AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Mechanics

Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 A spring of natural length 50 cm and modulus of elasticity λ newtons has an elastic potential energy of 4 J when compressed by 5 cm.

Find the value of λ

Circle your answer.

[1 mark]

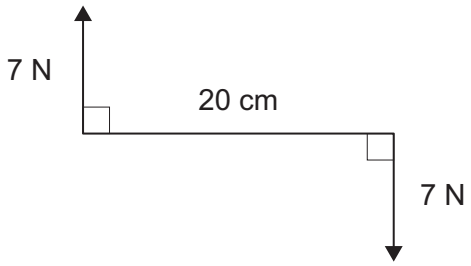
8 16 800 (1600)

$EPE = \lambda x^2/(2l)$: $4 = \lambda \times 0.05^2 \div (2 \times 0.5) \Rightarrow \lambda = 1600$

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2 A force of magnitude 7 N acts at each end of a rod of length 20 cm, forming a couple.

The forces act at right angles to the rod, as shown in the diagram below.



Find the magnitude of the resultant moment of the couple.

Circle your answer.

[1 mark]

(1.4 N m) 2.8 N m 140 N m 280 N m

Moment of couple = 7 × 0.2 = 1.4 N m

Turn over for the next question

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3 A ball has mass 0.4 kg and is hit by a wooden bat.

The speed of the ball just before it is hit by the bat is $6\,\text{m}\,\text{s}^{-1}$

The velocity of the ball immediately after being hit by the bat is perpendicular to its initial velocity.

The speed of the ball just after it is hit by the bat is $8\,\text{m}\,\text{s}^{-1}$

Show that the impulse on the ball has magnitude 4 N s

[3 marks]

Let initial velocity $u = 6i$ and final velocity $v = 8j$ (perpendicular)

Impulse = change in momentum

$I = mv - mu = 0.4 \times 8j - 0.4 \times 6i$

$= -2.4i + 3.2j$

$|I| = \sqrt{(2.4^2 + 3.2^2)}$

$= \sqrt{(5.76 + 10.24)} = \sqrt{16}$

$= 4$ N s as required

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4 A spring has stiffness k

4 (a) Determine the dimensions of k

[1 mark]

$[k] = [\text{force}] \div [\text{extension}] = MLT^{-2} \div L = MT^{-2}$

4 (b) One end of the spring is attached to a fixed point. A particle of mass m kg is attached to the other end of the spring.

The particle is set into vertical motion and moves up and down, taking t seconds to complete one oscillation.

A possible model for t is

$$t = pm^a g^b k^c$$

where p is a dimensionless constant and $g\,\text{m}\,\text{s}^{-2}$ is the acceleration due to gravity.

Find the values of a, b and c for this model to be dimensionally consistent.

[3 marks]

$t = p\, m^a\, g^b\, k^c$

$[t] = T, \quad [m] = M, \quad [g] = LT^{-2}, \quad [k] = MT^{-2}$

$T = M^a (LT^{-2})^b (MT^{-2})^c = M^{a+c} L^b T^{-2b-2c}$

M: $a + c = 0$

L: $b = 0$

T: $-2b - 2c = 1 \Rightarrow c = -1/2$

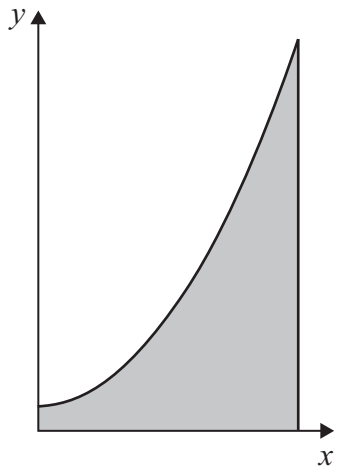
$a = -c = 1/2$

$\therefore \quad a = 1/2, \quad b = 0, \quad c = -1/2$

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5 A uniform lamina has the shape of the region enclosed by the curve $y = x^2 + 1$ and the lines $x = 0$, $x = 4$ and $y = 0$

The diagram below shows the lamina.



5 (a) Find the coordinates of the centre of mass of the lamina, giving your answer in exact form.

[4 marks]

Area $= \int_0^4 (x^2 + 1)\,dx = [x^3/3 + x]_0^4 = 64/3 + 4 = 76/3$

Mass $= (76/3)\rho$ (ρ = mass per unit area)

$\bar{x} \times (76/3)\rho = \rho\int_0^4 x(x^2 + 1)\,dx = \rho\int_0^4 (x^3 + x)\,dx$

$= \rho[x^4/4 + x^2/2]_0^4 = \rho(64 + 8) = 72\rho$

$(76/3)\bar{x} = 72 \Rightarrow \bar{x} = 216/76 = 54/19$

$\bar{y} \times (76/3)\rho = \rho\int_0^4 \tfrac{1}{2}(x^2 + 1)^2\,dx$

$= \tfrac{1}{2}\rho\int_0^4 (x^4 + 2x^2 + 1)\,dx$

$= \tfrac{1}{2}\rho[x^5/5 + 2x^3/3 + x]_0^4$

$= \tfrac{1}{2}\rho(1024/5 + 128/3 + 4)$

$= \tfrac{1}{2}\rho \times 3772/15 = (3772/30)\rho$

$(76/3)\bar{y} = 3772/30 \Rightarrow \bar{y} = (3772 \times 3)/(30 \times 76)$

$= 11316/2280 = 943/190$

Centre of mass is at $(54/19, 943/190)$

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5 (b) The lamina is suspended from the point where the curve intersects the line $x = 4$ and hangs in equilibrium.

Find the angle between the vertical and the longest straight edge of the lamina, giving your answer correct to the nearest degree.

[3 marks]

Longest straight edge is x = 4, from (4, 0) to (4, 17)

Suspended from (4, 17), so G is vertically below it.

Horizontal distance of G from x = 4:

4 − 54/19 = 22/19

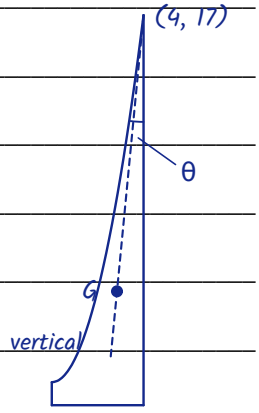
Vertical distance of G below (4, 17):

17 − 943/190 = 2287/190

$\tan\theta = (22/19) \div (2287/190) = 220/2287$

$\theta = \tan^{-1}(0.096196...) = 5.4947...°$

Angle = 5° (nearest degree)



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6 A ball of mass m kg is held at rest at a height h metres above a horizontal surface.

The ball is released and bounces on the surface.

The coefficient of restitution between the ball and the surface is e

Prove that the kinetic energy lost during the first bounce is given by

$$mgh(1 - e^2)$$

[4 marks]

Let U = speed just before impact (falls h from rest):

$v^2 = u^2 + 2as \Rightarrow U^2 = 0^2 + 2gh \Rightarrow U = \sqrt{(2gh)}$

Let V = rebound speed. Newton's law of restitution:

$V = eU = e\sqrt{(2gh)}$

KE lost $= \frac{1}{2}mU^2 - \frac{1}{2}mV^2$

$= \frac{1}{2}m(2gh) - \frac{1}{2}m(e^2 \times 2gh)$

$= mgh - mghe^2$

$= mgh(1 - e^2)$ as required

Turn over ▶

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7 A light string has length 1.5 metres.

A small sphere is attached to one end of the string.

The other end of the string is attached to a fixed point O

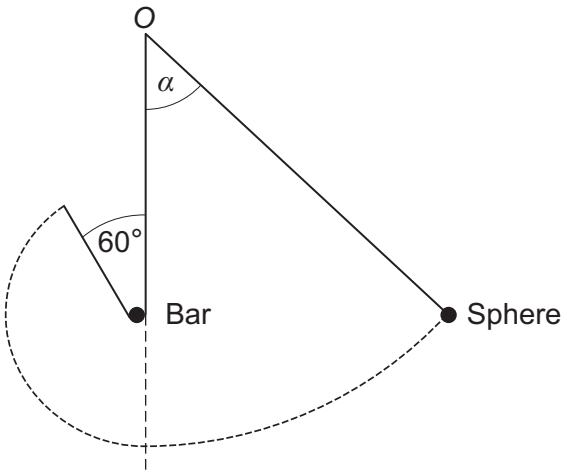
A thin horizontal bar is positioned 0.9 metres directly below O

The bar is perpendicular to the plane in which the sphere moves.

The sphere is released from rest with the string taut and at an angle α to the downward vertical through O

The string becomes slack when the angle between the two sections of the string is 60°

Ben draws the diagram below to show the initial position of the sphere, the bar and the path of the sphere.



7 (a) State two reasons why Ben's diagram is not a good representation of the situation.

[2 marks]

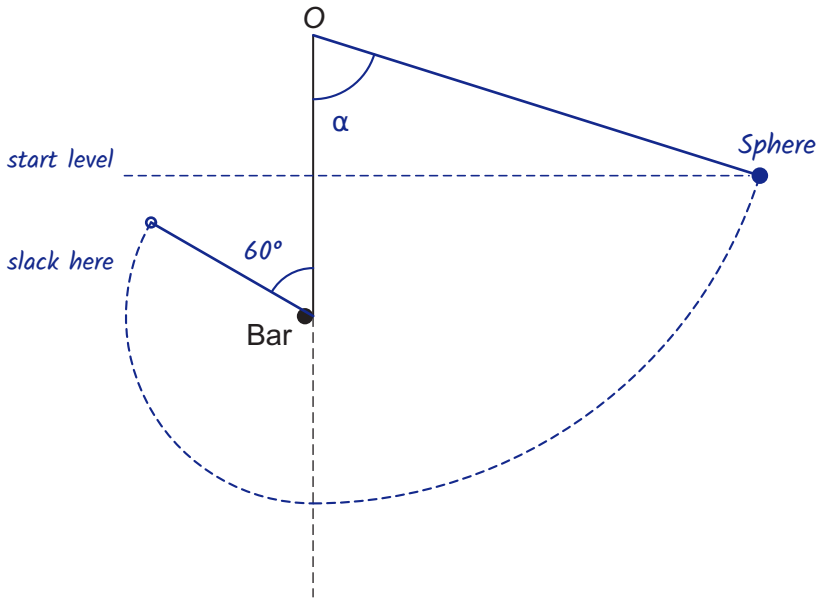
Reason 1 The 60° angle is drawn too small: the angle between the two sections of string in the diagram is much less than 60°.

Reason 2 The sphere must start at a higher level than the point where the string becomes slack (it still has KE there), but Ben shows it starting lower.

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7 (b) Using your answer to part **(a)**, sketch an improved diagram.

[1 mark]



Question 7 continues on the next page

Turn over ▶

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7 (c) Find α, giving your answer to the nearest degree.

[6 marks]

Let mass = m, and speed when the string becomes slack = v

Length of string below the bar = 1.5 − 0.9 = 0.6 m

When slack, T = 0. Resolving towards the bar (N2L):

$mg\cos60° = mv^2/0.6$

$v^2 = 0.6g\cos60° = 0.3g$

Height of slack point above bar = $0.6\cos60° = 0.3$ m

Take GPE = 0 at the bar; let h = initial height above the bar.

Energy: $mgh = \frac{1}{2}m(0.3g) + mg(0.3)$

$h = 0.15 + 0.3 = 0.45$ m

Initially the sphere is $1.5\cos\alpha$ below O, and the bar is 0.9 below O:

$0.9 - 1.5\cos\alpha = 0.45$

$\cos\alpha = 0.45 \div 1.5 = 0.3$

$\alpha = \cos^{-1}(0.3) = 72.542...°$

$\alpha = 73°$ (nearest degree)

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8 **In this question use $g = 9.8\,\text{m s}^{-2}$**

A lift is used to raise a crate of mass 250 kg

The lift exerts an upward force of magnitude P newtons on the crate.

When the crate is at a height of x metres above its initial position

$$P = k(x + 1)(12 - x) + 2450$$

where k is a constant.

The crate is initially at rest, at the point where $x = 0$

8 (a) Show that the work done by the upward force as the crate rises to a height of 12 metres is given by

$$29\,400 + 360k$$

[3 marks]

$WD = \int_0^{12} P\,dx = \int_0^{12} [k(x + 1)(12 - x) + 2450]\,dx$

$= k\int_0^{12} (-x^2 + 11x + 12)\,dx + \int_0^{12} 2450\,dx$

$= k[-x^3/3 + 11x^2/2 + 12x]_0^{12} + [2450x]_0^{12}$

$= k(-576 + 792 + 144) + 29400$

$= 360k + 29400$

$\therefore$ work done $= 29400 + 360k$ as required

8 (b) The speed of the crate is $3\,\text{m s}^{-1}$ when it has risen to a height of 12 metres.

Find the speed of the crate when it has risen to a height of 15 metres.

[5 marks]

From $x = 0$ to 12: WD = gain in GPE + gain in KE

$29400 + 360k = 250 \times 9.8 \times 12 + \frac{1}{2} \times 250 \times 3^2$

$29400 + 360k = 29400 + 1125$

$360k = 1125 \Rightarrow k = 25/8$

WD from $x = 0$ to 15: $(25/8)[-x^3/3 + 11x^2/2 + 12x]_0^{15} + 2450 \times 15$

$= (25/8)(-1125 + 1237.5 + 180) + 36750$

$= (25/8)(585/2) + 36750 = 37664.0625$ J

Energy: $37664.0625 = 250 \times 9.8 \times 15 + \frac{1}{2} \times 250 \times v^2$

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$125v^2 = 37664.0625 - 36750 = 914.0625$

$v^2 = 7.3125$

$v = 2.7041...$

Speed = $2.7\ m\ s^{-1}$ (2 s.f.)

8 (c) Find the height of the crate when its speed becomes zero.

[2 marks]

Speed zero when KE = 0, so WD by P = gain in GPE:

$(25/8)[-x^3/3 + 11x^2/2 + 12x]_0^x + 2450x = 250 \times 9.8 \times x$

$(25/8)x(12 + 11x/2 - x^2/3) = 0$

$x \neq 0$, so $x^2/3 - 11x/2 - 12 = 0 \Rightarrow 2x^2 - 33x - 72 = 0$

$x = (33 \pm \sqrt{(33^2 + 576)}) \div 4 = 18.451...$ or -1.95

$x > 0$, so height = 18.451... = 18 m (2 s.f.)

8 (d) Air resistance has been ignored.

Explain why this is reasonable in this context.

[1 mark]

The crate moves at low speeds (a few $m\ s^{-1}$), so air resistance is very small.

Turn over ▶

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9 **In this question use $g = 9.81\,\text{m s}^{-2}$**

A conical pendulum is made from an elastic string and a sphere of mass 0.2 kg

The string has natural length 1.6 metres and modulus of elasticity 200 N

The sphere describes a horizontal circle of radius 0.5 metres at a speed of $v\,\text{m s}^{-1}$

The angle between the elastic string and the vertical is α

9 (a) Show that

$$62.5 - 200\sin\alpha = 1.962\tan\alpha$$

[5 marks]

Let the extension be e. Radius = (1.6 + e)sinα = 0.5

e = 0.5/sinα − 1.6

Hooke's law: T = λe/l = 200(0.5/sinα − 1.6) ÷ 1.6

T = 62.5/sinα − 200

Resolving vertically: T cosα = 0.2 × 9.81 = 1.962

T = 1.962/cosα

Equating: 1.962/cosα = 62.5/sinα − 200

Multiply by sinα: 1.962 sinα/cosα = 62.5 − 200sinα

Since tanα = sinα/cosα:

62.5 − 200sinα = 1.962tanα as required

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9 (b) Use your calculator to find α

[1 mark]

$\alpha = 18.0176...° = 18.0°$

9 (c) Find the value of v

[4 marks]

$T = 1.962/\cos(18.0176...°) = 2.06317...$

Resolving horizontally (towards the centre):

$T \sin\alpha = mv^2/r$

$2.06317... \times \sin(18.0176...°) = 0.2v^2/0.5$

$v^2 = 2.06317... \times \sin(18.0176...°) \div 0.4 = 1.59539...$

$v = 1.26309...$

$v = 1.26 \text{ m s}^{-1}$ (3 s.f.)

END OF QUESTIONS

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