AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Paper 1

Wednesday 5 June 2019 Morning Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Answer **all** questions in the spaces provided.

1 Given that $a > 0$, determine which of these expressions is **not** equivalent to the others.

Circle your answer.

[1 mark]

$-2\log_{10}\left(\frac{1}{a}\right)$ $\quad$ $2\log_{10}(a)$ $\quad$ $\log_{10}(a^2)$ $\quad$ $-4\log_{10}(\sqrt{a})$

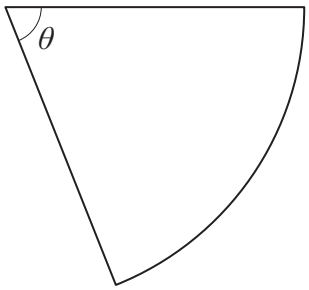
2 Given $y = e^{kx}$, where k is a constant, find $\frac{dy}{dx}$

Circle your answer.

[1 mark]

$\frac{dy}{dx} = e^{kx}$ $\quad$ $\frac{dy}{dx} = ke^{kx}$ $\quad$ $\frac{dy}{dx} = kxe^{kx-1}$ $\quad$ $\frac{dy}{dx} = \frac{e^{kx}}{k}$

3 The diagram below shows a sector of a circle.



The radius of the circle is 4 cm and $\theta = 0.8$ radians.

Find the area of the sector.

Circle your answer.

[1 mark]

1.28 cm^2 $\quad$ 3.2 cm^2 $\quad$ 6.4 cm^2 $\quad$ 12.8 cm^2

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4 The point A has coordinates $(-1, a)$ and the point B has coordinates $(3, b)$

The line AB has equation $5x + 4y = 17$

Find the equation of the perpendicular bisector of the points A and B.

[4 marks]

$5x + 4y = 17 \Rightarrow y = -\frac{5}{4}x + \frac{17}{4}$ so gradient of $AB = -\frac{5}{4}$

Gradient of perpendicular $= \frac{4}{5}$

$A(-1, a)$: $-5 + 4a = 17 \Rightarrow a = 5.5$

$B(3, b)$: $15 + 4b = 17 \Rightarrow b = 0.5$

Midpoint of $AB = \left(\frac{-1+3}{2}, \frac{5.5+0.5}{2}\right) = (1, 3)$

$y - 3 = \frac{4}{5}(x - 1)$

$5y - 15 = 4x - 4$

$5y - 4x = 11 \quad \left(y = \frac{4}{5}x + \frac{11}{5}\right)$

Turn over for the next question

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5 An arithmetic sequence has first term a and common difference d.

The sum of the first 16 terms of the sequence is 260

5 (a) Show that $4a + 30d = 65$

[2 marks]

$S_{16} = \frac{16}{2}(2a + 15d) = 260$

$8(2a + 15d) = 260$

$2a + 15d = 32.5$

$4a + 30d = 65$ as required

5 (b) Given that the sum of the first 60 terms is 315, find the sum of the first 41 terms.

[3 marks]

$S_{60} = \frac{60}{2}(2a + 59d) = 315$

$30(2a + 59d) = 315 \Rightarrow 2a + 59d = 10.5 \Rightarrow 4a + 118d = 21$

Subtract $4a + 30d = 65$: $88d = -44 \Rightarrow d = -0.5$

$4a = 65 + 15 = 80 \Rightarrow a = 20$

$S_{41} = \frac{41}{2}(2 \times 20 + 40 \times (-0.5))$

$= \frac{41}{2} \times 20$

$= 410$

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5 (c) S_n is the sum of the first n terms of the sequence.

Explain why the value you found in part **(b)** is the maximum value of S_n

[2 marks]

$u_n = a + (n - 1)d = 20 - 0.5(n - 1) = 20.5 - 0.5n$

For $n < 41$, $u_n > 0$, so every term up to the 40th adds to the sum.

$u_{41} = 20.5 - 20.5 = 0$, so $S_{41} = S_{40}$

For $n > 41$, $u_n < 0$, so every later term makes the sum smaller.

Hence the sum increases up to $n = 40$ or 41 and then decreases,

so $S_{41} = 410$ is the maximum value of S_n

Turn over for the next question

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6 The function f is defined by

$$\mathrm{f}(x) = \frac{1}{2}(x^2 + 1),\ x \geq 0$$

6 (a) Find the range of f.

[1 mark]

$f(x) \geq \frac{1}{2}$

6 (b) (i) Find $\mathrm{f}^{-1}(x)$

[3 marks]

$y = \frac{1}{2}(x^2 + 1)$

$2y = x^2 + 1$

$x^2 = 2y - 1$

$x = \sqrt{(2y - 1)} \quad (x \geq 0)$

$f^{-1}(x) = \sqrt{(2x - 1)}, \quad x \geq \frac{1}{2}$

6 (b) (ii) State the range of $\mathrm{f}^{-1}(x)$

[1 mark]

$f^{-1}(x) \geq 0$

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6 (c) State the transformation which maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}^{-1}(x)$

[1 mark]

Reflection in the line y = x

6 (d) Find the coordinates of the point of intersection of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$

[2 marks]

The graphs meet on the line y = x, so

$x = \frac{1}{2}(x^2 + 1)$

$x^2 - 2x + 1 = 0$

$(x - 1)^2 = 0 \Rightarrow x = 1, \ y = 1$

Point of intersection (1, 1)

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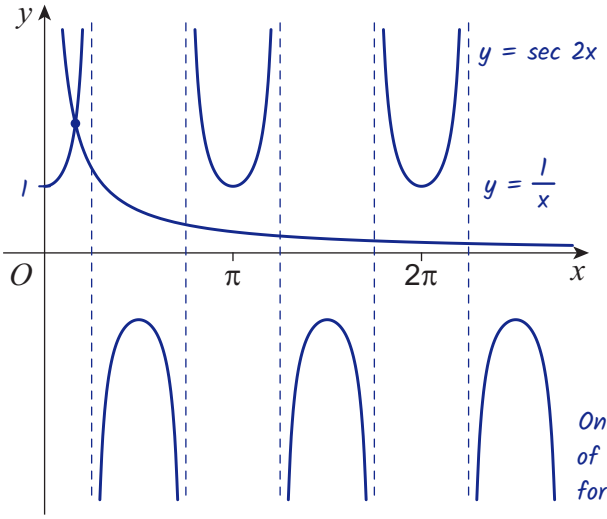
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7 (a) By sketching the graphs of $y = \frac{1}{x}$ and $y = \sec 2x$ on the axes below, show that the equation

$$\frac{1}{x} = \sec 2x$$

has exactly one solution for $x > 0$

[3 marks]



Only one point of intersection for $x > 0$, so exactly one solution

7 (b) By considering a suitable change of sign, show that the solution to the equation lies between 0.4 and 0.6

[2 marks]

$f(x) = \frac{1}{x} - \sec 2x$, continuous for $0.4 \leq x \leq 0.6$ $\left(2x < \frac{\pi}{2}\right)$

$f(0.4) = 2.5 - \frac{1}{\cos 0.8} = 1.0646... > 0$

$f(0.6) = 1.6666... - \frac{1}{\cos 1.2} = -1.0930... < 0$

Change of sign, so the solution lies between 0.4 and 0.6

7 (c) Show that the equation can be rearranged to give

$$x = \frac{1}{2}\cos^{-1} x$$

[2 marks]

$\frac{1}{x} = \frac{1}{\cos 2x} \Rightarrow \cos 2x = x$

$2x = \cos^{-1} x$

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$x = \frac{1}{2}\cos^{-1} x$ as required

7 (d) (i) Use the iterative formula

$$x_{n+1} = \frac{1}{2}\cos^{-1} x_n$$

with $x_1 = 0.4$, to find x_2, x_3 and x_4, giving your answers to four decimal places.

[2 marks]

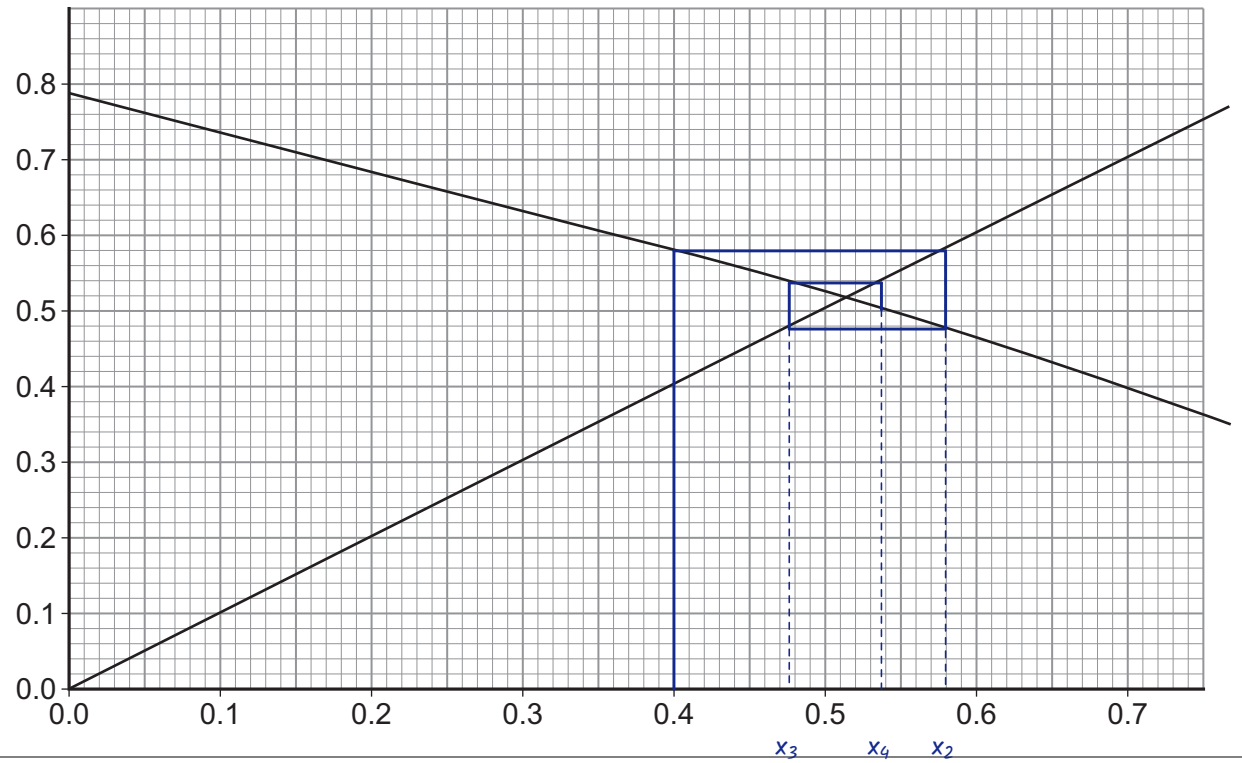
$x_2 = \frac{1}{2}\cos^{-1}(0.4) = 0.57963\ldots = 0.5796$

$x_3 = \frac{1}{2}\cos^{-1}(0.5796\ldots) = 0.47625\ldots = 0.4763$

$x_4 = \frac{1}{2}\cos^{-1}(0.4762\ldots) = 0.53720\ldots = 0.5372$

7 (d) (ii) On the graph below, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2, x_3 and x_4.

[2 marks]



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8 $\mathrm{P}(n) = \sum_{k=0}^{n} k^3 - \sum_{k=0}^{n-1} k^3$ where n is a positive integer.

8 (a) Find $\mathrm{P}(3)$ and $\mathrm{P}(10)$

[2 marks]

$P(n) = (0^3 + 1^3 + \ldots + n^3) - (0^3 + 1^3 + \ldots + (n-1)^3) = n^3$

$P(3) = 3^3 = 27$

$P(10) = 10^3 = 1000$

8 (b) Solve the equation $\mathrm{P}(n) = 1.25 \times 10^8$

[2 marks]

$n^3 = 1.25 \times 10^8$

$n = \sqrt[3]{(1.25 \times 10^8)} = 500$

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9 Prove that the sum of a rational number and an irrational number is always irrational.

[5 marks]

Assume, for contradiction, that m is rational, n is irrational and their sum $m + n$ is rational.

Then $m = \frac{a}{b}$ and $m + n = \frac{c}{d}$ where a, b, c, d are integers, $b, d \neq 0$

$$\frac{a}{b} + n = \frac{c}{d}$$

$$n = \frac{c}{d} - \frac{a}{b} = \frac{bc - ad}{bd}$$

$bc - ad$ and bd are integers ($bd \neq 0$), so n is rational.

This contradicts n being irrational.

So the assumption is false, and the sum of a rational number and an irrational number is always irrational.

Turn over for the next question

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10 The volume of a spherical bubble is increasing at a constant rate.

Show that the rate of increase of the radius, r, of the bubble is inversely proportional to r^2

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

[4 marks]

Volume increases at a constant rate: $\frac{dV}{dt} = k$ (k constant)

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$k = 4\pi r^2 \times \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{k}{4\pi r^2} = \frac{k}{4\pi} \times \frac{1}{r^2}$

$\frac{k}{4\pi}$ is a constant, so $\frac{dr}{dt} \propto \frac{1}{r^2}$

The rate of increase of r is inversely proportional to r^2

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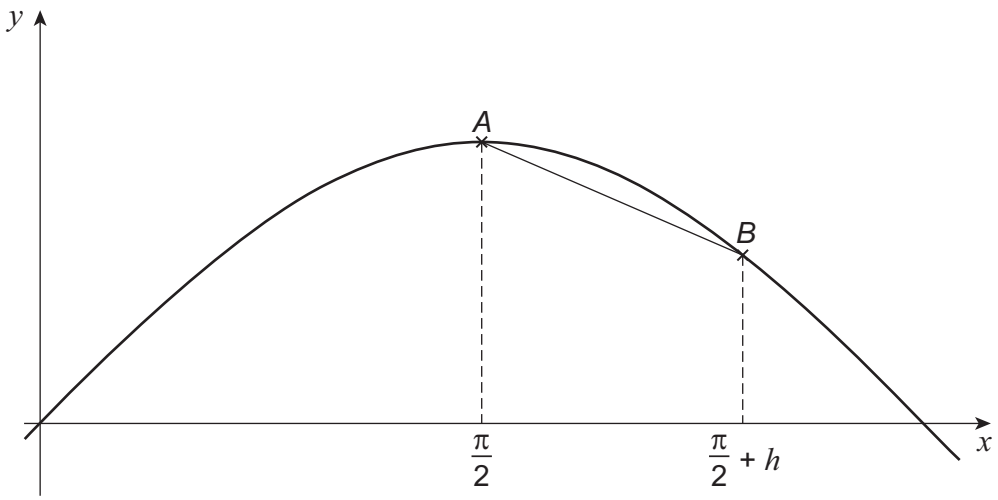
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11 Jodie is attempting to use differentiation from first principles to prove that the gradient of $y = \sin x$ is zero when $x = \frac{\pi}{2}$

Jodie's teacher tells her that she has made mistakes starting in Step 4 of her working. Her working is shown below.



Step 1 Gradient of chord $AB = \dfrac{\sin\left(\frac{\pi}{2} + h\right) - \sin\left(\frac{\pi}{2}\right)}{h}$

Step 2 $= \dfrac{\sin\left(\frac{\pi}{2}\right)\cos(h) + \cos\left(\frac{\pi}{2}\right)\sin(h) - \sin\left(\frac{\pi}{2}\right)}{h}$

Step 3 $= \sin\left(\frac{\pi}{2}\right)\left(\dfrac{\cos(h) - 1}{h}\right) + \cos\left(\frac{\pi}{2}\right)\dfrac{\sin(h)}{h}$

Step 4 For gradient of curve at *A*,

let $h = 0$ then

$\dfrac{\cos(h) - 1}{h} = 0$ and $\dfrac{\sin(h)}{h} = 0$

Step 5 Hence the gradient of the curve at *A* is given by

$\sin\left(\frac{\pi}{2}\right) \times 0 + \cos\left(\frac{\pi}{2}\right) \times 0 = 0$

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Complete Steps 4 and 5 of Jodie's working below, to correct her proof.

[4 marks]

Step 4 For gradient of curve at A,

let $h \to 0$ then

$\frac{\cos(h) - 1}{h} \to 0$ and $\frac{\sin(h)}{h} \to 1$

(for small h, $\sin h \approx h$ and $\cos h \approx 1 - \frac{h^2}{2}$)

Step 5 Hence the gradient of the curve at A is given by

$\sin\left(\frac{\pi}{2}\right) \times 0 + \cos\left(\frac{\pi}{2}\right) \times 1$

$= 1 \times 0 + 0 \times 1 = 0$

Turn over for the next question

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12 (a) Show that the equation

$$2\cot^2 x + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$$

can be written in the form

$$a\operatorname{cosec}^2 x + b\operatorname{cosec} x + c = 0$$

[2 marks]

$\cot^2 x = \operatorname{cosec}^2 x - 1$

$2(\operatorname{cosec}^2 x - 1) + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$

$4\operatorname{cosec}^2 x - 2 - 1 - 4\operatorname{cosec} x = 0$

$4\operatorname{cosec}^2 x - 4\operatorname{cosec} x - 3 = 0 \quad (a = 4, b = -4, c = -3)$

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12 (b) Hence, given x is obtuse and

$$2\cot^2 x + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$$

find the exact value of $\tan x$

Fully justify your answer.

[5 marks]

$(2\operatorname{cosec} x - 3)(2\operatorname{cosec} x + 1) = 0$

$\operatorname{cosec} x = \frac{3}{2}$ or $\operatorname{cosec} x = -\frac{1}{2}$

$|\operatorname{cosec} x| \geq 1$ for all x (since $|\sin x| \leq 1$), so reject $\operatorname{cosec} x = -\frac{1}{2}$

$\operatorname{cosec} x = \frac{3}{2}$

$\cot^2 x = \operatorname{cosec}^2 x - 1 = \frac{9}{4} - 1 = \frac{5}{4}$

$\cot x = \pm \frac{\sqrt{5}}{2}$ so $\tan x = \pm \frac{2}{\sqrt{5}}$

x is obtuse ($90° < x < 180°$), so $\tan x < 0$

$\tan x = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

Turn over for the next question

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13 A curve, C, has equation

$$y = \frac{e^{3x-5}}{x^2}$$

Show that C has exactly one stationary point.

Fully justify your answer.

[7 marks]

$y = \frac{e^{3x-5}}{x^2}$, $x \neq 0$ since y is undefined at $x = 0$

Quotient rule: $u = e^{3x-5}$, $u' = 3e^{3x-5}$; $v = x^2$, $v' = 2x$

$\frac{dy}{dx} = \frac{3x^2e^{3x-5} - 2xe^{3x-5}}{x^4}$

$= \frac{xe^{3x-5}(3x-2)}{x^4} = \frac{e^{3x-5}(3x-2)}{x^3}$

Stationary points occur where $\frac{dy}{dx} = 0$

$\frac{e^{3x-5}(3x-2)}{x^3} = 0 \Rightarrow e^{3x-5}(3x-2) = 0$

$e^{3x-5} > 0$ for all x, so $e^{3x-5} \neq 0$

so $3x - 2 = 0 \Rightarrow x = \frac{2}{3}$ (which is not 0, so y is defined there)

This is the only solution of $\frac{dy}{dx} = 0$, so C has exactly one

stationary point, at $x = \frac{2}{3}$

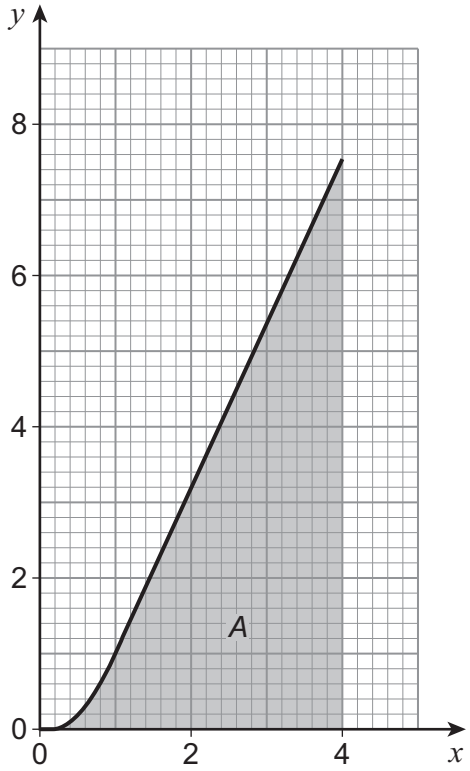
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14 The graph of $y = \dfrac{2x^3}{x^2 + 1}$ is shown for $0 \leq x \leq 4$



Caroline is attempting to approximate the shaded area, A, under the curve using the trapezium rule by splitting the area into n trapezia.

14 (a) When $n = 4$

14 (a) (i) State the number of ordinates that Caroline uses.

[1 mark]

5 ordinates

14 (a) (ii) Calculate the area that Caroline should obtain using this method.

Give your answer correct to two decimal places.

[3 marks]

$h = 1$: $x = 0, 1, 2, 3, 4$

$y = 0, 1, 3.2, 5.4, \frac{128}{17} = 7.5294...$

Area $\approx \frac{1}{2} \times 1 \times [0 + 7.5294... + 2(1 + 3.2 + 5.4)]$

$= \frac{1}{2} \times 26.7294...$

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$= 13.3647...$

$= 13.36$

14 (b) Show that the exact area of A is

$$16 - \ln 17$$

Fully justify your answer.

[5 marks]

$A = \int_0^4 \frac{2x^3}{x^2 + 1}\, dx$

Let $u = x^2 + 1$, $\frac{du}{dx} = 2x$, $x^2 = u - 1$

$x = 0 \Rightarrow u = 1$, $x = 4 \Rightarrow u = 17$

$A = \int_1^{17} \frac{2x^3}{u} \times \frac{du}{2x} = \int_1^{17} \frac{x^2}{u}\, du = \int_1^{17} \frac{u - 1}{u}\, du$

$= \int_1^{17} \left(1 - \frac{1}{u}\right) du$

$= [u - \ln u]_1^{17}$

$= (17 - \ln 17) - (1 - \ln 1)$

$= 17 - \ln 17 - 1 + 0$

$= 16 - \ln 17$ as required

Question 14 continues on the next page

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14 (c) Explain what would happen to Caroline's answer to part **(a)(ii)** as $n \to \infty$

[1 mark]

As $n \to \infty$ the trapezium rule answer gets closer and closer to the exact area, so it tends to $16 - \ln 17$ (= 13.17 to 2 d.p.)

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15 (a) At time t hours **after a high tide**, the height, h metres, of the tide and the velocity, v knots, of the tidal flow can be modelled using the parametric equations

$$v = 4 - \left(\frac{2t}{3} - 2\right)^2$$

$$h = 3 - 2\sqrt[3]{t - 3}$$

High tides and low tides occur alternately when the velocity of the tidal flow is zero.

A high tide occurs at 2 am.

15 (a) (i) Use the model to find the height of this high tide.

[1 mark]

High tide at t = 0: $h = 3 - 2\sqrt[3]{(0 - 3)} = 3 + 2\sqrt[3]{3} = 5.884... = 5.88$ m

15 (a) (ii) Find the time of the first **low** tide after 2 am.

[3 marks]

Low tide when v = 0: $4 - (\frac{2t}{3} - 2)^2 = 0$

$(\frac{2t}{3} - 2)^2 = 4 \Rightarrow \frac{2t}{3} - 2 = \pm 2$

$\frac{2t}{3} = 0$ or $4 \Rightarrow t = 0$ (the high tide) or $t = 6$

First low tide is 6 hours after 2 am

Time = 8 am

15 (a) (iii) Find the height of this low tide.

[1 mark]

t = 6: $h = 3 - 2\sqrt[3]{(6 - 3)} = 3 - 2\sqrt[3]{3} = 0.1155... = 0.12$ m

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15 (b) Use the model to find the height of the tide when it is flowing with maximum velocity.

[3 marks]

$v = 4 - (\frac{2t}{3} - 2)^2$ is greatest when $(\frac{2t}{3} - 2)^2 = 0$

$\Rightarrow \quad t = 3 \quad (v_{max} = 4 \text{ knots})$

$h = 3 - 2\sqrt[3]{(3 - 3)} = 3 - 0$

$h = 3$ metres

15 (c) Comment on the validity of the model.

[2 marks]

The model is only valid for one tidal cycle ($0 \leq t \leq 6$).

$v = 0$ only at $t = 0$ and $t = 6$, so it predicts no more high or low tides,

and after 6 hours the model shows the height continuing to decrease

(eventually negative), which is not realistic as tides repeat.

Turn over for the next question

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16 (a) $y = \mathrm{e}^{-x}(\sin x + \cos x)$

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

Simplify your answer.

[3 marks]

Product rule:

$\frac{dy}{dx} = -e^{-x}(\sin x + \cos x) + e^{-x}(\cos x - \sin x)$

$= e^{-x}(-\sin x - \cos x + \cos x - \sin x)$

$= -2e^{-x} \sin x$

16 (b) Hence, show that

$$\int \mathrm{e}^{-x} \sin x \, \mathrm{d}x = a\mathrm{e}^{-x}(\sin x + \cos x) + c$$

where a is a rational number.

[2 marks]

From (a): $\int -2e^{-x} \sin x \, dx = e^{-x}(\sin x + \cos x) + k$

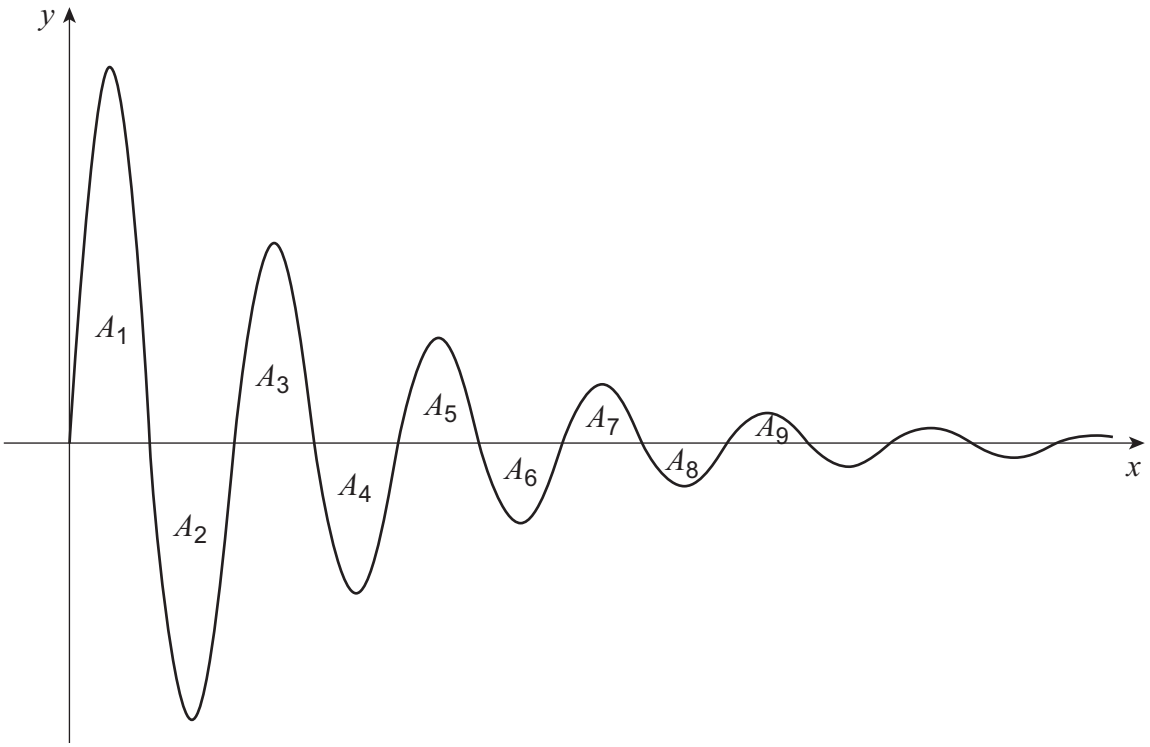
$\int e^{-x} \sin x \, dx = -\frac{1}{2} e^{-x}(\sin x + \cos x) + c$

so $a = -\frac{1}{2}$

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16 (c) A sketch of the graph of $y = e^{-x} \sin x$ for $x \geq 0$ is shown below.

The areas of the finite regions bounded by the curve and the x-axis are denoted by $A_1, A_2, ..., A_n, ...$



16 (c) (i) Find the exact value of the area A_1

[3 marks]

$A_1 = \int_0^{\pi} e^{-x} \sin x \, dx = \left[-\frac{1}{2} e^{-x}(\sin x + \cos x) \right]_0^{\pi}$

$= -\frac{1}{2} \left[e^{-\pi}(\sin \pi + \cos \pi) - e^{0}(\sin 0 + \cos 0) \right]$

$= -\frac{1}{2} \left[e^{-\pi}(0 - 1) - 1(0 + 1) \right]$

$= -\frac{1}{2}(-e^{-\pi} - 1)$

$= \frac{1 + e^{-\pi}}{2}$

Question 16 continues on the next page

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16 (c) (ii) Show that

$$\frac{A_2}{A_1} = e^{-\pi}$$

[4 marks]

$\int_{\pi}^{2\pi} e^{-x} \sin x \, dx = \left[-\frac{1}{2} e^{-x}(\sin x + \cos x) \right]_{\pi}^{2\pi}$

$= -\frac{1}{2}\left[e^{-2\pi}(\sin 2\pi + \cos 2\pi) - e^{-\pi}(\sin \pi + \cos \pi) \right]$

$= -\frac{1}{2}\left[e^{-2\pi}(0 + 1) - e^{-\pi}(0 - 1) \right]$

$= -\frac{1}{2}(e^{-2\pi} + e^{-\pi}) = -\frac{e^{-\pi}(e^{-\pi} + 1)}{2}$

A_2 lies below the x-axis, so $A_2 = \frac{e^{-\pi}(1 + e^{-\pi})}{2}$

$\frac{A_2}{A_1} = \frac{e^{-\pi}(1 + e^{-\pi})}{2} \div \frac{1 + e^{-\pi}}{2}$

$= e^{-\pi}$ as required

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16 (c) (iii) Given that

$$\frac{A_{n+1}}{A_n} = e^{-\pi}$$

show that the exact value of the total area enclosed between the curve and the x-axis is

$$\frac{1 + e^{\pi}}{2(e^{\pi} - 1)}$$

[4 marks]

$A_{n+1} = e^{-\pi}A_n$, so $A_1, A_2, A_3, \ldots$ form a geometric series

first term $a = A_1 = \frac{1 + e^{-\pi}}{2}$, common ratio $r = e^{-\pi}$

$|r| = e^{-\pi} < 1$, so the series converges

Total area $= S_\infty = \frac{a}{1 - r} = \frac{1 + e^{-\pi}}{2(1 - e^{-\pi})}$

Multiply numerator and denominator by e^{π}:

$= \frac{e^{\pi} + 1}{2(e^{\pi} - 1)}$

$= \frac{1 + e^{\pi}}{2(e^{\pi} - 1)}$ as required

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