AQA

Please write clearly in block capitals.

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A-level MATHEMATICS

Paper 3

Please note that question 13 uses the original Large Data Set "Family Food". This was replaced by the data set "Transport Stock Vehicle Database" in A-level exams from June 2020.
If you'd like to see the original data set, please contact maths@aqa.org.uk.

Friday 15 June 2018 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 A circle has equation $(x-4)^2+(y+4)^2=9$

What is the area of the circle?

Circle your answer.

[1 mark]

3π $\quad$ 9π $\quad$ 16π $\quad$ 81π

2 A curve has equation $y=x^5+4x^3+7x+q$ where q is a positive constant.

Find the gradient of the curve at the point where $x=0$

Circle your answer.

[1 mark]

0 $\quad$ 4 $\quad$ 7 $\quad$ q

3 The line L has equation $2x+3y=7$

Which one of the following is perpendicular to L?

Tick **one** box.

[1 mark]

$2x-3y=7$ ☐

$3x+2y=-7$ ☐

$2x+3y=-\frac{1}{7}$ ☐

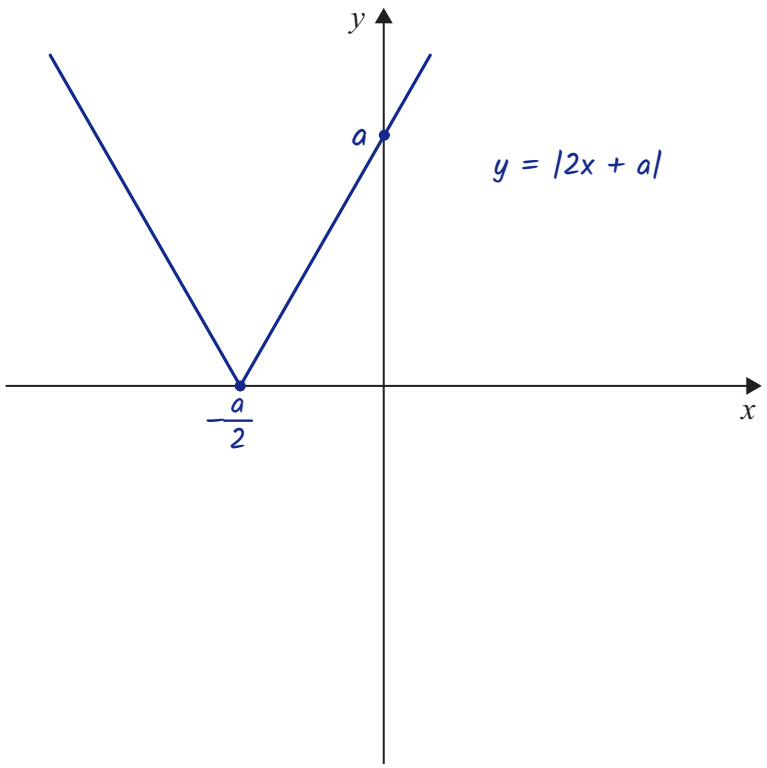
$3x-2y=7$ ☑

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4 Sketch the graph of $y = |2x + a|$, where a is a positive constant.

Show clearly where the graph intersects the axes.

[3 marks]



5 Show that, for small values of x, the graph of $y = 5 + 4\sin\frac{x}{2} + 12\tan\frac{x}{3}$ can be approximated by a straight line.

[3 marks]

For small x: sin θ ≈ θ and tan θ ≈ θ

$y \approx 5 + 4\left(\frac{x}{2}\right) + 12\left(\frac{x}{3}\right)$

$y \approx 5 + 2x + 4x$

$y \approx 6x + 5$

This is the equation of a straight line (gradient 6, y-intercept 5),

so for small values of x the graph can be approximated by a straight line.

Turn over ▶

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6 A function f is defined by $f(x) = \dfrac{x}{\sqrt{2x-2}}$

6 (a) State the maximum possible domain of f.

[2 marks]

Need $2x - 2 > 0$ (cannot square root a negative or divide by 0)

$\Rightarrow x > 1$

Domain: $\{x \in \mathbb{R} : x > 1\}$

6 (b) Use the quotient rule to show that $f'(x) = \dfrac{x-2}{(2x-2)^{\frac{3}{2}}}$

[3 marks]

$u = x \Rightarrow u' = 1$

$v = (2x-2)^{1/2} \Rightarrow v' = \frac{1}{2}(2x-2)^{-1/2} \times 2 = (2x-2)^{-1/2}$

$f'(x) = \dfrac{vu' - uv'}{v^2} = \dfrac{(2x-2)^{1/2} \times 1 - x(2x-2)^{-1/2}}{2x-2}$

multiply top and bottom by $(2x-2)^{1/2}$: $f'(x) = \dfrac{(2x-2) - x}{(2x-2)^{3/2}}$

$f'(x) = \dfrac{x-2}{(2x-2)^{3/2}}$ as required

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6 (c) Show that the graph of $y = f(x)$ has exactly one point of inflection.

[7 marks]

At a point of inflection $f''(x) = 0$ and $f''(x)$ changes sign.

$f'(x) = (x - 2)(2x - 2)^{-3/2}$

$f''(x) = 1 \times (2x - 2)^{-3/2} + (x - 2) \times (-3/2)(2x - 2)^{-5/2} \times 2$

$= (2x - 2)^{-3/2} - 3(x - 2)(2x - 2)^{-5/2}$

$= (2x - 2)^{-5/2} [\, (2x - 2) - 3(x - 2) \,]$

$= (2x - 2)^{-5/2} (4 - x)$ i.e. $f''(x) = \dfrac{4 - x}{(2x - 2)^{5/2}}$

$f''(x) = 0 \Rightarrow (2x - 2)^{-5/2} (4 - x) = 0$

$(2x - 2)^{-5/2} \neq 0$, and $x \neq 1$ since $x = 1$ is not in the domain,

so $4 - x = 0 \Rightarrow x = 4$ (the only solution)

Test either side of $x = 4$:

$f''(3) = \dfrac{4 - 3}{4^{5/2}} = \dfrac{1}{32} > 0$

$f''(5) = \dfrac{4 - 5}{8^{5/2}} = \dfrac{-\sqrt{2}}{256} < 0$

Since $(2x - 2)^{5/2} > 0$ for $x > 1$: $f''(x) > 0$ for $1 < x < 4$

and $f''(x) < 0$ for $x > 4$, so f'' changes sign only at $x = 4$.

Point of inflection at $\left(4, \dfrac{4}{\sqrt{6}}\right)$ and it is the only one,

$\therefore$ the graph of $y = f(x)$ has exactly one point of inflection.

6 (d) Write down the values of x for which the graph of $y = f(x)$ is convex.

[1 mark]

Convex where $f''(x) > 0$: $1 < x < 4$

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7 (a) Given that $\log_a y = 2\log_a 7 + \log_a 4 + \frac{1}{2}$, find y in terms of a.

[4 marks]

$\log_a y = 2\log_a 7 + \log_a 4 + \frac{1}{2}$

$\log_a y = \log_a 7^2 + \log_a 4 + \frac{1}{2}$

$\log_a y = \log_a (49 \times 4) + \frac{1}{2} = \log_a 196 + \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}\log_a a = \log_a a^{1/2} = \log_a \sqrt{a}$

$\log_a y = \log_a 196 + \log_a \sqrt{a}$

$\log_a y = \log_a (196\sqrt{a})$

$\therefore \quad y = 196\sqrt{a}$

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7 (b) When asked to solve the equation

$$2\log_a x = \log_a 9 - \log_a 4$$

a student gives the following solution:

$$2\log_a x = \log_a 9 - \log_a 4$$

$$\Rightarrow 2\log_a x = \log_a \frac{9}{4}$$

$$\Rightarrow \log_a x^2 = \log_a \frac{9}{4}$$

$$\Rightarrow x^2 = \frac{9}{4}$$

$$\therefore x = \frac{3}{2} \text{ or } -\frac{3}{2}$$

Explain what is wrong with the student's solution.

[1 mark]

$\log_a x$ is only defined for $x > 0$, so $x = -\frac{3}{2}$ must be rejected, as $\log_a(-\frac{3}{2})$ cannot be evaluated. The only solution is $x = \frac{3}{2}$.

Turn over for the next question

Do not write outside the box

8 (a) Prove the identity $\dfrac{\sin 2x}{1+\tan^2 x} \equiv 2\sin x \cos^3 x$

[3 marks]

Using $\sin 2x \equiv 2 \sin x \cos x$ and $1 + \tan^2 x \equiv \sec^2 x$:

$$\text{LHS} \equiv \frac{\sin 2x}{1 + \tan^2 x} \equiv \frac{2 \sin x \cos x}{\sec^2 x}$$

$$\equiv 2 \sin x \cos x \times \cos^2 x$$

$$\equiv 2 \sin x \cos^3 x \equiv \text{RHS}$$

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8 (b) Hence find $\int \frac{4\sin 4\theta}{1+\tan^2 2\theta}\,\mathrm{d}\theta$

[6 marks]

Using (a) with $x = 2\theta$: $\frac{\sin 4\theta}{1+\tan^2 2\theta} = 2\sin 2\theta \cos^3 2\theta$

$\int \frac{4\sin 4\theta}{1+\tan^2 2\theta}\,d\theta = \int 8\sin 2\theta \cos^3 2\theta\,d\theta$

Let $u = \cos 2\theta$: $\frac{du}{d\theta} = -2\sin 2\theta \Rightarrow \sin 2\theta\,d\theta = -\frac{1}{2}\,du$

$\int 8\sin 2\theta \cos^3 2\theta\,d\theta = \int 8u^3 \times \left(-\frac{1}{2}\right) du$

$= -4\int u^3\,du$

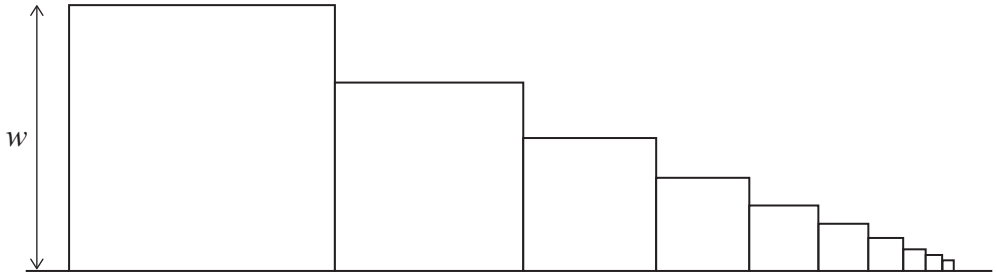
$= -4 \times \frac{u^4}{4} + c$

$= -u^4 + c$

$= -\cos^4 2\theta + c$

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9 Helen is creating a mosaic pattern by placing square tiles next to each other along a straight line.



The area of each tile is half the area of the previous tile, and the sides of the largest tile have length w centimetres.

9 (a) Find, in terms of w, the length of the sides of the second largest tile.

[1 mark]

Area of 2nd tile $= \frac{w^2}{2} \Rightarrow$ side $= \frac{w}{\sqrt{2}} = \frac{\sqrt{2}}{2} w$ cm

9 (b) Assume the tiles are in contact with adjacent tiles, but do not overlap.

Show that, no matter how many tiles are in the pattern, the total length of the series of tiles will be less than $3.5w$.

[4 marks]

Each side is $1/\sqrt{2}$ times the previous side, so the lengths form a geometric series with first term $a = w$ and common ratio $r = 1/\sqrt{2}$.

Since $|r| = 1/\sqrt{2} < 1$ the sum to infinity exists.

$S_\infty = \frac{a}{1-r} = \frac{w}{1-1/\sqrt{2}} = (2+\sqrt{2})w$

$= 3.4142...w \approx 3.41w$

For any number of tiles n, total length $S_n < S_\infty = 3.41w < 3.5w$

so no matter how many tiles, the total length is less than $3.5w$.

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9 (c) Helen decides the pattern will look better if she leaves a 3 millimetre gap between adjacent tiles.

Explain how you could refine the model used in part **(b)** to account for the 3 millimetre gap, and state how the total length of the series of tiles will be affected.

[2 marks]

Refine the model by adding 3 mm (0.3 cm) for each gap between adjacent tiles: total length of n tiles = S_n + 0.3(n − 1) cm.

The total length will increase and will no longer have an upper limit: it grows without bound as n increases, so it may now exceed 3.5w.

Turn over for the next question

Do not write outside the box

10 Prove by contradiction that $\sqrt[3]{2}$ is an irrational number.

[7 marks]

Assume, for contradiction, that $\sqrt[3]{2}$ is rational.

Then $\sqrt[3]{2} = \frac{a}{b}$ where a and b are integers, $b \neq 0$,

and a and b have no common factors (the fraction is in lowest terms).

Cubing: $2 = \frac{a^3}{b^3} \Rightarrow a^3 = 2b^3$

So a^3 is even, and hence a is even (the cube of an odd number is odd).

Let $a = 2k$ for some integer k:

$(2k)^3 = 2b^3 \Rightarrow 8k^3 = 2b^3 \Rightarrow b^3 = 4k^3 = 2(2k^3)$

So b^3 is even, and hence b is even.

Therefore a and b are both even, so they have a common factor of 2.

This contradicts the assumption that a and b have no common factors.

So the assumption that $\sqrt[3]{2}$ is rational is false.

$\therefore$ $\sqrt[3]{2}$ is an irrational number.

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Section B

Answer **all** questions in the spaces provided.

11 The table below shows the probability distribution for a discrete random variable X.

x	1	2	3	4	5
$\mathbf{P}(X = x)$	k	$2k$	$4k$	$2k$	k

Find the value of k.

Circle your answer.

[1 mark]

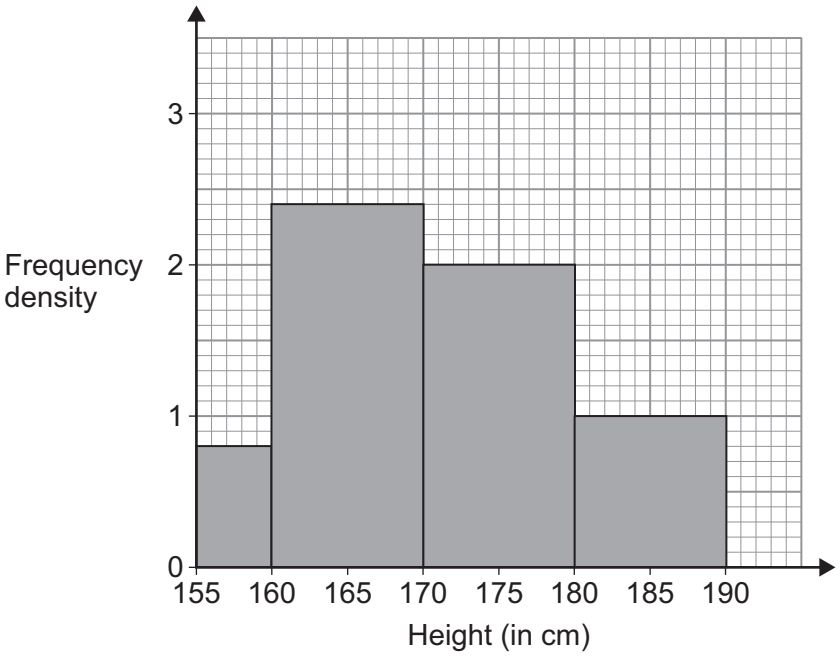
$\frac{1}{2}$ $\quad$ $\frac{1}{4}$ $\quad$ $\frac{1}{10}$ $\quad$ 1

$k + 2k + 4k + 2k + k = 10k = 1 \Rightarrow k = 1/10$

Turn over for the next question

Do not write outside the box

12 The histogram below shows the heights, in cm, of male A-level students at a particular school.



Which class interval contains the median height?

Circle your answer.

[1 mark]

[155, 160) [160, 170) [170, 180) [180, 190]

Frequencies: $5 \times 0.8 = 4$, $10 \times 2.4 = 24$, $10 \times 2 = 20$, $10 \times 1 = 10$

Total = 58, median is the 29.5th value: cumulative 4, 28, 48, ...

so the median lies in [170, 180)

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13 The table below shows an extract from the Large Data Set.

Year	2011	2012	2013	2014	% change since 2011
Other takeaway food brought home	0	0	0	0	−29

Sarah claims that the −29% change since 2011 is incorrect, as there is no change between 2011 and 2014.

Using your knowledge of the Large Data Set to justify your answer, explain whether Sarah's claim is correct.

[3 marks]

In the Large Data Set the values for 'Other takeaway food brought home' are rounded to the nearest whole number (grams per person per week), so the values shown as 0 are actually small non-zero amounts. The data set gives the actual values to many decimal places; the % change is calculated from these unrounded values, which do show a 29% decrease from 2011 to 2014, so the −29% figure is correct. Hence Sarah's claim is incorrect.

Turn over for the next question

Do not write outside the box

14 A teacher in a college asks her mathematics students what other subjects they are studying.

She finds that, of her 24 students:

12 study physics
8 study geography
4 study geography and physics

14 (a) A student is chosen at random from the class.

Determine whether the event 'the student studies physics' and the event 'the student studies geography' are independent.

[2 marks]

$P(P) = \frac{12}{24} = \frac{1}{2}$, $P(G) = \frac{8}{24} = \frac{1}{3}$

$P(P) \times P(G) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$P(P \cap G) = \frac{4}{24} = \frac{1}{6}$

$P(P \cap G) = P(P) \times P(G)$, so the events are independent.

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14 (b) It is known that for the whole college:

the probability of a student studying mathematics is $\frac{1}{5}$

the probability of a student studying biology is $\frac{1}{6}$

the probability of a student studying biology given that they study mathematics is $\frac{3}{8}$

Calculate the probability that a student studies mathematics or biology or both.

[4 marks]

$P(M) = \frac{1}{5}, \quad P(B) = \frac{1}{6}, \quad P(B \mid M) = \frac{3}{8}$

$P(M \cap B) = P(M) \times P(B \mid M) = \frac{1}{5} \times \frac{3}{8} = \frac{3}{40}$

$P(M \cup B) = P(M) + P(B) - P(M \cap B)$

$= \frac{1}{5} + \frac{1}{6} - \frac{3}{40}$

$= \frac{24}{120} + \frac{20}{120} - \frac{9}{120} = \frac{35}{120}$

$P(M \cup B) = \frac{7}{24}$

Turn over for the next question

Do not write outside the box

15 Abu visits his local hardware store to buy six light bulbs.

He knows that 15% of all bulbs at this store are faulty.

15 (a) State a distribution which can be used to model the number of faulty bulbs he buys.

[1 mark]

X = number of faulty bulbs out of 6: $X \sim B(6, 0.15)$

15 (b) Find the probability that all of the bulbs he buys are faulty.

[1 mark]

$P(X = 6) = 0.15^6 = 0.00001139... = 0.0000114$ (3 s.f.)

15 (c) Find the probability that at least two of the bulbs he buys are faulty.

[2 marks]

$P(X \geq 2) = 1 - P(X \leq 1)$

$P(X \leq 1) = 0.85^6 + 6 \times 0.15 \times 0.85^5 = 0.776484...$

$P(X \geq 2) = 1 - 0.776484... = 0.223515...$

$= 0.224$ (3 s.f.)

15 (d) Find the mean of the distribution stated in part **(a)**.

[1 mark]

$E(X) = np = 6 \times 0.15 = 0.9$

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15 (e) State two necessary assumptions in context so that the distribution stated in part **(a)** is valid.

[2 marks]

1. The probability that a bulb is faulty is fixed at 0.15 for each bulb he buys.

2. Whether a bulb is faulty is independent of whether any other bulb he buys is faulty.

Turn over for the next question

Do not write outside the box

16 A survey of 120 adults found that the volume, X litres per person, of carbonated drinks they consumed in a week had the following results:

$$\sum x = 165.6 \qquad \sum x^2 = 261.8$$

16 (a) (i) Calculate the mean of X.

[1 mark]

$\bar{x} = \frac{\Sigma x}{n} = \frac{165.6}{120} = 1.38$ litres

16 (a) (ii) Calculate the standard deviation of X.

[2 marks]

$\sigma = \sqrt{\Sigma x^2/n - \bar{x}^2} = \sqrt{261.8/120 - 1.38^2} = \sqrt{0.277266...}$

$= 0.52656...$

$= 0.527$ litres (3 s.f.)

16 (b) Assuming that X can be modelled by a normal distribution find

16 (b) (i) $P(0.5 < X < 1.5)$

[2 marks]

$X \sim N(1.38, 0.52656^2)$

$P(0.5 < X < 1.5) = P(-1.671 < Z < 0.2279)$

$= 0.54279...$

$= 0.543$ (3 s.f.)

Do not write outside the box

16 (b) (ii) P($X = 1$)

[1 mark]

X is continuous, so P(X = 1) = 0

16 (c) Determine with a reason, whether a normal distribution is suitable to model this data.

[2 marks]

$\bar{x} - 3\sigma = 1.38 - 3 \times 0.52656 = -0.1997$ which is less than 0.

A normal model would give a noticeable probability of negative values,

but the volume of drink consumed cannot be negative,

so a normal distribution might not be suitable to model this data.

16 (d) It is known that the volume, Y litres per person, of energy drinks consumed in a week may be modelled by a normal distribution with standard deviation 0.21

Given that P($Y > 0.75$) = 0.10, find the value of μ, correct to three significant figures.

[4 marks]

$Y \sim N(\mu, 0.21^2)$, $P(Y > 0.75) = 0.10$

$P(Z > \frac{0.75 - \mu}{0.21}) = 0.10$

$P(Z < z) = 0.90 \Rightarrow z = 1.2816$ (inverse normal)

$\frac{0.75 - \mu}{0.21} = 1.2816$

$\mu = 0.75 - 1.2816 \times 0.21 = 0.48087...$

$\mu = 0.481$ (3 s.f.)

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17 Suzanne is a member of a sports club.

For each sport she competes in, she wins half of the matches.

17 (a) After buying a new tennis racket Suzanne plays 10 matches and wins 7 of them.

Investigate, at the 10% level of significance, whether Suzanne's new racket has made a difference to the probability of her winning a match.

[7 marks]

Let X = number of matches won out of 10, p = P(she wins a match)

H_0: $p = 0.5$

H_1: $p \neq 0.5$ (two-tailed test at the 10% level)

Under H_0, $X \sim B(10, 0.5)$

$P(X \geq 7) = 1 - P(X \leq 6)$

$= 1 - 0.828125 = 0.171875 = 0.172$ (3 s.f.)

Two-tailed test, so compare with 0.05 in each tail:

$0.172 > 0.05$, so the result is not significant. Accept H_0.

There is not sufficient evidence at the 10% level to suggest that Suzanne's new racket has made a difference to the probability of her winning a match.

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17 (b) After buying a new squash racket, Suzanne plays 20 matches. Find the minimum number of matches she must win for her to conclude, at the 10% level of significance, that the new racket has improved her performance.

[5 marks]

Let Y = number of matches won out of 20.

H_0: $p = 0.5$, H_1: $p > 0.5$ (one-tailed, 10% level)

Under H_0, $Y \sim B(20, 0.5)$

Need the smallest c such that $P(Y \geq c) < 0.10$

$P(Y \geq 13) = 1 - P(Y \leq 12) = 1 - 0.8684 = 0.1316 > 0.10$

$P(Y \geq 14) = 1 - P(Y \leq 13) = 1 - 0.9423 = 0.0577 < 0.10$

So the critical region is $Y \geq 14$.

Minimum number of matches she must win = 14

Turn over for the next question

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18 In a region of England, the government decides to use an advertising campaign to encourage people to eat more healthily.

Before the campaign, the mean consumption of chocolate per person per week was known to be 66.5 g, with a standard deviation of 21.2 g

18 (a) After the campaign, the first 750 available people from this region were surveyed to find out their average consumption of chocolate.

18 (a) (i) State the sampling method used to collect the survey.

[1 mark]

Opportunity (convenience) sampling

18 (a) (ii) Explain why this sample should not be used to conduct a hypothesis test.

[1 mark]

The sample is not random (only the first 750 available people), so it may be biased and not representative of the population.

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18 (b) A second sample of 750 people revealed that the mean consumption of chocolate per person per week was 65.4 g

Investigate, at the 10% level of significance, whether the advertising campaign has decreased the mean consumption of chocolate per person per week.

Assume that an appropriate sampling method was used and that the consumption of chocolate is normally distributed with an unchanged standard deviation.

[6 marks]

Let μ = mean consumption of chocolate per person per week (g)

H_0: $\mu = 66.5$

H_1: $\mu < 66.5$ (one-tailed test at the 10% level)

Under H_0, $\bar{X} \sim N\left(66.5, \frac{21.2^2}{750}\right)$

$z = \frac{65.4 - 66.5}{21.2 \div \sqrt{750}} = -1.42097\ldots = -1.42$

Critical value: $P(Z < z) = 0.10 \Rightarrow z = -1.2816$

$-1.42 < -1.2816$, so the test statistic is in the critical region.

(p-value = $P(Z < -1.42) = 0.0777 < 0.10$) Reject H_0.

There is sufficient evidence at the 10% level to suggest that the advertising campaign has decreased the mean consumption of chocolate per person per week.

END OF QUESTIONS

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