AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Statistics

Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The discrete uniform distribution X can take values 1, 2, 3, …, 10

Find $P(X \geq 7)$

Circle your answer.

[1 mark]

0.3 (0.4) 0.6 0.7

2 The random variable X has variance $\text{Var}(X)$

Which of the following expressions is equal to $\text{Var}(aX + b)$, where a and b are non-zero constants?

Circle your answer.

[1 mark]

$a\,\text{Var}(X)$ $a\,\text{Var}(X) + b$ ($a^2\,\text{Var}(X)$) $a^2\,\text{Var}(X) + b$

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3 In a game, it is only possible to score 10, 20 or 30 points.

The probability of scoring 20 points is twice the probability of scoring 30 points.

The probability of scoring 20 points is half the probability of scoring 10 points.

3 (a) Find the mean points scored when the game is played once, giving your answer to two decimal places.

[3 marks]

Let $P(X = 30) = a$, then $P(X = 20) = 2a$ and $P(X = 10) = 4a$

x:	10	20	30
$P(X = x)$:	$4a$	$2a$	a

$4a + 2a + a = 1 \Rightarrow 7a = 1 \Rightarrow a = \frac{1}{7}$

$E(X) = 10 \times \frac{4}{7} + 20 \times \frac{2}{7} + 30 \times \frac{1}{7}$

$= \frac{110}{7} = 15.714...$

Mean = 15.71 points (2 d.p.)

3 (b) Mina plays the game.

Her father, Michael, tells her that he will multiply her score by 5 and then subtract 10

He will then give her the value he has calculated in pence rounded to the nearest penny.

Calculate the expected value in pence that Mina receives.

[2 marks]

Amount received = $5X - 10$ pence

$E(5X - 10) = 5E(X) - 10 = 5 \times 15.714... - 10$

$= 68.571... \approx 69$ pence

Turn over ▶

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4 Oscar is studying the daily maximum temperature in °C in a village during the month of June.

He constructs a 95% confidence interval of width 0.8 °C using a random sample of 150 days.

He assumes that the daily maximum temperature has a normal distribution.

4 (a) Find the standard deviation of Oscar's sample, giving your answer to three significant figures.

[3 marks]

95% confidence $\Rightarrow$ $z = 1.96$

Width $= 2 \times 1.96 \times \frac{s}{\sqrt{150}} = 0.8$

$1.96 \times \frac{s}{\sqrt{150}} = 0.4$

$s = \frac{0.4 \times \sqrt{150}}{1.96} = 2.4994...$

$s = 2.50$ °C (3 s.f.)

4 (b) Oscar calculates the mean of his sample to be 25.3 °C

He claims that the population mean is 26.0 °C

Explain whether or not his confidence interval supports his claim.

[2 marks]

Upper limit of the interval = 25.3 + 0.4 = 25.7 °C

Interval is (24.9, 25.7); 26.0 > 25.7, so 26.0 is not in the interval.

So the confidence interval does not support Oscar's claim.

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4 (c) Explain how Oscar could reduce the width of his 95% confidence interval.

[1 mark]

Oscar could take a larger random sample (more than 150 days),

since the width is proportional to $1/\sqrt{n}$.

Turn over for the next question

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5 The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & x \leq 1 \\ \frac{1}{10}x - \frac{1}{10} & 1 < x \leq 6 \\ \frac{1}{90}x^2 + \frac{1}{10} & 6 < x \leq 9 \\ 1 & x > 9 \end{cases}$$

5 (a) Find the probability density function $f(x)$

[2 marks]

$$f(x) = F'(x) = \begin{cases} \frac{1}{10} & 1 < x \leq 6 \\ \frac{x}{45} & 6 < x \leq 9 \\ 0 & \text{otherwise} \end{cases}$$

5 (b) Show that $\text{Var}(X) = \frac{6737}{1200}$

[4 marks]

$E(X) = \int_1^6 \frac{x}{10}\,dx + \int_6^9 \frac{x^2}{45}\,dx$

$= \left[\frac{x^2}{20}\right]_1^6 + \left[\frac{x^3}{135}\right]_6^9 = \frac{35}{20} + \frac{513}{135} = 1.75 + 3.8 = 5.55$

$E(X^2) = \int_1^6 \frac{x^2}{10}\,dx + \int_6^9 \frac{x^3}{45}\,dx$

$= \left[\frac{x^3}{30}\right]_1^6 + \left[\frac{x^4}{180}\right]_6^9 = \frac{215}{30} + \frac{5265}{180} = \frac{437}{12}$

$\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{437}{12} - 5.55^2$

$= \frac{43700}{1200} - \frac{36963}{1200} = \frac{6737}{1200}$ as required

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6 Danai is investigating the number of speeding offences in different towns in a country.

She carries out a hypothesis test to test for association between town and number of speeding offences per year.

6 (a) State the hypotheses for this test.

[1 mark]

H_0: There is no association between town and number of speeding offences per year.

H_1: There is an association between town and number of offences.

6 (b) The observed frequencies, O, have been collected and the expected frequencies, E, have been calculated in an $n \times m$ contingency table, where $n > 3$ and $m > 3$

One of the values of E is less than 5

6 (b) (i) Explain what steps Danai should take before calculating the test statistic.

[2 marks]

Merge the row (or column) containing the expected frequency less than 5 with another suitable row (or column), so that all expected frequencies are at least 5. This must be done for both the observed frequencies and the expected frequencies (and the degrees of freedom reduced accordingly).

6 (b) (ii) State an expression for the test statistic Danai should calculate.

[1 mark]

$$X^2 = \Sigma \frac{(O - E)^2}{E}$$

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6 (c) Danai correctly calculates the value of the test statistic to be 45.22

The number of degrees of freedom for the test is 25

Determine the outcome of Danai's test, using the 1% level of significance.

[3 marks]

$\nu = 25$: 1% critical value of $\chi^2_{25} = 44.314$

$45.22 > 44.314$, so the test statistic is in the critical region

Reject H_0

There is significant evidence at the 1% level to suggest that there is an association between town and number of speeding offences per year.

Turn over for the next question

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7 The random variable X has an exponential distribution with parameter λ

7 (a) Prove that $\mathrm{E}(X) = \frac{1}{\lambda}$

[4 marks]

$E(X) = \int_0^\infty \lambda x\, e^{-\lambda x}\, dx = \lim_{k\to\infty} \int_0^k \lambda x\, e^{-\lambda x}\, dx$

By parts: $u = x$, $dv/dx = \lambda e^{-\lambda x}$ $\Rightarrow$ $du/dx = 1$, $v = -e^{-\lambda x}$

$\int_0^k \lambda x\, e^{-\lambda x}\, dx = \left[-x\, e^{-\lambda x} \right]_0^k + \int_0^k e^{-\lambda x}\, dx$

$= -k\, e^{-\lambda k} + \left[-\frac{1}{\lambda} e^{-\lambda x} \right]_0^k$

$= -k\, e^{-\lambda k} - \frac{1}{\lambda} e^{-\lambda k} + \frac{1}{\lambda}$

As $k \to \infty$, $k\, e^{-\lambda k} \to 0$ and $e^{-\lambda k} \to 0$ (since $\lambda > 0$)

$\therefore E(X) = 0 - 0 + \frac{1}{\lambda} = \frac{1}{\lambda}$ as required

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7 (b) Prove that $\text{Var}(X) = \frac{1}{\lambda^2}$

[7 marks]

$E(X^2) = \int_0^\infty \lambda x^2 e^{-\lambda x}\, dx = \lim_{k\to\infty} \int_0^k \lambda x^2 e^{-\lambda x}\, dx$

By parts: $u = x^2$, $dv/dx = \lambda e^{-\lambda x}$ $\Rightarrow$ $du/dx = 2x$, $v = -e^{-\lambda x}$

$\int_0^k \lambda x^2 e^{-\lambda x}\, dx = \left[-x^2 e^{-\lambda x} \right]_0^k + \int_0^k 2x\, e^{-\lambda x}\, dx$

By parts again: $u = 2x$, $v = -\frac{1}{\lambda} e^{-\lambda x}$

$\int_0^k 2x\, e^{-\lambda x}\, dx = \left[-\frac{2x}{\lambda} e^{-\lambda x} \right]_0^k + \int_0^k \frac{2}{\lambda} e^{-\lambda x}\, dx$

$= -\frac{2k}{\lambda} e^{-\lambda k} + \left[-\frac{2}{\lambda^2} e^{-\lambda x} \right]_0^k$

$= -\frac{2k}{\lambda} e^{-\lambda k} - \frac{2}{\lambda^2} e^{-\lambda k} + \frac{2}{\lambda^2}$

So $E(X^2) = \lim_{k\to\infty} \left(-k^2 e^{-\lambda k} - \frac{2k}{\lambda} e^{-\lambda k} - \frac{2}{\lambda^2} e^{-\lambda k} + \frac{2}{\lambda^2} \right)$

As $k \to \infty$, $k^2 e^{-\lambda k} \to 0$, $k\, e^{-\lambda k} \to 0$ and $e^{-\lambda k} \to 0$

$\therefore E(X^2) = \frac{2}{\lambda^2}$

$\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda} \right)^2$

$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$ as required

Turn over ▶

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8 A company records the number of complaints, X, that it receives over 60 months. The summarised results are

$$\sum x = 102 \quad \text{and} \quad \sum (x - \bar{x})^2 = 103.25$$

8 (a) Using this data, explain why it may be appropriate to model the number of complaints received by the company per month by a Poisson distribution with mean 1.7

[3 marks]

$\bar{x} = \frac{\Sigma x}{n} = \frac{102}{60} = 1.7$

$s^2 = \frac{\Sigma(x - \bar{x})^2}{n - 1} = \frac{103.25}{59} = 1.75$

The sample mean (1.7) is approximately equal to the sample variance (1.75), as required for a Poisson distribution (mean = variance), so a Poisson distribution with mean 1.7 may be appropriate.

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8 (b) The company also receives enquiries as well as complaints. The number of enquiries received is independent of the number of complaints received.

The company models the number of complaints per month with a Poisson distribution with mean 1.7 and the number of enquiries per month with a Poisson distribution with mean 5.2

The company starts selling a new product.

The company records a **total** of 3 complaints and enquiries in one randomly chosen month.

Investigate if the mean total number of complaints and enquiries received per month has changed following the introduction of the new product, using the 10% level of significance.

[6 marks]

T = total number of complaints and enquiries in a month

Before: $T \sim Po(1.7 + 5.2) = Po(6.9)$ (sum of independent Poissons)

H_0: $\lambda = 6.9$ H_1: $\lambda \neq 6.9$

(λ = mean total number of complaints and enquiries per month)

Two-tailed test at 10%, so compare with 0.05 in each tail

Under H_0, $T \sim Po(6.9)$: $P(T \leq 3) = 0.087129...$

$0.0871 > 0.05$, so $T = 3$ is not in the critical region

Do not reject H_0

There is no significant evidence at the 10% level to suggest that the mean total number of complaints and enquiries received per month has changed following the introduction of the new product.

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8 (c) It is later found that the mean total number of complaints and enquiries received per month is 6.1

Find the power of the test carried out in part **(b)**, giving your answer to four decimal places.

[4 marks]

Critical region for the test in (b), $T \sim Po(6.9)$, 0.05 in each tail:

$P(T \leqslant 2) = 0.031951... < 0.05$, $P(T \leqslant 3) = 0.0871 > 0.05$

$P(T \geqslant 12) = 1 - P(T \leqslant 11) = 0.048960... < 0.05$

$P(T \geqslant 11) = 1 - P(T \leqslant 10) = 0.091574... > 0.05$

Critical region: $T \leqslant 2$ or $T \geqslant 12$

Power = $P(\text{reject } H_0 \text{ when } \lambda = 6.1)$, $Y \sim Po(6.1)$

$= P(Y \leqslant 2) + P(Y \geqslant 12) = 0.057652... + 0.022440...$

$= 0.080093...$

Power = 0.0801 (4 d.p.)

END OF QUESTIONS

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