = 0.39 \times 0.28 = 0.1092$

$P(G \cap H) = 0.21$

$P(G \cap H) \neq P(G) \times P(H)$ since $0.21 \neq 0.1092$

Hence G and H are not independent.

Turn over for the next question

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19 It is known that 80% of all diesel cars registered in 2017 had carbon monoxide (CO) emissions less than 0.3 g/km.

Talat decides to investigate whether the proportion of diesel cars registered in 2022 with CO emissions less than 0.3 g/km has **changed**.

Talat will carry out a hypothesis test at the 10% significance level on a random sample of 25 diesel cars registered in 2022.

19 (a) (i) State suitable null and alternative hypotheses for Talat's test.

[1 mark]

$H_0: p = 0.8$ $H_1: p \neq 0.8$

19 (a) (ii) Using a 10% level of significance, find the critical region for Talat's test.

[5 marks]

Under H_0, $X \sim B(25, 0.8)$

Two-tailed test at 10%: 5% in each tail

$P(X \leq 16) = 0.0468 < 0.05$

$P(X \leq 17) = 0.1091 > 0.05$

$P(X \geq 24) = 1 - P(X \leq 23) = 0.0274 < 0.05$

$P(X \geq 23) = 1 - P(X \leq 22) = 0.0982 > 0.05$

Critical region: $X \leq 16$ or $X \geq 24$

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19 (a) (iii) In his random sample, Talat finds 18 cars with CO emissions less than 0.3 g/km.

State Talat's conclusion in context.

[1 mark]

18 is not in the critical region, so do not reject H_0. There is insufficient evidence to suggest that the proportion of diesel cars registered in 2022 with CO emissions less than 0.3 g/km has changed from 80%.

19 (b) Talat now wants to use his random sample of 25 diesel cars, registered in 2022, to investigate whether the proportion of diesel cars in England with CO emissions more than 0.5 g/km has changed from the proportion given by the Large Data Set.

Using your knowledge of the Large Data Set, give **two** reasons why it is not possible for Talat to do this.

[2 marks]

No diesel cars in the Large Data Set have CO emissions of more than 0.5 g/km, so there is no proportion to compare with.

Also, the Large Data Set only contains cars from some regions of England (and some makes), so it is not representative of all diesel cars in England.

END OF QUESTIONS

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