AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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A-level MATHEMATICS

Paper 3

Friday 14 June 2019 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 $\mathrm{f}(x) = \arcsin x$

State the maximum possible domain of f

Tick (✓) **one** box.

[1 mark]

$\{x \in \mathbb{R} : -1 \leq x \leq 1\}$ ✓

$\left\{x \in \mathbb{R} : -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\right\}$

$\{x \in \mathbb{R} : -\pi \leq x \leq \pi\}$

$\{x \in \mathbb{R} : -90 \leq x \leq 90\}$

2 Find the value of $\frac{100!}{98! \times 3!}$

Circle your answer.

[1 mark]

$\frac{50}{147}$ 1650 3300 161 700

3 Given $u_1 = 1$, determine which one of the formulae below defines an increasing sequence for $n \geq 1$

Circle your answer.

[1 mark]

$u_{n+1} = 1 + \frac{1}{u_n}$ $u_n = 2 - 0.9^{n-1}$ $u_{n+1} = -1 + 0.5u_n$ $u_n = 0.9^{n-1}$

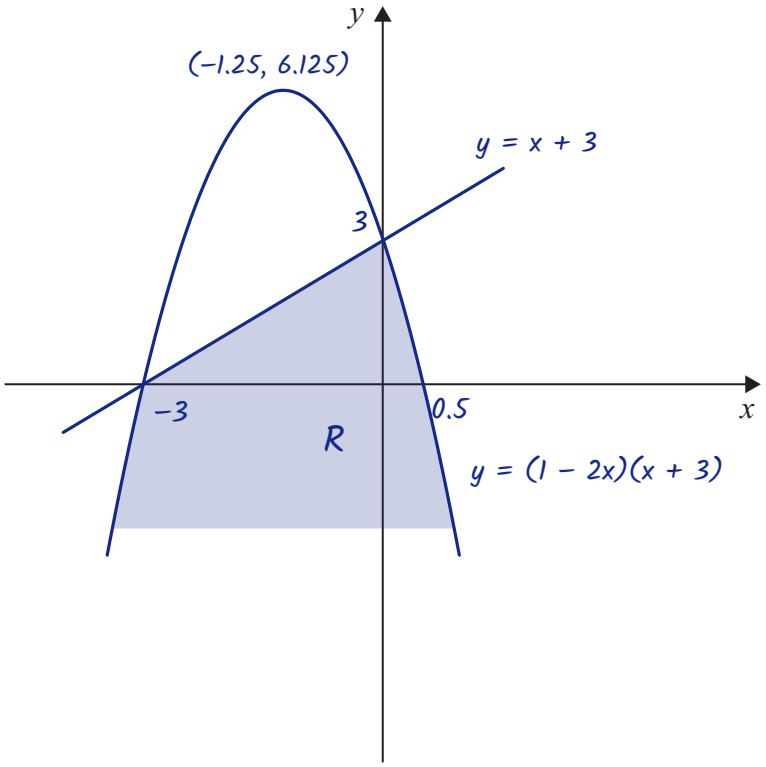
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4 Sketch the region defined by the inequalities

$$y \leq (1 - 2x)(x + 3) \text{ and } y - x \leq 3$$

Clearly indicate your region by shading it in and labelling it *R*.

[3 marks]



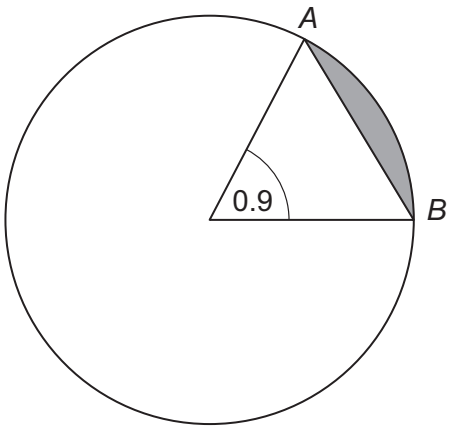
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5 A circle has equation $x^2 + y^2 - 6x - 8y = 264$

AB is a chord of the circle.

The angle at the centre of the circle, subtended by *AB*, is 0.9 radians, as shown in the diagram below.



Find the area of the minor segment shaded on the diagram.

Give your answer to three significant figures.

[5 marks]

$x^2 - 6x + y^2 - 8y = 264$

$(x - 3)^2 - 9 + (y - 4)^2 - 16 = 264$

$(x - 3)^2 + (y - 4)^2 = 289 \quad \Rightarrow \quad$ radius $r = 17$

Area of sector $= \frac{1}{2} r^2\theta = \frac{1}{2} \times 17^2 \times 0.9 = 130.05$

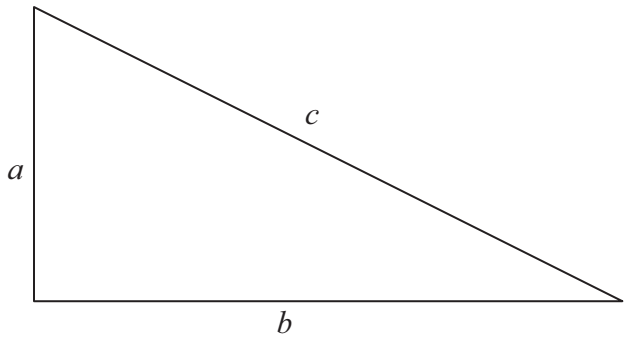
Area of triangle $= \frac{1}{2} r^2 \sin\theta = \frac{1}{2} \times 17^2 \times \sin 0.9 = 113.1907...$

Area of segment $= 130.05 - 113.1907... = 16.8592...$

Area of minor segment $= 16.9$ (3 s.f.)

Do not write outside the box

6 The three sides of a right-angled triangle have lengths a, b and c, where $a, b, c \in \mathbb{Z}$



6 (a) State an example where a, b and c are all even.

[1 mark]

$a = 6,\ b = 8,\ c = 10$ since $6^2 + 8^2 = 36 + 64 = 100 = 10^2$

6 (b) Prove that it is **not** possible for all of a, b and c to be odd.

[3 marks]

Assume, for contradiction, that a, b and c are all odd.

Let $a = 2m + 1$ and $b = 2n + 1$, where $m, n \in \mathbb{Z}$.

By Pythagoras: $c^2 = a^2 + b^2 = (2m + 1)^2 + (2n + 1)^2$

$= 4m^2 + 4m + 1 + 4n^2 + 4n + 1$

$= 2(2m^2 + 2m + 2n^2 + 2n + 1)$, which is even.

c^2 even $\Rightarrow$ c is even (the square of an odd number is odd).

This contradicts c being odd, so it is not possible for all of a, b and c to be odd.

Turn over for the next question

Do not write outside the box

7 (a) Express $\dfrac{4x+3}{(x-1)^2}$ in the form $\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}$

[3 marks]

$\frac{4x+3}{(x-1)^2} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2}$

$4x + 3 \equiv A(x-1) + B$

$x = 1$: $7 = B$

$x = 0$: $3 = -A + B \Rightarrow A = 7 - 3 = 4$

$A = 4, \quad B = 7$

$\frac{4x+3}{(x-1)^2} = \frac{4}{x-1} + \frac{7}{(x-1)^2}$

Do not write outside the box

7 (b) Show that

$$\int_3^4 \frac{4x+3}{(x-1)^2}\,\mathrm{d}x = p + \ln q$$

where p and q are rational numbers.

[5 marks]

$\int_3^4 \frac{4x+3}{(x-1)^2}\,dx = \int_3^4 \left(\frac{4}{x-1} + \frac{7}{(x-1)^2}\right) dx$

$= \left[4\ln(x-1) - \frac{7}{x-1}\right]_3^4$

$= \left(4\ln 3 - \frac{7}{3}\right) - \left(4\ln 2 - \frac{7}{2}\right)$

$= 4\ln 3 - 4\ln 2 + \frac{7}{2} - \frac{7}{3}$

$= 4\ln\frac{3}{2} + \frac{7}{6}$

$= \ln\frac{3^4}{2^4} + \frac{7}{6} = \frac{7}{6} + \ln\frac{81}{16}$

$\therefore\ p = \frac{7}{6},\quad q = \frac{81}{16}$

Turn over for the next question

Do not write outside the box

8 A student is conducting an experiment in a laboratory to investigate how quickly liquids cool to room temperature.

A beaker containing a hot liquid at an initial temperature of 75 °C cools so that the temperature, θ °C, of the liquid at time t minutes can be modelled by the equation

$$\theta = 5(4 + \lambda e^{-kt})$$

where λ and k are constants.

After 2 minutes the temperature falls to 68 °C.

8 (a) Find the temperature of the liquid after 15 minutes.

Give your answer to three significant figures.

[7 marks]

$t = 0, \theta = 75$: $75 = 5(4 + \lambda e^{0})$

$15 = 4 + \lambda \Rightarrow \lambda = 11$

$t = 2, \theta = 68$: $68 = 5(4 + 11e^{-2k})$

$13.6 = 4 + 11e^{-2k} \Rightarrow 11e^{-2k} = 9.6$

$e^{-2k} = \frac{9.6}{11} = \frac{48}{55}$

$k = -\frac{1}{2} \ln \frac{48}{55} = 0.068066...$

$t = 15$: $\theta = 5(4 + 11e^{-0.068066... \times 15})$

$= 5(4 + 11 \times 0.360237...)$

$= 39.8131...$

Temperature after 15 minutes = 39.8 °C (3 s.f.)

Do not write outside the box

8 (b) (i) Find the room temperature of the laboratory, giving a reason for your answer.

[2 marks]

As $t \to \infty$, $e^{-kt} \to 0$, so $\theta \to 5(4 + 0) = 20$

The liquid cools towards room temperature, so the temperature predicted by the model approaches room temperature as t increases.

Room temperature = 20 °C

8 (b) (ii) Find the time taken in minutes for the liquid to cool to 1 °C above the room temperature of the laboratory.

[2 marks]

1 °C above room temperature: $\theta = 21$

$5(4 + 11e^{-kt}) = 21 \Rightarrow 11e^{-kt} = 0.2 \Rightarrow e^{-kt} = 1/55$

$t = (\ln 55) \div 0.068066... = 58.874...$

Time taken = 58.9 minutes (about 59 minutes)

8 (c) Explain why the model might need to be changed if the experiment was conducted in a different place.

[1 mark]

The room temperature may be different in a different place (higher or lower), so the limiting value 20 °C (and the rate of cooling k) would change.

Turn over for the next question

Do not write outside the box

9 A curve has equation

$$x^2y^2 + xy^4 = 12$$

9 (a) Prove that the curve does not intersect the coordinate axes.

[2 marks]

When $x = 0$: $0^2y^2 + 0 \times y^4 = 0 \neq 12$

When $y = 0$: $x^2 \times 0^2 + x \times 0^4 = 0 \neq 12$

This contradicts $x^2y^2 + xy^4 = 12$, so no point on the curve has $x = 0$ or $y = 0$. Hence the curve does not intersect either coordinate axis.

9 (b) (i) Show that $\frac{dy}{dx} = -\frac{2xy + y^3}{2x^2 + 4xy^2}$

[5 marks]

Differentiating implicitly with respect to x (product rule):

$$2xy^2 + 2x^2y\frac{dy}{dx} + y^4 + 4xy^3\frac{dy}{dx} = 0$$

$$(2x^2y + 4xy^3)\frac{dy}{dx} = -(2xy^2 + y^4)$$

$$\frac{dy}{dx} = -\frac{2xy^2 + y^4}{2x^2y + 4xy^3} = -\frac{y(2xy + y^3)}{y(2x^2 + 4xy^2)}$$

Since $y \neq 0$ (part (a)), cancel the factor y:

$$\frac{dy}{dx} = -\frac{2xy + y^3}{2x^2 + 4xy^2}$$

Do not write outside the box

9 (b) (ii) Prove that the curve has no stationary points.

[4 marks]

At a stationary point $\frac{dy}{dx} = 0 \Rightarrow 2xy + y^3 = 0$

$y(2x + y^2) = 0$, and $y \neq 0$ (part (a)), so $y^2 = -2x$

Substitute into the curve, writing $xy^4 = x(y^2)y^2$:

$x^2y^2 + x(-2x)y^2 = 12$

$-x^2y^2 = 12$

But $x^2y^2 > 0$ ($x \neq 0$, $y \neq 0$), so $-x^2y^2 < 0$ and cannot equal 12.

This is a contradiction, so the curve has no stationary points.

9 (b) (iii) In the case when $x > 0$, find the equation of the tangent to the curve when $y = 1$

[4 marks]

$y = 1$: $x^2 + x = 12 \Rightarrow x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$, $x > 0 \Rightarrow x = 3$

$\frac{dy}{dx} = -\frac{2(3)(1) + 1^3}{2(3)^2 + 4(3)(1)^2} = -\frac{7}{30}$

$y - 1 = -\frac{7}{30}(x - 3)$

$30y - 30 = -7x + 21$

Tangent: $7x + 30y - 51 = 0$

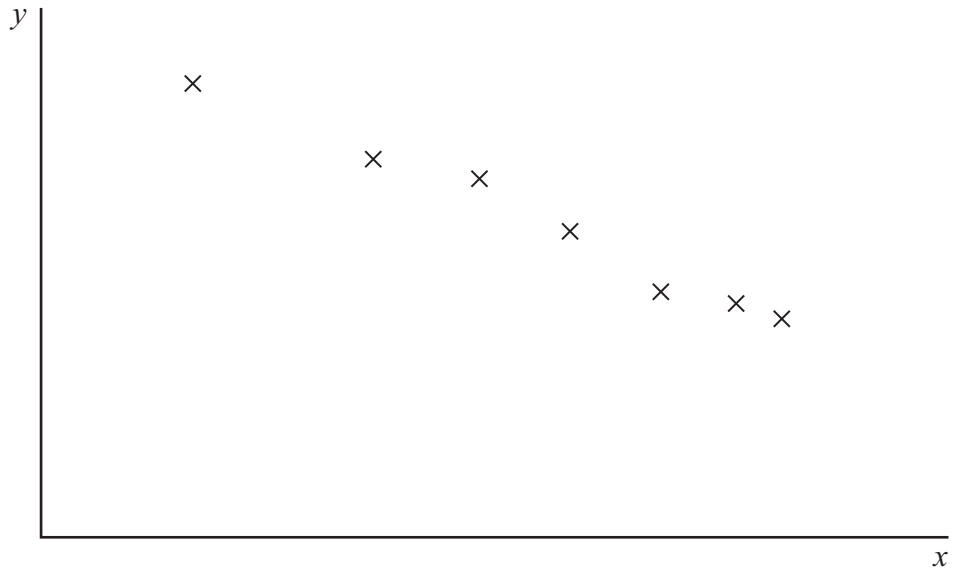
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Section B

Answer **all** questions in the spaces provided.

10 Which of the options below best describes the correlation shown in the diagram below?



Tick (✓) **one** box.

[1 mark]

- moderate positive ☐
- strong positive ☐
- moderate negative ☐
- strong negative ☑

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11 Lenny is one of a team of people interviewing shoppers in a town centre.

He is asked to survey 50 women between the ages of 18 and 29

Identify the name of this type of sampling.

Circle your answer.

[1 mark]

simple random stratified (quota) systematic

Turn over for the next question

Do not write outside the box

12 Amelia decides to analyse the heights of members of her school rowing club.

The heights of a random sample of 10 rowers are shown in the table below.

Rower	Jess	Nell	Liv	Neve	Ann	Tori	Maya	Kath	Darcy	Jen
Height (cm)	162	169	172	156	146	161	159	164	157	160

12 (a) Any value more than 2 standard deviations from the mean may be regarded as an outlier.

Verify that Ann's height is an outlier.

Fully justify your answer.

[4 marks]

$\Sigma x = 1606, \quad \Sigma x^2 = 258388, \quad n = 10$

$\bar{x} = 1606 \div 10 = 160.6$

$\sigma = \sqrt{(258388 \div 10 - 160.6^2)} = \sqrt{46.44} = 6.8146...$

$\bar{x} - 2\sigma = 160.6 - 2 \times 6.8146... = 146.97...$

Ann's height: $146 < 146.97...$

So Ann's height is more than 2 standard deviations below the mean, hence Ann's height is an outlier.

Do not write outside the box

12 (b) Amelia thinks she may have written down Ann's height incorrectly.

If Ann's height were discarded, state with a reason what, if any, difference this would make to the mean and standard deviation.

[2 marks]

The mean would increase (to 162.2 cm), because Ann's height is the lowest value and is well below the mean.

The standard deviation would decrease (to 5.03 cm), because the outlier is removed so the data are less spread out.

Turn over for the next question

Do not write outside the box

13 Patrick is practising his skateboarding skills. On each day, he has 30 attempts at performing a difficult trick.

Every time he attempts the trick, there is a probability of 0.2 that he will fall off his skateboard.

Assume that the number of times he falls off on any given day may be modelled by a binomial distribution.

13 (a) (i) Find the mean number of times he falls off in a day.

[1 mark]

$X \sim B(30, 0.2)$: $\text{mean} = np = 30 \times 0.2 = 6$

13 (a) (ii) Find the variance of the number of times he falls off in a day.

[1 mark]

$\text{Variance} = np(1 - p) = 30 \times 0.2 \times 0.8 = 4.8$

13 (b) (i) Find the probability that, on a particular day, he falls off exactly 10 times.

[2 marks]

$P(X = 10) = {}^{30}C_{10} \times 0.2^{10} \times 0.8^{20}$

$= 0.035470...$

$P(X = 10) = 0.0355$ (3 s.f.)

Do not write outside the box

13 (b) (ii) Find the probability that, on a particular day, he falls off 5 or more times.

[3 marks]

$P(X \geq 5) = 1 - P(X \leq 4)$

$P(X \leq 4) = 0.255233...$ (binomial cumulative, $n = 30$, $p = 0.2$)

$P(X \geq 5) = 1 - 0.255233... = 0.744766...$

$P(X \geq 5) = 0.745$ (3 s.f.)

13 (c) Patrick has 30 attempts to perform the trick on each of 5 consecutive days.

13 (c) (i) Calculate the probability that he will fall off his skateboard at least 5 times on each of the 5 days.

[2 marks]

The 5 days are independent, so

$P(\text{at least 5 falls on each day}) = 0.744766...^5$

$= 0.22914...$

$= 0.229$ (3 s.f.)

13 (c) (ii) Explain why it may be unrealistic to use the same value of 0.2 for the probability of falling off for all 5 days.

[1 mark]

Patrick is likely to improve with practice over the 5 days, so the probability of falling off may decrease from day to day.

Turn over for the next question

Do not write outside the box

14 A survey was conducted into the health of 120 teachers.

The survey recorded whether or not they had suffered from a range of four health issues in the past year.

In addition, their physical exercise level was categorised as low, medium or high.

50 teachers had a low exercise level, 40 teachers had a medium exercise level and 30 teachers had a high exercise level.

The results of the survey are shown in the table below.

	Low exercise	**Medium exercise**	**High exercise**
Back trouble	14	7	10
Stress	38	14	5
Depression	9	2	1
Headache/Migraine	4	5	5

14 (a) Find the probability that a randomly selected teacher:

14 (a) (i) suffers from back trouble and has a high exercise level;

[1 mark]

$P(\text{back trouble and high exercise}) = \frac{10}{120} = \frac{1}{12}$

14 (a) (ii) suffers from depression.

[2 marks]

Number with depression $= 9 + 2 + 1 = 12$

$P(\text{depression}) = \frac{12}{120} = \frac{1}{10}$

Do not write outside the box

14 (a) (iii) suffers from stress, given that they have a low exercise level.

[2 marks]

$P(\text{stress} \mid \text{low exercise}) = \frac{38}{50} = \frac{19}{25} \quad (= 0.76)$

14 (b) For teachers in the survey with a low exercise level, explain why the events 'suffers from back trouble' and 'suffers from stress' are **not** mutually exclusive.

[2 marks]

Of the 50 low exercise teachers, 14 have back trouble and 38 have stress: $14 + 38 = 52 > 50$

So at least 2 of these teachers suffer from both, i.e. the events can happen together, so they are not mutually exclusive.

Turn over for the next question

Do not write outside the box

15 Jamal, a farmer, claims that the larger the rainfall, the greater the yield of wheat from his farm.

He decides to investigate his claim, at the 5% level of significance.

He measures the rainfall in centimetres and the yield in kilograms for a random sample of ten years.

He correctly calculates the product moment correlation coefficient between rainfall and yield for his sample to be 0.567

The table below shows the critical values for correlation coefficients for a sample size of 10 for different significance levels, for both 1- and 2-tailed tests.

1-tailed test significance level	5%	2.5%	1%	0.5%
2-tailed test significance level	10%	5%	2%	1%
Critical value	0.549	0.632	0.716	0.765

Determine what Jamal's conclusion to his investigation should be, justifying your answer.

[3 marks]

H_0: $\rho = 0$ H_1: $\rho > 0$ (1-tailed test at 5% level)

Critical value for 1-tailed 5%, $n = 10$: 0.549

$r = 0.567 > 0.549$, so r is in the critical region: reject H_0

There is sufficient evidence at the 5% level of positive correlation between rainfall and yield, so the evidence supports Jamal's claim that the larger the rainfall, the greater the yield.

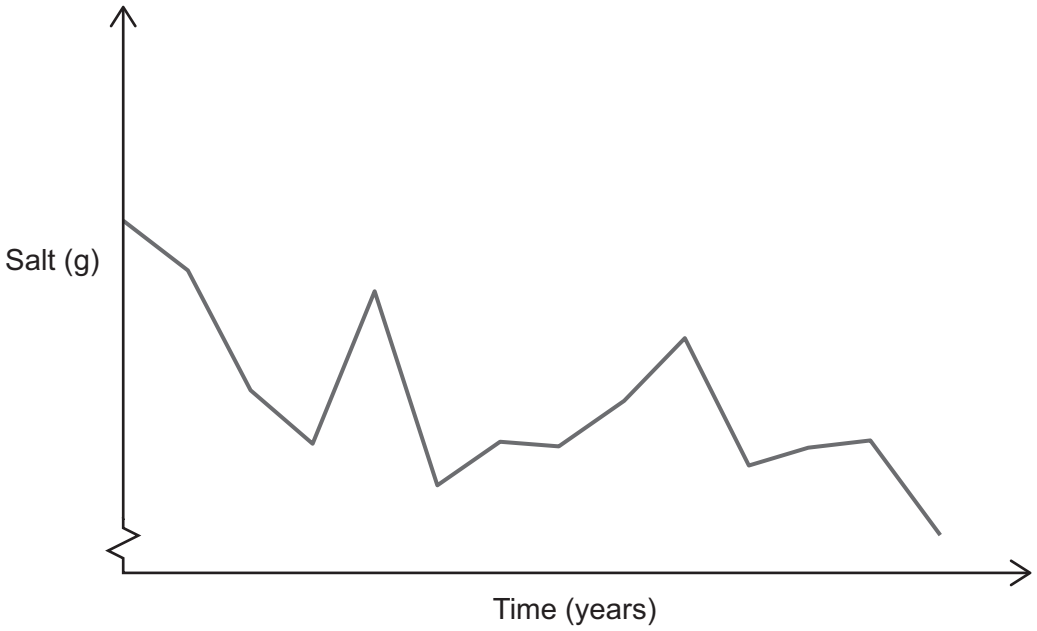
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16 (a) The graph below shows the amount of salt, in grams, purchased per person per week in England between 2001–02 and 2014, based upon the Large Data Set.



Meera and Gemma are arguing about what this graph shows.

Meera believes that the amount of salt consumed by people decreased greatly during this period.

Gemma says that this is not the case.

Using your knowledge of the Large Data Set, give **two** reasons why Gemma may be correct.

[2 marks]

1. The vertical (salt) axis does not start at zero, so the decrease looks much greater than it really is.

2. The Large Data Set records salt purchased (as a separate food item), not salt consumed, so it does not show the amount of salt consumed (e.g. salt in processed food is not included).

Do not write outside the box

16 (b) It is known that the mean amount of sugar purchased per person in England in 2014 was 78.9 grams, with a standard deviation of 25.0 grams.

In 2018, a sample of 918 people had a mean of 80.4 grams of sugar purchased per person.

Investigate, at the 5% level of significance, whether the mean amount of sugar purchased per person in England has changed between 2014 and 2018.

Assume that the survey data is a random sample taken from a normal distribution and that the standard deviation has remained the same.

[6 marks]

μ = population mean amount of sugar purchased per person (g)

H_0: $\mu = 78.9$ H_1: $\mu \neq 78.9$ (two-tailed, 5% level)

Under H_0: $\bar{x} \sim N\left(78.9, \frac{25.0^2}{918}\right)$

Test statistic $z = \frac{80.4 - 78.9}{25.0 \div \sqrt{918}} = 1.8179...$

Critical values: $z = \pm 1.96$ (2.5% in each tail)

$1.82 < 1.96$, so z is not in the critical region

(or p-value: $P(Z \geq 1.82) = 0.0345 > 0.025$)

Accept H_0

There is insufficient evidence at the 5% level to suggest that the mean amount of sugar purchased per person in England has changed between 2014 and 2018.

Question 16 continues on the next page

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16 (c) Another test is performed to determine whether the mean amount of fat purchased per person has changed between 2014 and 2018.

At the 10% significance level, the null hypothesis is rejected.

With reference to the 10% significance level, explain why it is not necessarily true that there has been a change.

[2 marks]

The 10% significance level is the probability of rejecting H_0 when H_0 is actually true. So there is a 10% chance that H_0 has been rejected in error (a Type I error), i.e. the mean amount of fat purchased may not actually have changed.

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17 Elizabeth's Bakery makes brownies.

It is known that the mass, X grams, of a brownie may be modelled by a normal distribution.

10% of the brownies have a mass less than 30 grams.

80% of the brownies have a mass greater than 32.5 grams.

17 (a) Find the mean and standard deviation of X.

[7 marks]

$X \sim N(\mu, \sigma^2)$

$P(X < 30) = 0.1 \Rightarrow P(Z < \frac{30 - \mu}{\sigma}) = 0.1$

$\frac{30 - \mu}{\sigma} = -1.2816 \Rightarrow 30 - \mu = -1.2816\sigma$ (1)

$P(X > 32.5) = 0.8 \Rightarrow P(X < 32.5) = 0.2$

$\frac{32.5 - \mu}{\sigma} = -0.8416 \Rightarrow 32.5 - \mu = -0.8416\sigma$ (2)

(2) − (1): $2.5 = 0.4400\sigma$

$\sigma = 2.5 \div 0.44003... = 5.6827...$

$\mu = 30 + 1.2816 \times 5.6827... = 37.2826...$

Mean $\mu = 37.3$ g (3 s.f.)

Standard deviation $\sigma = 5.68$ g (3 s.f.)

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17 (b) (i) Find $P(X \neq 35)$

[1 mark]

$P(X \neq 35) = 1$ (X is continuous, so $P(X = 35) = 0$)

17 (b) (ii) Find $P(X < 35)$

[2 marks]

$P(X < 35) = P\left(Z < \frac{35 - 37.2826...}{5.6827...}\right)$

$= P(Z < -0.40169...) = 0.34395...$

$P(X < 35) = 0.344$ (3 s.f.)

17 (c) Brownies are baked in batches of 13.

Calculate the probability that, in a batch of brownies, no more than 3 brownies are less than 35 grams.

You may assume that the masses of brownies are independent of each other.

[2 marks]

Y = number of brownies less than 35 g in a batch of 13

$Y \sim B(13, 0.344)$

$P(Y \leq 3) = 0.29375...$

$P(Y \leq 3) = 0.294$ (3 s.f.)

END OF QUESTIONS

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