AQA

Please write clearly in block capitals.

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A-level MATHEMATICS

Paper 2

Wednesday 13 June 2018 Morning Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Which of these statements is correct?

Tick **one** box.

[1 mark]

$x = 2 \Rightarrow x^2 = 4$ ☑

$x^2 = 4 \Rightarrow x = 2$ ☐

$x^2 = 4 \Leftrightarrow x = 2$ ☐

$x^2 = 4 \Rightarrow x = -2$ ☐

2 Find the coefficient of x^2 in the expansion of $(1 + 2x)^7$

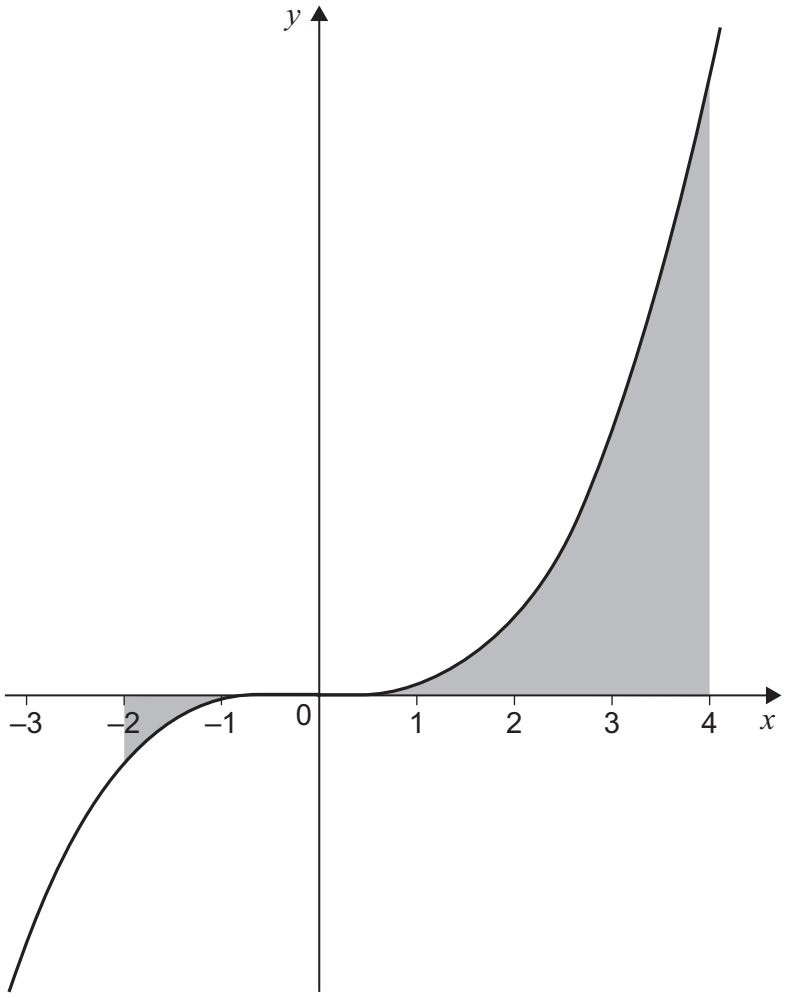
Circle your answer.

[1 mark]

42 4 21 (84)

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3 The graph of $y = x^3$ is shown.



Find the total shaded area.

Circle your answer.

[1 mark]

-68 60 68 128

Turn over ▶

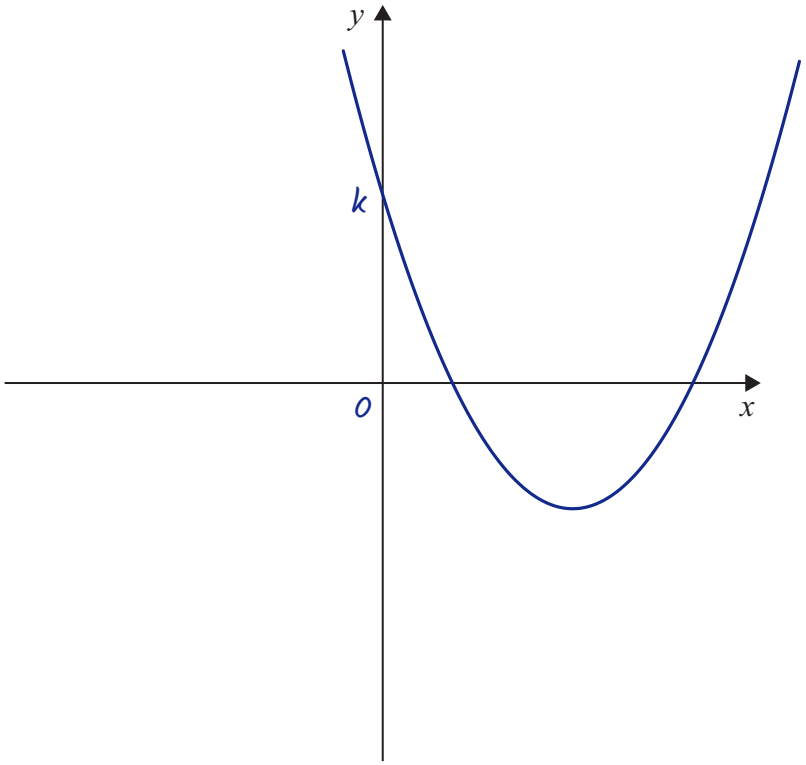
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4 A curve, C, has equation $y = x^2 - 6x + k$, where k is a constant.

The equation $x^2 - 6x + k = 0$ has two distinct positive roots.

4 (a) Sketch C on the axes below.

[2 marks]



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4 (b) Find the range of possible values for k.

Fully justify your answer.

[4 marks]

Two distinct real roots: $b^2 - 4ac > 0$

$(-6)^2 - 4 \times 1 \times k > 0$

$36 - 4k > 0 \quad \Rightarrow \quad k < 9$

Both roots positive: k is the y-intercept of C. The curve is u-shaped with both x-intercepts positive, so it must cross the y-axis above the origin, i.e. $k > 0$ (also product of roots $= k > 0$, sum $= 6 > 0$)

$\therefore \quad 0 < k < 9$

Turn over for the next question

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5 Prove that 23 is a prime number.

[2 marks]

$\sqrt{23} \approx 4.80$, so if 23 had a factor pair, one factor would be ≤ 4.80

so only need to check the primes 2 and 3.

23 is odd, so it is not divisible by 2.

$23 \div 3 = 7$ remainder 2, so it is not divisible by 3.

So 23 has no factors other than 1 and 23.

$\therefore$ 23 is a prime number.

Do not write outside the box

6 Find the coordinates of the stationary point of the curve with equation

$$(x + y - 2)^2 = e^y - 1$$

[7 marks]

Differentiate implicitly with respect to x:

$2(x + y - 2)(1 + dy/dx) = e^y \, dy/dx$

At a stationary point $dy/dx = 0$:

$2(x + y - 2)(1 + 0) = 0$

$\Rightarrow \quad x + y - 2 = 0$

Substitute into the equation of the curve:

$0^2 = e^y - 1$

$e^y = 1 \quad \Rightarrow \quad y = 0$

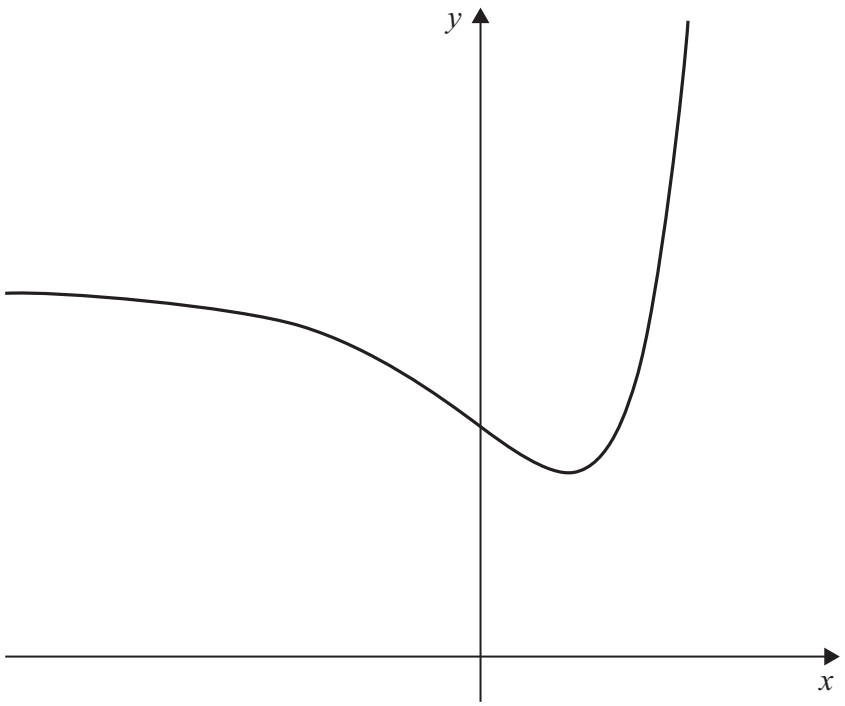
$x + 0 - 2 = 0 \quad \Rightarrow \quad x = 2$

Stationary point is (2, 0)

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7 A function f has domain $\mathbb{R}$ and range $\{y \in \mathbb{R} : y \geq \mathrm{e}\}$

The graph of $y = \mathrm{f}(x)$ is shown.



The gradient of the curve at the point (x, y) is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = (x - 1)\mathrm{e}^x$

Find an expression for $\mathrm{f}(x)$.

Fully justify your answer.

[8 marks]

$y = \int (x - 1)e^x \, dx$

By parts: $u = x - 1$, $dv/dx = e^x \Rightarrow du/dx = 1$, $v = e^x$

$y = (x - 1)e^x - \int e^x \, dx$

$y = (x - 1)e^x - e^x + c = (x - 2)e^x + c$

Stationary point: $dy/dx = 0 \Rightarrow (x - 1)e^x = 0$, $e^x \neq 0 \Rightarrow x = 1$

This is the minimum point shown on the graph. The range is $y \geq e$,

so the least value of y is e, i.e. the minimum value of $f(x)$ is e.

$\therefore$ the curve passes through $(1, e)$

$e = (1 - 2)e^1 + c \Rightarrow e = -e + c$

$c = 2e$

$\therefore f(x) = (x - 2)e^x + 2e$

Do not write outside the box

Turn over for the next question

Do not write outside the box

8 (a) Determine a sequence of transformations which maps the graph of $y = \sin x$ onto the graph of $y = \sqrt{3}\sin x - 3\cos x + 4$

Fully justify your answer.

[7 marks]

$\sqrt{3}\sin x - 3\cos x = R\sin(x - \alpha) = R\sin x\cos\alpha - R\cos x\sin\alpha$

$R\cos\alpha = \sqrt{3}, \quad R\sin\alpha = 3$

$R^2 = (\sqrt{3})^2 + 3^2 = 12 \Rightarrow R = \sqrt{12} = 2\sqrt{3}$

$\tan\alpha = 3/\sqrt{3} = \sqrt{3} \Rightarrow \alpha = \pi/3$

$\therefore y = \sqrt{3}\sin x - 3\cos x + 4 = 2\sqrt{3}\sin(x - \pi/3) + 4$

Transformations, in this order, starting from $y = \sin x$:

1. Translation by vector $\begin{bmatrix} \pi/3 \\ 0 \end{bmatrix}$ ($\pi/3$ in the positive x-direction)

$y = \sin x \rightarrow y = \sin(x - \pi/3)$

2. Stretch in the y-direction, scale factor $2\sqrt{3}$

$y = \sin(x - \pi/3) \rightarrow y = 2\sqrt{3}\sin(x - \pi/3)$

3. Translation by vector $\begin{bmatrix} 0 \\ 4 \end{bmatrix}$ (4 in the positive y-direction)

$y = 2\sqrt{3}\sin(x - \pi/3) \rightarrow y = 2\sqrt{3}\sin(x - \pi/3) + 4$

Do not write outside the box

8 (b) (i) Show that the least value of $\dfrac{1}{\sqrt{3}\sin x - 3\cos x + 4}$ is $\dfrac{2-\sqrt{3}}{2}$

[2 marks]

$1/(\sqrt{3}\sin x - 3\cos x + 4) = 1/(2\sqrt{3}\sin(x - \pi/3) + 4)$

Least value when the denominator is greatest, i.e. $\sin(x - \pi/3) = 1$

Least value $= 1/(2\sqrt{3} + 4)$

$= (4 - 2\sqrt{3})/((4 + 2\sqrt{3})(4 - 2\sqrt{3})) = (4 - 2\sqrt{3})/(16 - 12)$

$= (4 - 2\sqrt{3})/4$

$= (2 - \sqrt{3})/2$ as required

8 (b) (ii) Find the greatest value of $\dfrac{1}{\sqrt{3}\sin x - 3\cos x + 4}$

[1 mark]

Greatest when $\sin(x - \pi/3) = -1$: denominator $= 4 - 2\sqrt{3} > 0$

Greatest value $= 1/(4 - 2\sqrt{3}) = (4 + 2\sqrt{3})/(16 - 12)$

$= (2 + \sqrt{3})/2$

Turn over for the next question

Do not write outside the box

9 A market trader notices that daily sales are dependent on two variables:

number of hours, t, after the stall opens

total sales, x, in pounds since the stall opened.

The trader models the rate of sales as directly proportional to $\frac{8-t}{x}$

After two hours the rate of sales is £72 per hour and total sales are £336

9 (a) Show that

$$x\frac{\mathrm{d}x}{\mathrm{d}t} = 4032(8-t)$$

[3 marks]

$dx/dt \propto (8 - t)/x \quad \Rightarrow \quad dx/dt = k(8 - t)/x$

When $t = 2$: $dx/dt = 72$ and $x = 336$

$72 = k(8 - 2)/336$

$k = 72 \times 336 \div 6 = 4032$

$dx/dt = 4032(8 - t)/x$

$\therefore \; x\, dx/dt = 4032(8 - t)$ as required

Do not write outside the box

9 (b) Hence, show that

$$x^2 = 4032t(16 - t)$$

[3 marks]

Separate variables: $\int x\,dx = \int 4032(8 - t)\,dt$

$\frac{1}{2}x^2 = 4032(8t - \frac{1}{2}t^2) + c$

$t = 2, x = 336$: $\frac{1}{2} \times 336^2 = 4032(16 - 2) + c$

$56448 = 56448 + c \Rightarrow c = 0$

$x^2 = 2 \times 4032(8t - \frac{1}{2}t^2) = 4032(16t - t^2)$

$\therefore x^2 = 4032t(16 - t)$ as required

Question 9 continues on the next page

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9 (c) The stall opens at 09.30.

9 (c) (i) The trader closes the stall when the rate of sales falls below £24 per hour.

Using the results in parts **(a)** and **(b)**, calculate the earliest time that the trader closes the stall.

[6 marks]

Rate of sales: $dx/dt = 24$

From (a): $24x = 4032(8 - t) \Rightarrow x = 168(8 - t)$

Substitute into $x^2 = 4032t(16 - t)$:

$168^2(8 - t)^2 = 4032t(16 - t)$

$28224(64 - 16t + t^2) = 4032(16t - t^2)$

$7(64 - 16t + t^2) = 16t - t^2$

$448 - 112t + 7t^2 = 16t - t^2$

$8t^2 - 128t + 448 = 0 \Rightarrow t^2 - 16t + 56 = 0$

$t = (16 \pm \sqrt{(256 - 224)})/2 = 8 \pm \sqrt{8}$

$t = 5.1715...$ or $t = 10.828...$

For $t > 8$, $dx/dt < 0$ ($x = 168(8 - t) < 0$), so reject $t = 10.828...$

Rate first falls below £24 per hour at $t = 5.1715...$ hours

$0.1715... \times 60 = 10.29...$ minutes $\Rightarrow$ 5 hours 10 minutes

09:30 + 5 h 10 min

Earliest closing time = 14:40

Do not write outside the box

9 (c) (ii) Explain why the model used by the trader is not valid at 09.30.

[2 marks]

At 09:30, $t = 0$ and the stall has just opened, so there have been zero sales, i.e. $x = 0$.

Then $dx/dt = 4032(8 - t)/x$ has a denominator of zero, so dx/dt is undefined at $t = 0$ and the model cannot be valid at 09:30.

Turn over for Section B

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Section B

Answer **all** questions in the spaces provided.

10 A garden snail moves in a straight line from rest to $1.28\,\text{cm}\,\text{s}^{-1}$, with a constant acceleration in 1.8 seconds.

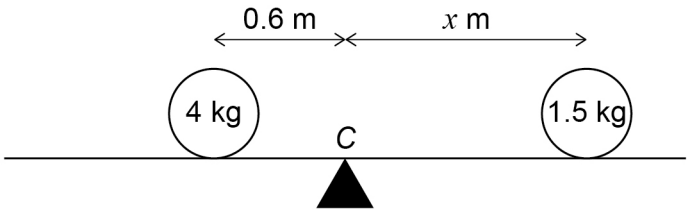
Find the acceleration of the snail.

Circle your answer.

[1 mark]

$2.30\,\text{m}\,\text{s}^{-2}$ $0.71\,\text{m}\,\text{s}^{-2}$ $0.0071\,\text{m}\,\text{s}^{-2}$ $0.023\,\text{m}\,\text{s}^{-2}$

11 A uniform rod, *AB*, has length 4 metres.
The rod is resting on a support at its midpoint *C*.
A particle of mass 4 kg is placed 0.6 metres to the left of *C*.
Another particle of mass 1.5 kg is placed x metres to the right of *C*, as shown.



The rod is balanced in equilibrium at *C*.

Find x.

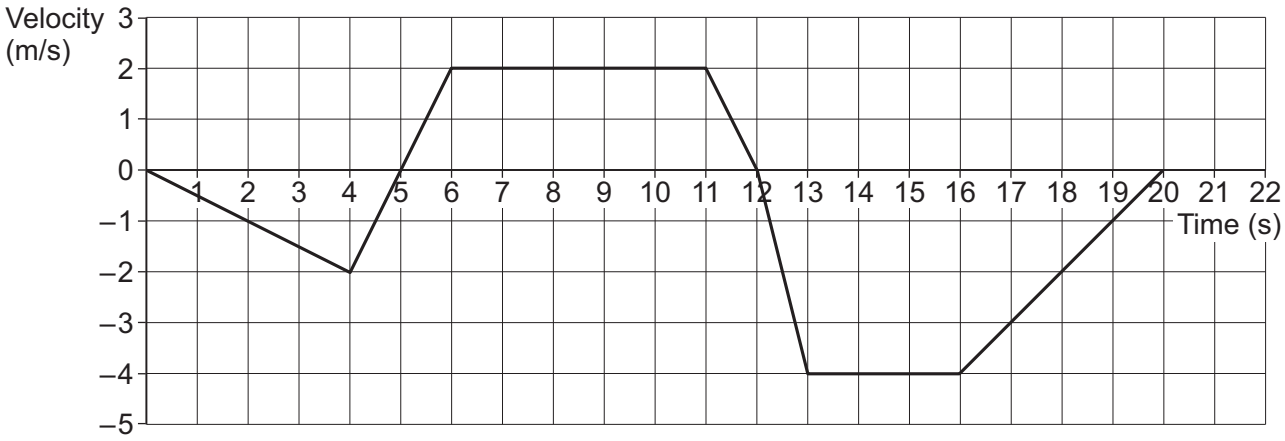
Circle your answer.

[1 mark]

1.8 m 1.5 m 1.75 m 1.6 m

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12 The graph below shows the velocity of an object moving in a straight line over a 20 second journey.



12 (a) Find the maximum magnitude of the acceleration of the object.

[1 mark]

Steepest section: $t = 12$ to 13, gradient $= (-4 - 0)/1 = -4$

Maximum magnitude of acceleration $= 4\ m\ s^{-2}$

12 (b) The object is at its starting position at times 0, t_1 and t_2 seconds.

Find t_1 and t_2

[4 marks]

Displacement = area under the v–t graph (below the axis negative)

$0 \leq t \leq 4$: $\frac{1}{2} \times 4 \times (-2) = -4$; $4 \leq t \leq 5$: $\frac{1}{2} \times 1 \times (-2) = -1$

$5 \leq t \leq 6$: $\frac{1}{2} \times 1 \times 2 = +1$ $\Rightarrow$ displacement at $t = 6$ is -4

Need $+4$ more at $v = 2$: $4 \div 2 = 2$ s $\Rightarrow$ $t_1 = 6 + 2 = 8$

$8 \leq t \leq 11$: $3 \times 2 = +6$ $\Rightarrow$ displacement at $t = 11$ is $+6$

$11 \leq t \leq 12$: $\frac{1}{2} \times 1 \times 2 = +1$ $\Rightarrow$ displacement at $t = 12$ is $+7$

$12 \leq t \leq 13$: $\frac{1}{2} \times 1 \times (-4) = -2$ $\Rightarrow$ displacement at $t = 13$ is $+5$

Need -5 more at $v = -4$: $5 \div 4 = 1.25$ s

$t_2 = 13 + 1.25 = 14.25$

$\therefore\ t_1 = 8$ s, $t_2 = 14.25$ s

Do not write outside the box

13 **In this question use $g = 9.8\,\text{m s}^{-2}$**

A boy attempts to move a wooden crate of mass 20 kg along horizontal ground. The coefficient of friction between the crate and the ground is 0.85

13 (a) The boy applies a horizontal force of 150 N. Show that the crate remains stationary.

[3 marks]

Resolving vertically: $R = mg = 20 \times 9.8 = 196$ N

Maximum friction $F_{max} = \mu R = 0.85 \times 196 = 166.6$ N

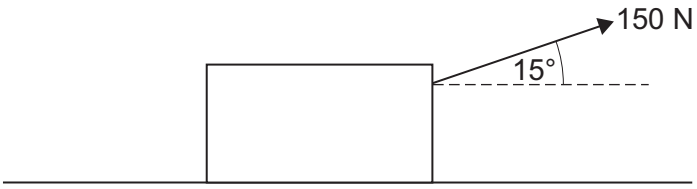
Applied force 150 N < 166.6 N = maximum friction

so friction can balance the 150 N pull (friction is not limiting)

$\therefore$ the crate remains stationary.

Do not write outside the box

13 (b) Instead, the boy uses a handle to pull the crate forward. He exerts a force of 150 N, at an angle of 15° above the horizontal, as shown in the diagram.



Determine whether the crate remains stationary.

Fully justify your answer.

[5 marks]

Resolving vertically: $R + 150 \sin 15° = 20 \times 9.8$

$R = 196 - 150 \sin 15° = 196 - 38.822... = 157.177...$ N

Maximum (limiting) friction $F_{max} = \mu R = 0.85 \times 157.177...$

$= 133.60...$ N

Horizontal component of the pull = $150 \cos 15° = 144.88...$ N

144.9 N > 133.6 N, so the horizontal pull is greater than the maximum friction force.

∴ the crate does not remain stationary: it begins to move.

Do not write outside the box

14 A quadrilateral has vertices A, B, C and D with position vectors given by

$$\overrightarrow{OA} = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}, \overrightarrow{OB} = \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}, \overrightarrow{OC} = \begin{bmatrix} 0 \\ 7 \\ 6 \end{bmatrix} \text{ and } \overrightarrow{OD} = \begin{bmatrix} 4 \\ 10 \\ 0 \end{bmatrix}$$

14 (a) Write down the vector $\overrightarrow{AB}$

[1 mark]

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{bmatrix} -4 \\ -3 \\ 6 \end{bmatrix}$

14 (b) Show that $ABCD$ is a parallelogram, but not a rhombus.

[5 marks]

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$ $\quad \overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$

$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD} = \begin{bmatrix} -4 \\ -3 \\ 6 \end{bmatrix}$

$\overrightarrow{AB} = \overrightarrow{DC}$ and $\overrightarrow{BC} = \overrightarrow{AD}$, so opposite sides are parallel and equal in length $\therefore$ $ABCD$ is a parallelogram.

$|AB| = \sqrt{((-4)^2 + (-3)^2 + 6^2)} = \sqrt{61}$

$|AD| = \sqrt{(1^2 + 5^2 + (-1)^2)} = \sqrt{27} = 3\sqrt{3}$

$\sqrt{61} \neq 3\sqrt{3}$, so $|AB| \neq |AD|$: adjacent sides are not equal in length

$\therefore$ $ABCD$ is a parallelogram but not a rhombus.

Do not write outside the box

15 A driver is road-testing two minibuses, A and B, for a taxi company.

The performance of each minibus along a straight track is compared.

A flag is dropped to indicate the start of the test.

Each minibus starts from rest.

The acceleration in $\mathrm{m\,s^{-2}}$ of each minibus is modelled as a function of time, t seconds, after the flag is dropped:

$$\text{The acceleration of A} = 0.138t^2$$
$$\text{The acceleration of B} = 0.024t^3$$

15 (a) Find the time taken for A to travel 100 metres.

Give your answer to four significant figures.

[4 marks]

$v = \int 0.138t^2\,dt = 0.046t^3 + c$

$t = 0,\ v = 0 \quad \Rightarrow \quad c = 0$

$s = \int 0.046t^3\,dt = 0.0115t^4 + k$

$t = 0,\ s = 0 \quad \Rightarrow \quad k = 0$

$0.0115t^4 = 100$

$t^4 = 8695.65\ldots$

$t = 9.65663\ldots$

$t = 9.657\ \text{s}\ \ (4\ \text{s.f.})$

Question 15 continues on the next page

Do not write outside the box

15 (b) The company decides to buy the minibus which travels 100 metres in the shortest time.

Determine which minibus should be bought.

[4 marks]

Minibus B: $v = \int 0.024t^3\,dt = 0.006t^4 + c$

$t = 0, v = 0 \Rightarrow c = 0$

$s = \int 0.006t^4\,dt = 0.0012t^5 + k$

$t = 0, s = 0 \Rightarrow k = 0$

$0.0012t^5 = 100$

$t^5 = 83333.3...$

$t = 9.64202... = 9.642$ s

9.642 s < 9.657 s, so B travels 100 m in the shorter time

∴ the company should buy minibus B.

15 (c) The models assume that both minibuses start moving immediately when $t = 0$

In light of this, explain why the company may, in reality, make the wrong decision.

[1 mark]

The times differ by only 0.015 s. If driver B has a longer reaction time than driver A, then A could travel 100 m faster than B in reality.

Do not write outside the box

16 **In this question use $g = 9.81\,\text{m s}^{-2}$**

A particle is projected with an initial speed u, at an angle of 35° above the horizontal.

It lands at a point 10 metres vertically below its starting position.

The particle takes 1.5 seconds to reach the highest point of its trajectory.

16 (a) Find u.

[3 marks]

Vertical motion, upwards positive. At the highest point $v_y = 0$:

$v = u + at$: $0 = u \sin 35° - 9.81 \times 1.5$

$u = 14.715 \div \sin 35° = 25.654...$

$u = 25.7\ \text{m s}^{-1}$ (3 s.f.)

16 (b) Find the total time that the particle is in flight.

[3 marks]

Vertically, $s = -10$: $s = ut + \frac{1}{2}at^2$

$-10 = (25.654... \sin 35°)t - \frac{1}{2} \times 9.81t^2$

$-10 = 14.715t - 4.905t^2 \Rightarrow 4.905t^2 - 14.715t - 10 = 0$

$t = (14.715 + \sqrt{(14.715^2 + 4 \times 4.905 \times 10)}) \div (2 \times 4.905)$

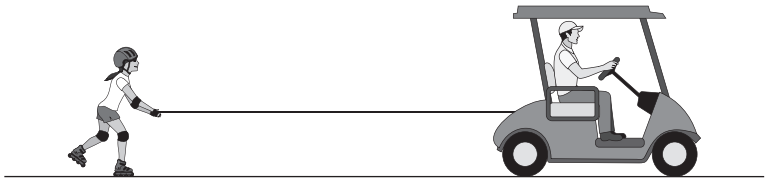
$t = 3.5709...$ (the other root is negative)

Time in flight = 3.57 seconds

Turn over ▶

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17 A buggy is pulling a roller-skater, in a straight line along a horizontal road, by means of a connecting rope as shown in the diagram.



The combined mass of the buggy and driver is 410 kg
A driving force of 300 N and a total resistance force of 140 N act on the buggy.

The mass of the roller-skater is 72 kg
A total resistance force of R newtons acts on the roller-skater.

The buggy and the roller-skater have an acceleration of $0.2\,\text{m}\,\text{s}^{-2}$

17 (a) (i) Find R.

[3 marks]

Whole system (buggy + skater), mass 410 + 72 = 482 kg

N2L: 300 − 140 − R = 482 × 0.2

160 − R = 96.4

R = 63.6 N

Do not write outside the box

17 (a) (ii) Find the tension in the rope.

[3 marks]

Roller-skater: $T - R = 72 \times 0.2$

$T - 63.6 = 14.4$

$T = 78$ N

17 (b) State a necessary assumption that you have made.

[1 mark]

The rope is light (has no mass) and inextensible.

Question 17 continues on the next page

Do not write outside the box

17 (c) The roller-skater releases the rope at a point A, when she reaches a speed of $6\,m\,s^{-1}$

She continues to move forward, experiencing the same resistance force.

The driver notices a change in motion of the buggy, and brings it to rest at a distance of 20 m from A.

17 (c) (i) Determine whether the roller-skater will stop before reaching the stationary buggy.

Fully justify your answer.

[5 marks]

After release, only the resistance $R = 63.6$ N acts horizontally on her:

$-63.6 = 72a \quad \Rightarrow \quad a = -0.8833...\ m\ s^{-2}$

$u = 6,\ v = 0,\ a = -0.8833...$

$v^2 = u^2 + 2as$: $\quad 0 = 6^2 - 2 \times 0.8833... \times s$

$s = 36 \div 1.7666... = 20.377...$ m

$s = 20.4$ m

20.4 m > 20 m, so she is still moving when she reaches the buggy.

∴ the roller-skater does not stop before reaching the buggy;

she will hit it.

Do not write outside the box

17 (c) (ii) Explain the change in motion that the driver noticed.

[2 marks]

When the rope is released there is no longer a tension pulling backwards on the buggy, so the resultant forward force on the buggy increases.

The driver notices an increase in the acceleration of the buggy.

END OF QUESTIONS

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