AQA

Please write clearly in block capitals.

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I declare this is my own work.

A-level MATHEMATICS

Paper 1

Tuesday 6 June 2023 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 Find the coefficient of x^7 in the expansion of $(2x - 3)^7$

Circle your answer.

[1 mark]

-2187 -128 2 128

2 Given that $y = 2x^3$ find $\frac{dy}{dx}$

Circle your answer.

[1 mark]

$\frac{dy}{dx} = 5x^2$ $\frac{dy}{dx} = 6x^2$ $\frac{dy}{dx} = \frac{x^4}{2}$ $\frac{dy}{dx} = 6x^3$

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3 The curve with equation $y = \ln x$ is transformed by a stretch parallel to the x-axis with scale factor 2

Find the equation of the transformed curve.

Circle your answer.

[1 mark]

$y = \frac{1}{2}\ln x$ $\quad$ $y = 2\ln x$ $\quad$ $y = \ln\frac{x}{2}$ $\quad$ $y = \ln 2x$

4 Given that θ is a small angle, find an approximation for $\cos 2\theta$

Circle your answer.

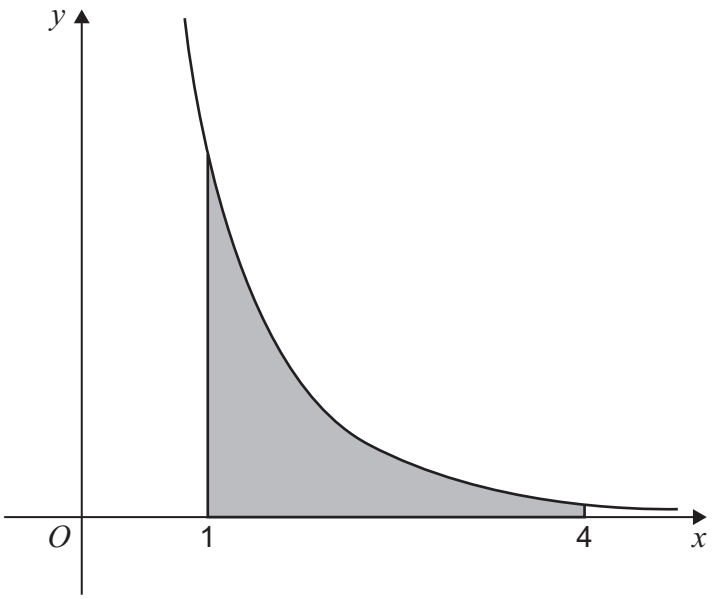
[1 mark]

$1 - \frac{\theta^2}{2}$ $\quad$ $2 - 2\theta^2$ $\quad$ $1 - 2\theta^2$ $\quad$ $1 - \theta^2$

Turn over for the next question

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5 The graph of $y = \dfrac{5}{e^x - 1}$ is shown in the diagram below.



The trapezium rule with 6 ordinates (5 strips) is to be used to find an approximation for the shaded area.

The values required to obtain this approximation are shown in the table below.

x	1	1.6	2.2	2.8	3.4	4
y	2.90988	1.26485	0.62305	0.32374	0.17263	0.09329

5 (a) Use the trapezium rule with 6 ordinates (5 strips) to find an approximate value for the shaded area.

Give your answer to four decimal places.

[3 marks]

$h = (4 - 1) \div 5 = 0.6$

$A \approx \frac{1}{2} \times 0.6 \times [2.90988 + 0.09329 + 2(1.26485 + 0.62305 + 0.32374 + 0.17263)]$

$= 0.3 \times [3.00317 + 2 \times 2.38427]$

$= 0.3 \times 7.77171$

$= 2.331513$

$= 2.3315$ (4 d.p.)

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5 (b) Using your answer to part **(a)** deduce an estimate for $\int_1^4 \frac{20}{e^x - 1}\,dx$

[1 mark]

$\int_1^4 \frac{20}{e^x - 1}\,dx = 4\int_1^4 \frac{5}{e^x - 1}\,dx \approx 4 \times 2.3315$

$= 9.326 \ (\approx 9.33)$

Turn over for the next question

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6 Show that the equation

$$2\log_{10} x = \log_{10} 4 + \log_{10} (x + 8)$$

has exactly one solution.

Fully justify your answer.

[5 marks]

$2\log_{10} x = \log_{10} 4 + \log_{10} (x + 8)$

$\log_{10} x^2 = \log_{10} [4(x + 8)]$

$x^2 = 4(x + 8) = 4x + 32$

$x^2 - 4x - 32 = 0$

$(x - 8)(x + 4) = 0$

$x = 8$ or $x = -4$

$x = -4$ is not a solution, since $\log_{10} x$ requires $x > 0$:

$\log_{10}(-4)$ is undefined (the log of a negative number has no real value).

Check $x = 8$: $2\log_{10} 8 = \log_{10} 64$ and $\log_{10} 4 + \log_{10} 16 = \log_{10} 64$ ✓

$\therefore$ the only solution is $x = 8$, so the equation has exactly one solution.

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7 (a) Given that n is a positive integer, express

$$\frac{7}{3+5\sqrt{n}} - \frac{7}{5\sqrt{n}-3}$$

as a single fraction not involving surds.

[3 marks]

$\frac{7}{3+5\sqrt{n}} - \frac{7}{5\sqrt{n}-3} = \frac{7(5\sqrt{n}-3) - 7(3+5\sqrt{n})}{(3+5\sqrt{n})(5\sqrt{n}-3)}$

$= \frac{35\sqrt{n} - 21 - 21 - 35\sqrt{n}}{15\sqrt{n} - 9 + 25n - 15\sqrt{n}}$

$= \frac{-42}{25n-9}$

so the expression $= -\frac{42}{25n-9}$

7 (b) Hence, deduce that

$$\frac{7}{3+5\sqrt{n}} - \frac{7}{5\sqrt{n}-3}$$

is a rational number for all positive integer values of n

[1 mark]

n is a positive integer, so 42 and $25n - 9$ are both integers (and $25n - 9 \neq 0$).

The expression is a ratio of two integers, so it is a rational number.

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8 Show that

$$\int_0^{\frac{\pi}{2}} (x \sin 4x)\, \mathrm{d}x = -\frac{\pi}{8}$$

[6 marks]

$u = x, \quad \frac{dv}{dx} = \sin 4x \quad \Rightarrow \quad \frac{du}{dx} = 1, \quad v = -\frac{1}{4}\cos 4x$

$\int_0^{\pi/2} x \sin 4x \, dx = \left[-\frac{1}{4} x \cos 4x\right]_0^{\pi/2} - \int_0^{\pi/2} \left(-\frac{1}{4}\cos 4x\right) dx$

$= \left[-\frac{1}{4} x \cos 4x + \frac{1}{16}\sin 4x\right]_0^{\pi/2}$

$= \left(-\frac{1}{4} \times \frac{\pi}{2} \times \cos 2\pi + \frac{1}{16}\sin 2\pi\right) - \left(0 \times \cos 0 + \frac{1}{16}\sin 0\right)$

$= \left(-\frac{\pi}{8} \times 1 + 0\right) - (0 + 0)$

$= -\frac{\pi}{8}$ as required

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Turn over for the next question

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9 The points P and Q have coordinates $(-6, 15)$ and $(12, 19)$ respectively.

9 (a) (i) Find the coordinates of the midpoint of PQ

[1 mark]

$\left(\frac{-6 + 12}{2}, \frac{15 + 19}{2}\right) = (3, 17)$

9 (a) (ii) Find the equation of the perpendicular bisector of PQ

Give your answer in the form $ax + by = c$ where a, b and c are integers.

[4 marks]

gradient of $PQ = \frac{19 - 15}{12 - (-6)} = \frac{4}{18} = \frac{2}{9}$

perpendicular gradient $= -\frac{9}{2}$

$y - 17 = -\frac{9}{2}(x - 3)$

$2y - 34 = -9x + 27$

$9x + 2y = 61$

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9 (b) (i) A circle passes through the points P and Q

The centre of the circle lies on the line with equation $2x - 5y = -30$

Find the equation of the circle.

[3 marks]

Centre lies on the perpendicular bisector of PQ and on $2x - 5y = -30$:

$9x + 2y = 61 \quad (\times 5) \Rightarrow 45x + 10y = 305$

$2x - 5y = -30 \quad (\times 2) \Rightarrow 4x - 10y = -60$

adding: $49x = 245 \Rightarrow x = 5, \quad y = (61 - 45) \div 2 = 8$

centre $(5, 8)$

$r^2 = (12 - 5)^2 + (19 - 8)^2 = 49 + 121 = 170$

$(x - 5)^2 + (y - 8)^2 = 170$

9 (b) (ii) The circle intersects the coordinate axes at n points.

State the value of n

[1 mark]

$x = 0$: $(y - 8)^2 = 145 > 0$, two points; $y = 0$: $(x - 5)^2 = 106 > 0$, two points

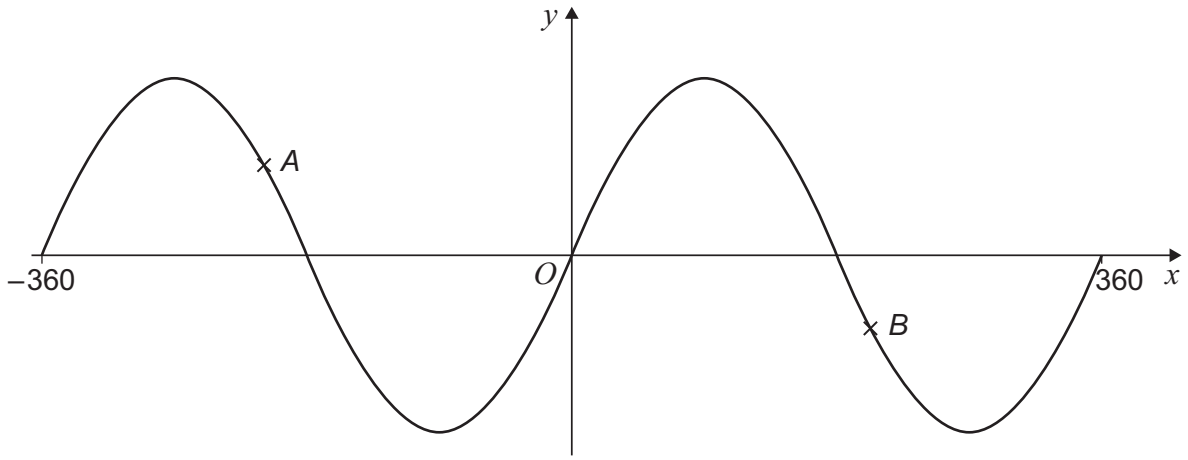
$n = 4$

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10 The curve with equation

$$y = \sin x°$$

for $-360 \leq x \leq 360$ is shown below.



10 (a) Point A on the curve has coordinates $(a, 0.5)$

10 (a) (i) Find the value of a

[2 marks]

sin 30° = 0.5, so sin x° = 0.5 at x = 30, 150, −210, −330

A is between −270 and −180 on the graph

a = −210

10 (a) (ii) State the value of $\sin(180° - a°)$

[1 mark]

sin(180° − a°) = sin a° = 0.5

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10 (b) Point B on the curve has coordinates $\left(b, -\frac{3}{7}\right)$

10 (b) (i) Find the exact value of $\sin(b° - 180°)$

[2 marks]

$\sin(b° - 180°) = -\sin(180° - b°) = -\sin b°$

$= -\left(-\frac{3}{7}\right) = \frac{3}{7}$

10 (b) (ii) Find the exact value of $\cos b°$

[3 marks]

$\cos^2 b° + \sin^2 b° = 1$

$\cos^2 b° = 1 - \left(-\frac{3}{7}\right)^2 = 1 - \frac{9}{49} = \frac{40}{49}$

$\cos b° = \pm\frac{\sqrt{40}}{7} = \pm\frac{2\sqrt{10}}{7}$

B is between 180 and 270 (third quadrant), so $\cos b° < 0$

$\cos b° = -\frac{2\sqrt{10}}{7}$

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11 The nth term of a sequence is u_n

The sequence is defined by

$$u_{n+1} = pu_n + 70$$

where $u_1 = 400$ and p is a constant.

11 (a) Find an expression, in terms of p, for u_2

[1 mark]

$u_2 = 400p + 70$

11 (b) It is given that $u_3 = 382$

11 (b) (i) Show that p satisfies the equation

$$200p^2 + 35p - 156 = 0$$

[3 marks]

$u_3 = pu_2 + 70$

$382 = p(400p + 70) + 70$

$382 = 400p^2 + 70p + 70$

$400p^2 + 70p - 312 = 0$

$\div 2$: $200p^2 + 35p - 156 = 0$ as required

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11 (b) (ii) It is given that the sequence is a decreasing sequence.

Find the value of u_4 and the value of u_5

[3 marks]

$(5p - 4)(40p + 39) = 0 \Rightarrow p = 0.8$ or $p = -0.975$

$p = -0.975$: $u_4 = -0.975 \times 382 + 70 = -302.45$, $u_5 = 364.88875$

$u_5 > u_4$, so not decreasing; reject $p = -0.975$

$p = 0.8$: $u_4 = 0.8 \times 382 + 70 = 375.6$

$u_5 = 0.8 \times 375.6 + 70 = 370.48$

$u_4 = 375.6$, $u_5 = 370.48$

11 (c) The limit of u_n as n tends to infinity is L

11 (c) (i) Write down an equation for L

[1 mark]

$L = 0.8L + 70$

11 (c) (ii) Find the value of L

[1 mark]

$0.2L = 70 \Rightarrow L = 350$

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12 One of the rides at a theme park is a room where the floor and ceiling both move up and down for 10π seconds.

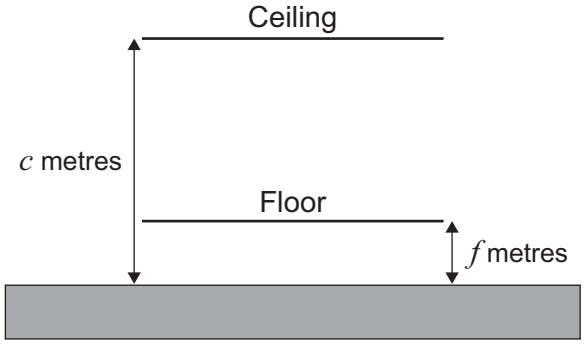
At time t seconds after the ride begins, the distance f metres of the floor above the ground is

$$f = 1 - \cos t$$

At time t seconds after the ride begins, the distance c metres of the ceiling above the ground is

$$c = 8 - 4\sin t$$

The ride is shown in the diagram below.



12 (a) Show that the initial distance between the floor and ceiling is 8 metres.

[1 mark]

$t = 0$: $c = 8 - 4\sin 0 = 8$, $f = 1 - \cos 0 = 0$

distance $= c - f = 8 - 0 = 8$ metres

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12 (b) Show that the distance d metres between the floor and ceiling at time t is given by

$$d = 7 + R\cos(t + \alpha)$$

where R and α are positive constants to be found.

[5 marks]

$d = c - f = 8 - 4\sin t - (1 - \cos t) = 7 + \cos t - 4\sin t$

$R\cos(t + \alpha) = R\cos t \cos \alpha - R\sin t \sin \alpha$

Comparing: $R\cos \alpha = 1$, $R\sin \alpha = 4$

$R = \sqrt{(1^2 + 4^2)} = \sqrt{17}$

$\tan \alpha = 4 \Rightarrow \alpha = 1.3258... = 1.33$ (3 s.f.)

$d = 7 + \sqrt{17}\cos(t + 1.33)$

12 (c) Hence, find the minimum distance between the ceiling and the floor.

Give your answer to the nearest centimetre.

[2 marks]

minimum when $\cos(t + 1.33) = -1$

$d_{min} = 7 - \sqrt{17} = 2.87689...$ m

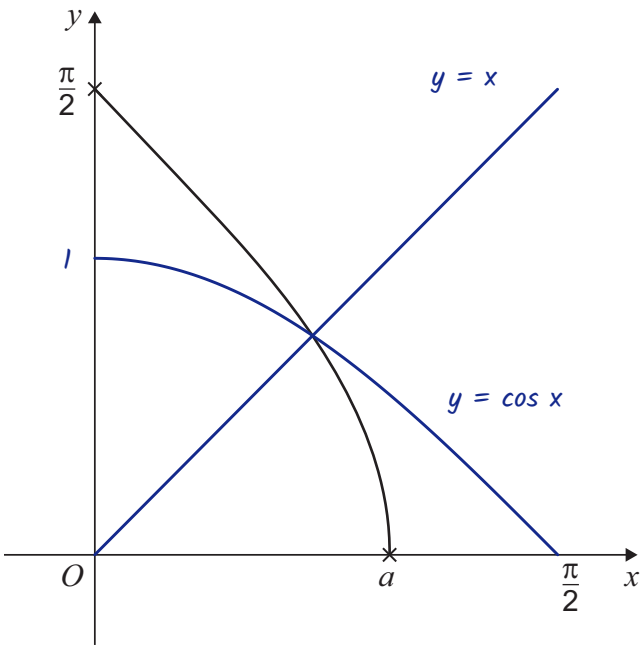
$= 2.88$ m (288 cm)

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13 The function f is defined by

$$\mathrm{f}(x) = \arccos x \quad \text{for} \quad 0 \leq x \leq a$$

The curve with equation $y = \mathrm{f}(x)$ is shown below.



13 (a) State the value of a

[1 mark]

a = 1

13 (b) (i) On the diagram above, sketch the curve with equation

$$y = \cos x \quad \text{for} \quad 0 \leq x \leq \frac{\pi}{2}$$

and

sketch the line with equation

$$y = x \quad \text{for} \quad 0 \leq x \leq \frac{\pi}{2}$$

[4 marks]

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13 (b) (ii) Explain why the solution to the equation

$$x - \cos x = 0$$

must also be a solution to the equation

$$\cos x = \arccos x$$

[1 mark]

y = cos x and y = arccos x are reflections of each other in the line y = x,

so all three graphs meet at the same point.

If x − cos x = 0 then x = cos x, so arccos x = x = cos x,

i.e. the solution also satisfies cos x = arccos x.

Question 13 continues on the next page

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13 (c) Use the Newton-Raphson method with $x_0 = 0$ to find an approximate solution, x_3, to the equation

$$x - \cos x = 0$$

Give your answer to four decimal places.

[3 marks]

$f(x) = x - \cos x, \quad f'(x) = 1 + \sin x$

$x_{n+1} = x_n - \dfrac{x_n - \cos x_n}{1 + \sin x_n}$

$x_1 = 0 - \dfrac{0 - \cos 0}{1 + \sin 0} = 0 - \dfrac{-1}{1} = 1$

$x_2 = 1 - \dfrac{1 - \cos 1}{1 + \sin 1} = 0.750363\ldots$

$x_3 = 0.750363\ldots - \dfrac{0.750363\ldots - \cos 0.750363\ldots}{1 + \sin 0.750363\ldots} = 0.739112\ldots$

$x_3 = 0.7391$ (4 d.p.)

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14 (a) (i) Given that

$$y = 2^x$$

write down $\frac{\mathrm{d}y}{\mathrm{d}x}$

[1 mark]

$\frac{dy}{dx} = 2^x \ln 2$

14 (a) (ii) Hence find

$$\int 2^x \, \mathrm{d}x$$

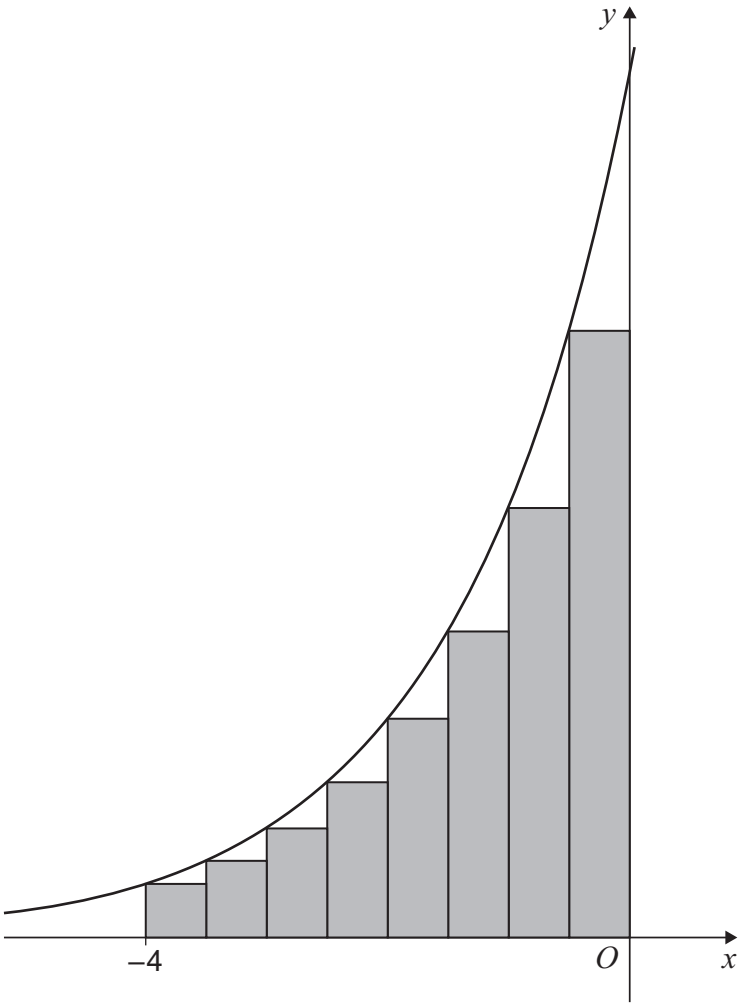
[2 marks]

From (a)(i), $\frac{d}{dx}(2^x) = 2^x \ln 2$, so $\frac{d}{dx}\left(\frac{2^x}{\ln 2}\right) = 2^x$

$\int 2^x \, dx = \frac{2^x}{\ln 2} + c$

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14 (b) The area, A, bounded by the curve with equation $y = 2^x$, the x-axis, the y-axis and the line $x = -4$ is approximated using eight rectangles of equal width as shown in the diagram below.



14 (b) (i) Show that the exact area of the largest rectangle is $\frac{\sqrt{2}}{4}$

[2 marks]

width of each rectangle = $4 \div 8 = 0.5$

largest rectangle: height = $2^{-0.5} = \frac{1}{\sqrt{2}}$ (value at $x = -0.5$)

area = $\frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

Question 14 continues on the next page

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14 (b) (ii) The areas of these rectangles form a geometric sequence with common ratio $\frac{\sqrt{2}}{2}$

Find the exact value of the total area of the eight rectangles.

Give your answer in the form $k(1+\sqrt{2})$ where k is a rational number.

[3 marks]

$a = \frac{\sqrt{2}}{4}, \quad r = \frac{\sqrt{2}}{2}, \quad n = 8, \quad r^8 = \frac{16}{256} = \frac{1}{16}$

$S_8 = \frac{a(1-r^8)}{1-r} = \frac{(\sqrt{2}/4)(1-1/16)}{1-\sqrt{2}/2} = \frac{15\sqrt{2}/64}{(2-\sqrt{2})/2} = \frac{15\sqrt{2}}{32(2-\sqrt{2})}$

$= \frac{15\sqrt{2}(2+\sqrt{2})}{32(2-\sqrt{2})(2+\sqrt{2})} = \frac{15(2\sqrt{2}+2)}{32 \times 2} = \frac{15+15\sqrt{2}}{32}$

$= \frac{15}{32}(1+\sqrt{2})$

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14 (b) (iii) More accurate approximations for A can be found by increasing the number, n, of rectangles used.

Find the exact value of the limit of the approximations for A as $n \to \infty$

[3 marks]

$A = \int_{-4}^{0} 2^x \, dx = \left[\frac{2^x}{\ln 2}\right]_{-4}^{0}$

$= \frac{1}{\ln 2}(2^0 - 2^{-4}) = \frac{1}{\ln 2}\left(1 - \frac{1}{16}\right)$

$= \frac{15}{16 \ln 2}$

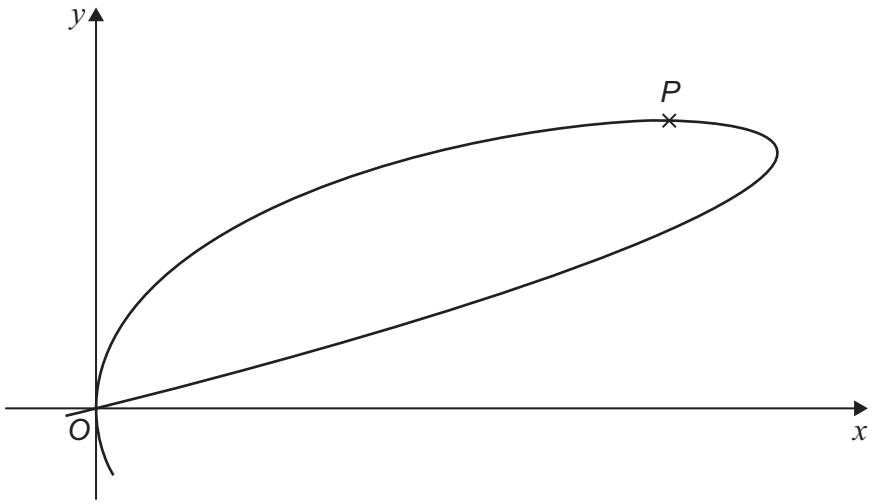
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15 The curve with equation

$$x^2 + 2y^3 - 4xy = 0$$

has a single stationary point at P as shown in the diagram below.



15 (a) Show that the y-coordinate of P satisfies the equation

$$y^2(y - 2) = 0$$

[7 marks]

$x^2 + 2y^3 - 4xy = 0$

Differentiating implicitly (product rule on $4xy$):

$2x + 6y^2\frac{dy}{dx} - (4y + 4x\frac{dy}{dx}) = 0$

$2x + 6y^2\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} = 0$

At the stationary point $\frac{dy}{dx} = 0$:

$2x - 4y = 0 \quad \Rightarrow \quad x = 2y$

Substitute into the curve: $(2y)^2 + 2y^3 - 4(2y)y = 0$

$4y^2 + 2y^3 - 8y^2 = 0$

$2y^3 - 4y^2 = 0$

$2y^2(y - 2) = 0$

$y^2(y - 2) = 0$ as required

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15 (b) Hence, find the coordinates of P

[2 marks]

$y = 0$ gives $x = 0$, which is the origin O, not P

so $y = 2$, $x = 2y = 4$

P is $(4, 2)$

Turn over for the next question

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16 (a) Given that

$$\frac{1}{16-9x^2} \equiv \frac{A}{4-3x} + \frac{B}{4+3x}$$

find the values of A and B

[3 marks]

$1 \equiv A(4+3x) + B(4-3x)$

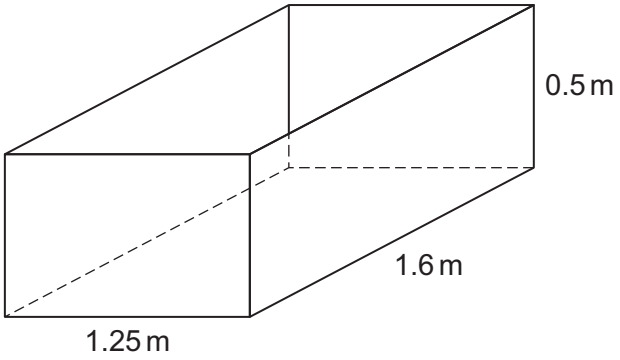
$x = \frac{4}{3}$: $1 = 8A \Rightarrow A = \frac{1}{8}$

$x = -\frac{4}{3}$: $1 = 8B \Rightarrow B = \frac{1}{8}$

$A = \frac{1}{8}$, $B = \frac{1}{8}$

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16 (b) An empty container, in the shape of a cuboid, has length 1.6 metres, width 1.25 metres and depth 0.5 metres, as shown in the diagram below.



The container has a small hole in the bottom.

Water is poured into the container at a rate of 0.16 cubic metres per minute.

At time t minutes after the container starts to be filled, the depth of water is d metres and water leaks out at a rate of $0.36d^2$ cubic metres per minute.

At time t minutes after the container starts to be filled, the volume of water in the container is V cubic metres.

16 (b) (i) Show that

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{16 - 9V^2}{100}$$

[4 marks]

$\frac{dV}{dt}$ = rate in − rate out = $0.16 - 0.36d^2$

$V = 1.6 \times 1.25 \times d = 2d \quad \Rightarrow \quad d = \frac{V}{2}$

$\frac{dV}{dt} = 0.16 - 0.36\left(\frac{V}{2}\right)^2$

$= 0.16 - 0.09V^2$

$= \frac{16 - 9V^2}{100}$ as required

Turn over ▶

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16 (b) (ii) Hence, find t in terms of V

[5 marks]

$\int \frac{100}{16 - 9V^2}\, dV = \int 1\, dt$

Using (a) with $x = V$: $100\int \left(\frac{1/8}{4 - 3V} + \frac{1/8}{4 + 3V}\right) dV = t$

$\frac{100}{8}\left(-\frac{1}{3}\ln(4 - 3V) + \frac{1}{3}\ln(4 + 3V)\right) = t + c$

$\frac{25}{6}(\ln(4 + 3V) - \ln(4 - 3V)) = t + c$

$t = 0, V = 0$: $\frac{25}{6}(\ln 4 - \ln 4) = 0 + c \Rightarrow c = 0$

$t = \frac{25}{6}\ln\left(\frac{4 + 3V}{4 - 3V}\right)$

16 (b) (iii) Determine how long it takes to fill the container with water.

Give your answer to the nearest minute.

[2 marks]

Full when $V = 1.6 \times 1.25 \times 0.5 = 1$

$t = \frac{25}{6}\ln\left(\frac{4 + 3}{4 - 3}\right) = \frac{25}{6}\ln 7 = 8.1079...$

= 8 minutes (to the nearest minute)

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