AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level MATHEMATICS

Paper 3

Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 State the range of values of x for which the binomial expansion of

$$\sqrt{1-\frac{x}{4}}$$

is valid.

Circle your answer.

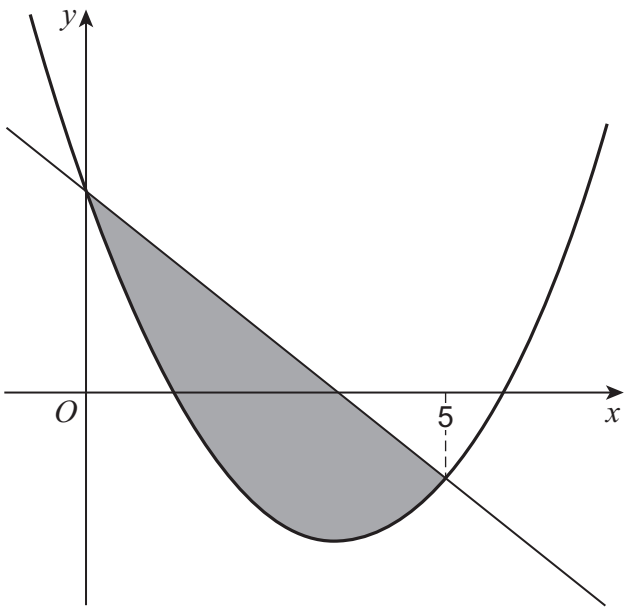
[1 mark]

$|x| < \frac{1}{4}$ $\quad$ $|x| < 1$ $\quad$ $|x| < 2$ $\quad$ (circled) $|x| < 4$

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2 The shaded region, shown in the diagram below, is defined by

$$x^2 - 7x + 7 \leq y \leq 7 - 2x$$



Identify which of the following gives the area of the shaded region.

Tick (✓) **one** box.

[1 mark]

$\int (7 - 2x)\,\mathrm{d}x - \int (x^2 - 7x + 7)\,\mathrm{d}x$ ☐

$\int_0^5 (x^2 - 5x)\,\mathrm{d}x$ ☐

$\int_0^5 (5x - x^2)\,\mathrm{d}x$ ☑

$\int_0^5 (x^2 - 9x + 14)\,\mathrm{d}x$ ☐

Turn over for the next question

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3 The function f is defined by

$$\mathrm{f}(x) = 2x + 1$$

Solve the equation

$$\mathrm{f}(x) = \mathrm{f}^{-1}(x)$$

Circle your answer.

[1 mark]

$x = -1$ $\quad$ $x = 0$ $\quad$ $x = 1$ $\quad$ $x = 2$

4 Find

$$\int \left(x^2 + x^{\frac{1}{2}} \right) \mathrm{d}x$$

[2 marks]

$\int (x^2 + x^{1/2})\, dx = \frac{x^3}{3} + \frac{x^{3/2}}{3/2} + c$

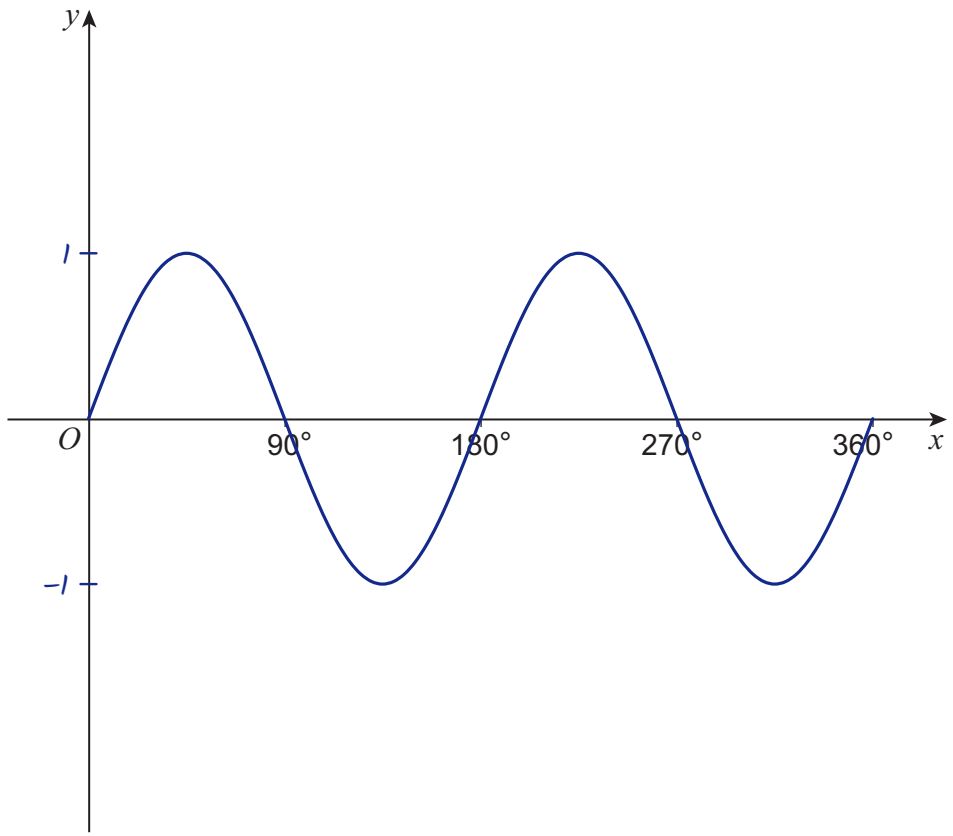
$= \frac{1}{3}x^3 + \frac{2}{3}x^{3/2} + c$

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5 (a) Sketch the graph of

$$y = \sin 2x$$

for $0° \leq x \leq 360°$



[2 marks]

5 (b) The equation

$$\sin 2x = A$$

has exactly two solutions for $0° \leq x \leq 360°$

State the possible values of A.

[1 mark]

sin 2x = 1 ⇒ x = 45°, 225° sin 2x = −1 ⇒ x = 135°, 315°

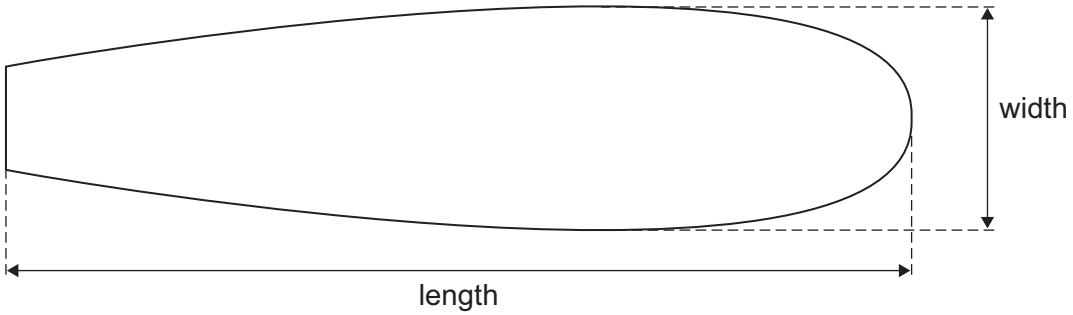
A = 1 or A = −1 (A = ±1)

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6 A design for a surfboard is shown in **Figure 1**.

Figure 1



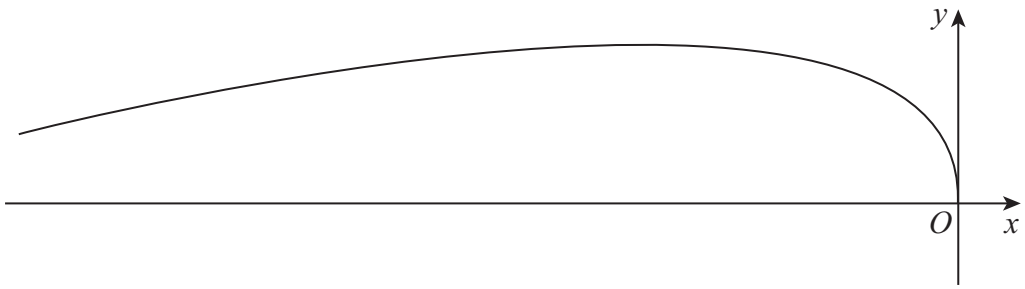
The curve of the **top half** of the surfboard can be modelled by the parametric equations

$$x = -2t^2$$

$$y = 9t - 0.7t^2$$

for $0 \leq t \leq 9.5$ as shown in **Figure 2**, where x and y are measured in centimetres.

Figure 2



6 (a) Find the length of the surfboard.

[2 marks]

The length is the magnitude of x at the end, t = 9.5

$x = -2 \times 9.5^2 = -180.5$

Length of surfboard = 180.5 cm

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6 (b) (i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

[3 marks]

$\frac{dx}{dt} = -4t$ $\frac{dy}{dt} = 9 - 1.4t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{9 - 1.4t}{-4t}$

6 (b) (ii) Hence, show that the width of the surfboard is approximately one third of its length.

[4 marks]

Maximum width where the top half is furthest from the x-axis:

$\frac{dy}{dx} = 0 \Rightarrow 9 - 1.4t = 0$

$t = \frac{9}{1.4} = \frac{45}{7} = 6.428...$

$y = 9 \times \frac{45}{7} - 0.7 \times (\frac{45}{7})^2 = 28.928...$ cm

Width $= 2y = 2 \times 28.928... = 57.857... \approx 58$ cm

One third of length $= 180.5 \div 3 = 60.16... \approx 60$ cm

$\frac{\text{width}}{\text{length}} = \frac{57.86}{180.5} = 0.32... \approx \frac{1}{3}$

Since $58 \approx 60$, the width of the surfboard is approximately one third of its length.

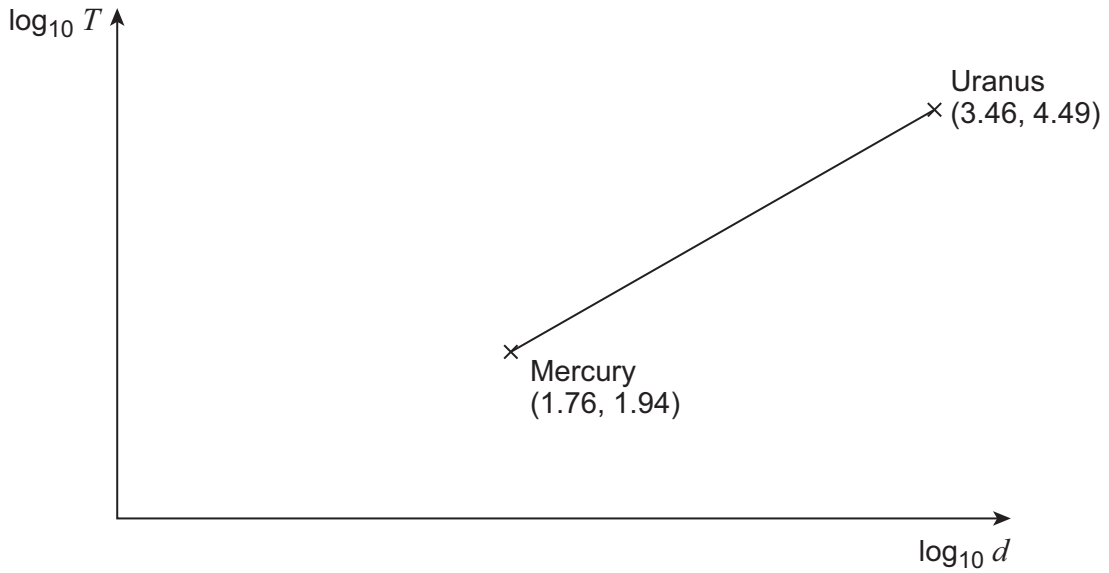
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7 A planet takes T days to complete one orbit of the Sun.

T is known to be related to the planet's average distance d, in millions of kilometres, from the Sun.

A graph of $\log_{10} T$ against $\log_{10} d$ is shown with data for Mercury and Uranus labelled.



7 (a) (i) Find the equation of the straight line in the form

$$\log_{10} T = a + b \log_{10} d$$

where a and b are constants to be found.

[3 marks]

b = gradient = $\frac{4.49 - 1.94}{3.46 - 1.76} = \frac{2.55}{1.70} = 1.5$

$\log_{10} T - 1.94 = 1.5(\log_{10} d - 1.76)$

$\log_{10} T = 1.5 \log_{10} d - 2.64 + 1.94$

$\log_{10} T = -0.7 + 1.5 \log_{10} d$

$a = -0.7$, $b = 1.5$

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7 (a) (ii) Show that

$$T = \mathrm{K}d^{\mathrm{n}}$$

where K and n are constants to be found.

[2 marks]

$\log_{10} T - 1.5 \log_{10} d = -0.7$

$\log_{10} T - \log_{10} d^{1.5} = -0.7$

$\log_{10} \left(\frac{T}{d^{1.5}}\right) = -0.7 \quad \Rightarrow \quad \frac{T}{d^{1.5}} = 10^{-0.7}$

$T = 10^{-0.7} d^{1.5}$ which is of the form $T = Kd^n$

with $K = 10^{-0.7} = 0.1995\ldots \approx 0.2$ and $n = 1.5$

7 (b) Neptune takes approximately 60 000 days to complete one orbit of the Sun.

Use your answer to **7(a)(ii)** to find an estimate for the average distance of Neptune from the Sun.

[2 marks]

$60000 = 10^{-0.7} \times d^{1.5}$

$d^{1.5} = 60000 \div 10^{-0.7} = 300\,708.5\ldots$

$d = (300\,708.5\ldots)^{2/3} = 4488.5\ldots$

Average distance of Neptune from the Sun

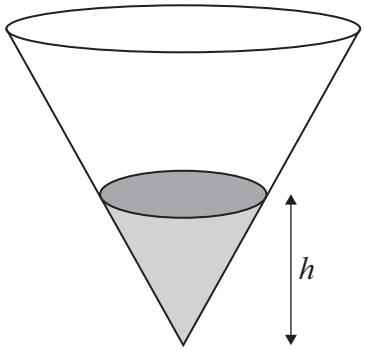
$\approx$ 4500 million km

Turn over for the next question

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8 Water is poured into an empty cone at a constant rate of $8\,\text{cm}^3/\text{s}$

After t seconds the depth of the water in the inverted cone is h cm, as shown in the diagram below.



When the depth of the water in the inverted cone is h cm, the volume, $V\,\text{cm}^3$, is given by

$$V = \frac{\pi h^3}{12}$$

8 (a) Show that when $t = 3$

$$\frac{\mathrm{d}V}{\mathrm{d}h} = 6\sqrt[3]{6\pi}$$

[4 marks]

$\frac{dV}{dt} = 8$, so when $t = 3$: $V = 8 \times 3 = 24\ cm^3$

$\frac{\pi h^3}{12} = 24 \Rightarrow h^3 = \frac{288}{\pi} \Rightarrow h = \left(\frac{288}{\pi}\right)^{1/3}$ $(= 4.509...)$

$\frac{dV}{dh} = \frac{3\pi h^2}{12} = \frac{\pi h^2}{4}$

When $t = 3$: $\frac{dV}{dh} = \frac{\pi}{4} \times \left(\frac{288}{\pi}\right)^{2/3}$

$= \frac{\pi}{4} \times \frac{288^{2/3}}{\pi^{2/3}} = \frac{\pi^{1/3} \times 82944^{1/3}}{4}$

$= \sqrt[3]{\left(\frac{82944\pi}{64}\right)} = \sqrt[3]{(1296\pi)}$

$= \sqrt[3]{(216 \times 6\pi)} = 6\sqrt[3]{(6\pi)}$ as required

Do not write outside the box

8 (b) Hence, find the rate at which the depth is increasing when $t = 3$

Give your answer to three significant figures.

[3 marks]

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$\frac{dh}{dt} = \frac{1}{6\sqrt[3]{(6\pi)}} \times 8 = \frac{8}{6\sqrt[3]{(6\pi)}}$

$= 0.501000...$

Rate of increase of depth = 0.501 cm s^{-1} (3 s.f.)

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9 Assume that a and b are integers such that

$$a^2 - 4b - 2 = 0$$

9 (a) Prove that a is even.

[2 marks]

$a^2 - 4b - 2 = 0 \Rightarrow a^2 = 4b + 2 = 2(2b + 1)$

$2b + 1$ is an integer, so a^2 has a factor of 2, i.e. a^2 is even.

If a were odd then a^2 would be odd, so a must be even.

9 (b) Hence, prove that $2b + 1$ is even and explain why this is a contradiction.

[3 marks]

Since a is even, let $a = 2p$ where p is an integer.

$(2p)^2 = 4b + 2$

$4p^2 = 2(2b + 1)$

$2p^2 = 2b + 1$

p^2 is an integer, so $2p^2$ is even, hence $2b + 1$ is even.

But b is an integer, so $2b$ is even and $2b + 1$ is odd.

$2b + 1$ cannot be both even and odd: this is a contradiction.

Do not write outside the box

9 (c) Explain what can be deduced about the solutions of the equation

$$a^2 - 4b - 2 = 0$$

[1 mark]

The assumption is false, so there are no solutions to $a^2 - 4b - 2 = 0$ where a and b are both integers.

Turn over for the next question

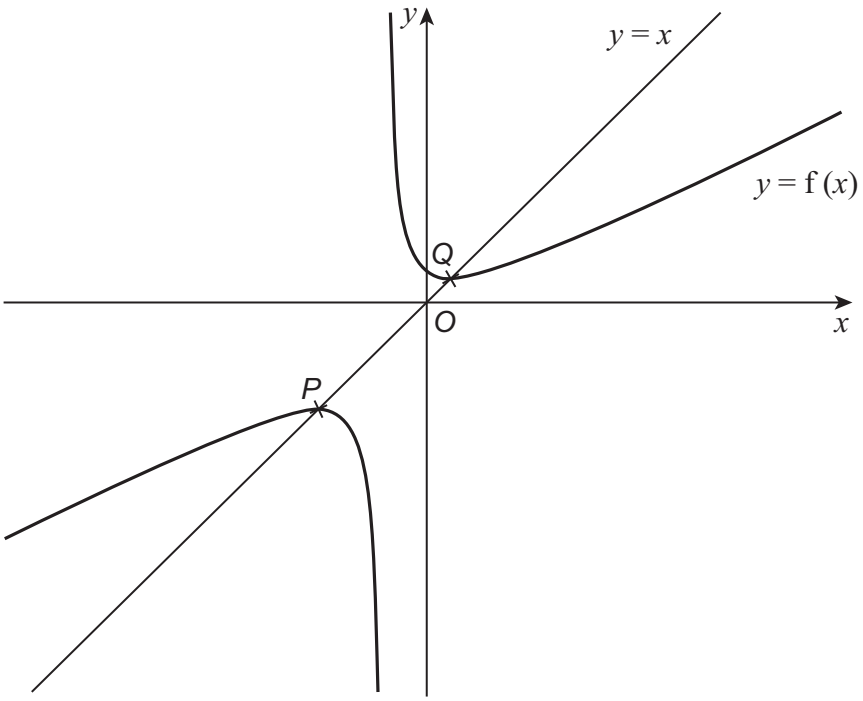
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10 The function f is defined by

$$\mathrm{f}(x) = \frac{x^2 + 10}{2x + 5}$$

where f has its maximum possible domain.

The curve $y = \mathrm{f}(x)$ intersects the line $y = x$ at the points P and Q as shown below.



10 (a) State the value of x which is not in the domain of f.

[1 mark]

$2x + 5 = 0 \Rightarrow x = -2.5$

$x = -2.5$ is not in the domain

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10 (b) Explain how you know that the function f is many-to-one.

[2 marks]

Many-to-one means two (or more) different values of x give the same value of f(x).

The line y = 2 meets the curve twice: f(0) = f(4) = 2, so f is many-to-one.

10 (c) (i) Show that the x-coordinates of P and Q satisfy the equation

$$x^2 + 5x - 10 = 0$$

[2 marks]

At P and Q the curve meets y = x, so

$x = \frac{x^2 + 10}{2x + 5}$

$x(2x + 5) = x^2 + 10$

$2x^2 + 5x = x^2 + 10$

$x^2 + 5x - 10 = 0$ as required

10 (c) (ii) Hence, find the exact x-coordinate of P and the exact x-coordinate of Q.

[1 mark]

$x = \frac{-5 \pm \sqrt{(5^2 + 40)}}{2} = \frac{-5 \pm \sqrt{65}}{2}$

P: $x = \frac{-5 - \sqrt{65}}{2}$ Q: $x = \frac{-5 + \sqrt{65}}{2}$

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10 (d) Show that P and Q are stationary points of the curve.

Fully justify your answer.

[5 marks]

$f'(x) = \frac{2x(2x + 5) - 2(x^2 + 10)}{(2x + 5)^2}$ (quotient rule)

$= \frac{4x^2 + 10x - 2x^2 - 20}{(2x + 5)^2} = \frac{2x^2 + 10x - 20}{(2x + 5)^2}$

$= \frac{2(x^2 + 5x - 10)}{(2x + 5)^2}$

Stationary points where $f'(x) = 0$:

$2(x^2 + 5x - 10) = 0 \Rightarrow x^2 + 5x - 10 = 0$

This is the same equation as in part (c)(i), with roots

$x = \frac{-5 - \sqrt{65}}{2}$ and $x = \frac{-5 + \sqrt{65}}{2}$, the x-coordinates of P and Q.

So $f'(x) = 0$ at the x-coordinates of P and Q (both in the domain as $x \neq -2.5$), hence P and Q are stationary points.

10 (e) Using set notation, state the range of f.

[2 marks]

P and Q lie on $y = x$, so their y-coordinates equal their x-coordinates.

$\{ y : y \leq \frac{-5 - \sqrt{65}}{2} \} \cup \{ y : y \geq \frac{-5 + \sqrt{65}}{2} \}$

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Section B

Answer **all** questions in the spaces provided.

11 $X \sim \text{N}(14, 0.35)$

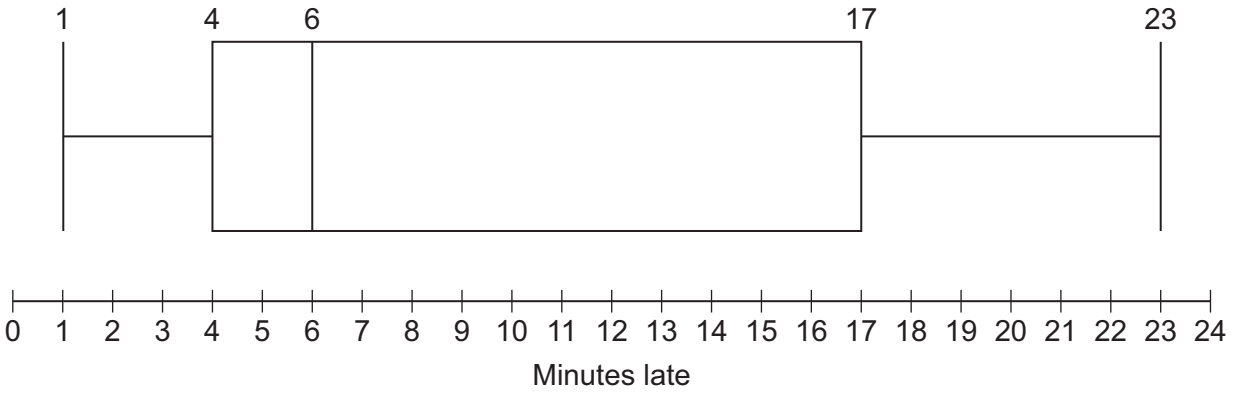
Find the standard deviation of X, correct to two decimal places.

Circle your answer.

[1 mark]

0.12 0.35 (0.59) 1.78

12 The box plot below shows summary data for the number of minutes late that buses arrived at a rural bus stop.



Identify which term best describes the distribution of this data.

Circle your answer.

[1 mark]

negatively skewed normal (positively skewed) symmetrical

Turn over for the next question

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13 A reporter is writing an article on the CO_2 emissions from vehicles using the Large Data Set.

The reporter claims that the Large Data Set shows that the CO_2 emissions from all vehicles in the UK have declined every year from 2002 to 2016.

Using your knowledge of the Large Data Set, give **two** reasons why this claim is invalid.

[2 marks]

1. The Large Data Set is only a sample of vehicles (not all regions or types of vehicle in the UK are included).

2. The Large Data Set only has data for 2002 and 2016, so nothing is known about the years in between.

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14 A customer service centre records every call they receive.

It is found that 30% of all calls made to this centre are complaints.

A sample of 20 calls is selected.

The number of calls in the sample which are complaints is denoted by the random variable X.

14 (a) State **two** assumptions necessary for X to be modelled by a binomial distribution.

[2 marks]

1. The probability that a call is a complaint is constant (0.3) for every call.

2. Calls are independent of each other (one call being a complaint does not affect another).

14 (b) Assume that X can be modelled by a binomial distribution.

14 (b) (i) Find $P(X = 1)$

[1 mark]

$X \sim B(20, 0.3)$: $P(X = 1) = 20 \times 0.3 \times 0.7^{19}$

$= 0.006839... = 0.00684$

14 (b) (ii) Find $P(X < 4)$

[2 marks]

$P(X < 4) = P(X \leq 3)$

$= 0.10708...$

$= 0.107$

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14 (b) (iii) Find $P(X \geq 10)$

[2 marks]

$P(X \geqslant 10) = 1 - P(X \leqslant 9)$

$= 1 - 0.95203...$

$= 0.04796... = 0.0480$

14 (c) In a random sample of 10 calls to a school, the number of calls which are complaints, Y, may be modelled by a binomial distribution

$$Y \sim B(10, p)$$

The standard deviation of Y is 1.5

Calculate the possible values of p.

[3 marks]

$Var(Y) = np(1-p)$ and $Var(Y) = 1.5^2 = 2.25$

$10p(1-p) = 2.25$

$10p^2 - 10p + 2.25 = 0$

$p = \frac{10 \pm \sqrt{(100 - 4 \times 10 \times 2.25)}}{20} = \frac{10 \pm \sqrt{10}}{20}$

$p = 0.342$ or $p = 0.658$ (3 s.f.)

Turn over for the next question

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15 Researchers are investigating the average time spent on social media by adults on the electoral register of a town.

They select every 100th adult from the electoral register for their investigation.

15 (a) Identify the population in their investigation.

[1 mark]

All adults on the electoral register of the town

15 (b) (i) State the name of this method of sampling.

[1 mark]

Systematic sampling

15 (b) (ii) Describe **one** advantage of this sampling method.

[1 mark]

It is quick and easy to carry out (just take every 100th name from the list).

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16 A sample of 240 households were asked which, if any, of the following animals they own as pets:

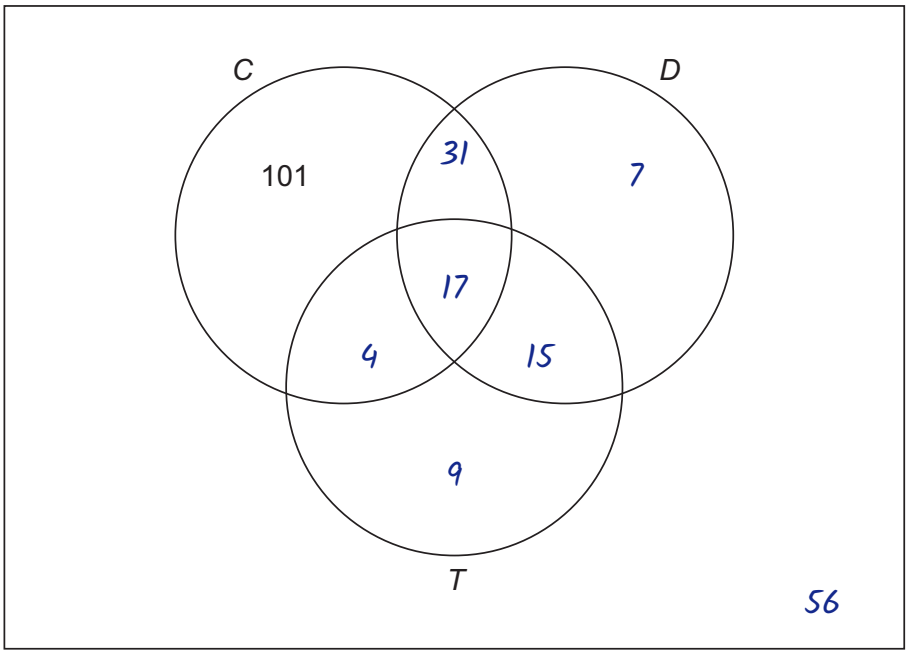
- cats (C)
- dogs (D)
- tortoises (T)

The results are shown in the table below.

Types of pet	C	D	T	C and D	C and T	D and T	C, D and T
Number of households	153	70	45	48	21	32	17

16 (a) Represent this information by fully completing the Venn diagram below.

[3 marks]



16 (b) A household is chosen at random from the sample.

16 (b) (i) Find the probability that the household owns a cat only.

[1 mark]

$P(\text{cat only}) = \frac{101}{240} = 0.421$

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16 (b) (ii) Find the probability that the household owns at least two of the three types of pet.

[2 marks]

$4 + 15 + 17 + 31 = 67$

$P(\text{at least two}) = \frac{67}{240} = 0.279$

16 (b) (iii) Find the probability that the household owns a cat or a dog or both, given that the household does not own a tortoise.

[2 marks]

$P(T') = \frac{240 - 45}{240} = \frac{195}{240}$

$P((C \cup D) \cap T') = \frac{101 + 31 + 7}{240} = \frac{139}{240}$

$P(C \cup D \mid T') = \frac{139}{195} = 0.7128... = 0.713$

16 (c) Determine whether a household owning a cat and a household owning a tortoise are independent of each other.

Fully justify your answer.

[2 marks]

$P(C) \times P(T) = \frac{153}{240} \times \frac{45}{240} = \frac{153}{1280} = 0.1195...$

$P(C \cap T) = \frac{21}{240} = 0.0875$

$0.0875 \neq 0.1195...$, so $P(C \cap T) \neq P(C) \times P(T)$

Hence owning a cat and owning a tortoise are not independent.

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17 The number of working hours per week of employees in a company is modelled by a normal distribution with mean of 34 hours and a standard deviation of 4.5 hours.

The manager claims that the mean working hours per week of the company's employees has increased.

A random sample of 30 employees in the company was found to have mean working hours per week of 36.2 hours.

Carry out a hypothesis test at the 2.5% significance level to investigate the manager's claim.

[6 marks]

Let μ = mean working hours per week of the employees

H_0: $\mu = 34$ H_1: $\mu > 34$

Under H_0: $\overline{X} \sim N\left(34, \frac{4.5^2}{30}\right)$

$z = \frac{36.2 - 34}{4.5 \div \sqrt{30}} = 2.6777... = 2.68$

$P(\overline{X} \geq 36.2) = 0.00370...$

$0.0037 < 0.025$ (equivalently $2.68 > 1.96$)

Reject H_0

There is sufficient evidence at the 2.5% level to suggest that the mean working hours per week of the employees has increased, so the manager's claim is supported.

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18 In a particular year, the height of a male athlete at the Summer Olympics has a mean 1.78 metres and standard deviation 0.23 metres.

The heights of 95% of male athletes are between 1.33 metres and 2.22 metres.

18 (a) Comment on whether a normal distribution may be suitable to model the height of a male athlete at the Summer Olympics in this particular year.

[3 marks]

$\mu \pm 2\sigma = 1.78 \pm 2 \times 0.23$: $1.78 - 0.46 = 1.32$, $1.78 + 0.46 = 2.24$

$1.32 \approx 1.33$ and $2.24 \approx 2.22$

For a normal distribution about 95% of values lie within two standard deviations of the mean, which matches 95% of heights being between 1.33 m and 2.22 m.

Height is also continuous data, so a normal distribution may be a suitable model.

18 (b) You may assume that the height of a male athlete at the Summer Olympics may be modelled by a normal distribution with mean 1.78 metres and standard deviation 0.23 metres.

18 (b) (i) Find the probability that the height of a randomly selected male athlete is 1.82 metres.

[1 mark]

$P(H = 1.82) = 0$ (height is continuous)

18 (b) (ii) Find the probability that the height of a randomly selected male athlete is between 1.70 metres and 1.90 metres.

[1 mark]

$H \sim N(1.78, 0.23^2)$

$P(1.70 < H < 1.90) = 0.33508... = 0.335$

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18 (b) (iii) Two male athletes are chosen at random.

Calculate the probability that **both** of their heights are between 1.70 metres and 1.90 metres.

[1 mark]

$P(\text{both}) = 0.33508...^2 = 0.11228... = 0.112$

18 (c) The summarised data for the heights, h metres, of a random sample of 40 male athletes at the Winter Olympics is given below.

$$\sum h = 69.2 \qquad \sum (h - \bar{h})^2 = 2.81$$

Use this data to calculate estimates of the mean and standard deviation of the heights of male athletes at the Winter Olympics.

[3 marks]

Mean: $\bar{h} = \frac{\Sigma h}{n} = \frac{69.2}{40} = 1.73 \text{ m}$

$s^2 = \frac{\Sigma(h - \bar{h})^2}{n - 1} = \frac{2.81}{39} = 0.07205...$

$s = \sqrt{0.07205...} = 0.26842...$

Standard deviation ≈ 0.268 m

18 (d) Using your answers from **part (c)**, compare the heights of male athletes at the Summer Olympics and male athletes at the Winter Olympics.

[2 marks]

On average, male athletes at the Summer Olympics are taller than male athletes at the Winter Olympics (1.78 m > 1.73 m).

The heights of the Summer Olympics athletes are less varied (less spread out) than those of the Winter athletes (0.23 < 0.268).

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19 A bank runs a campaign to promote Internet banking accounts to their customers.

Before the campaign, 42% of their customers had an Internet banking account.

One week after the campaign started, 35 customers were surveyed at random and 18 of them were found to have registered for an Internet banking account.

Using a binomial distribution, carry out a hypothesis test at the 10% significance level to investigate the claim that, since the campaign, there has been an increase in the proportion of customers registered for an Internet banking account.

[6 marks]

Let p = proportion of customers registered for Internet banking

H_0: $p = 0.42$ H_1: $p > 0.42$

Let X = number of customers in the sample with an account

Under H_0: $X \sim B(35, 0.42)$

$P(X \geq 18) = 1 - P(X \leq 17)$

$= 1 - 0.83136...$

$= 0.16863... = 0.169$

$0.169 > 0.10$

Do not reject H_0

There is insufficient evidence at the 10% level to suggest that there has been an increase in the proportion of customers registered for an Internet banking account since the campaign.

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