AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 2

Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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13	
TOTAL	

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Answer **all** questions in the spaces provided.

1 Which of the following matrices is singular?

Circle your answer.

[1 mark]

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}$

2 Find $\arg(-4-7\mathrm{i})$ to the nearest degree.

Circle your answer.

[1 mark]

$-120°$ $-60°$ $30°$ $60°$

Do not write outside the box

3 The line L has equation $\mathbf{r} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ -2 \\ 5 \end{bmatrix}$

Which of the following lines is perpendicular to the line L?

Tick (✓) **one** box.

[1 mark]

$\mathbf{r} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix}$ ☐

$\mathbf{r} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$ ☐

$\mathbf{r} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ ☐

$\mathbf{r} = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix} + \mu \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$ ☐

Turn over for the next question

Do not write outside the box

4 (a) Show that

$$(r+1)^2 - r^2 = 2r + 1$$

[1 mark]

4 (b) Use the method of differences to show that

$$\sum_{r=1}^{n} (2r+1) = n^2 + 2n$$

[3 marks]

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4 (c) Verify that using the formula for $\sum_{r=1}^{n} r$ gives the same result as that given in part **(b)**.

[3 marks]

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5 The equation

$$z^3 + 2z^2 - 5z - 3 = 0$$

has roots α, β and γ

Find a cubic equation with roots

$$\frac{1}{2}\alpha - 1, \ \frac{1}{2}\beta - 1 \ \text{ and } \ \frac{1}{2}\gamma - 1$$

[5 marks]

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Turn over for the next question

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6 The ellipse E_1 has equation

$$x^2 + \frac{y^2}{4} = 1$$

E_1 is translated by the vector $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ to give the ellipse E_2

6 (a) Write down the equation of E_2

[1 mark]

6 (b) The ellipse E_3 has equation

$$\frac{x^2}{4} + (y - 3)^2 = 1$$

Describe the transformation that maps E_2 to E_3

[1 mark]

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6 (c) Each of the lines L_A and L_B is a tangent to both E_2 and E_3

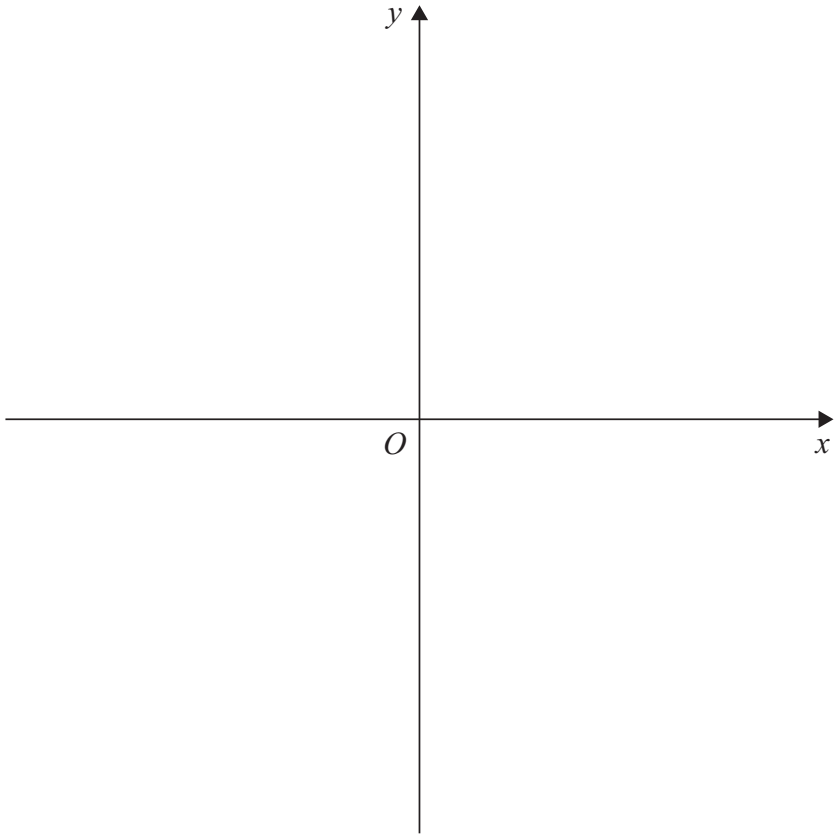
L_A is closer to the origin than L_B

E_2 and E_3 both lie between L_A and L_B

Sketch and label E_2, E_3, L_A and L_B on the axes below.

You do not need to show the values of the axis intercepts for L_A and L_B

[4 marks]



6 (d) Explain, without doing any calculations, why L_A has an equation of the form

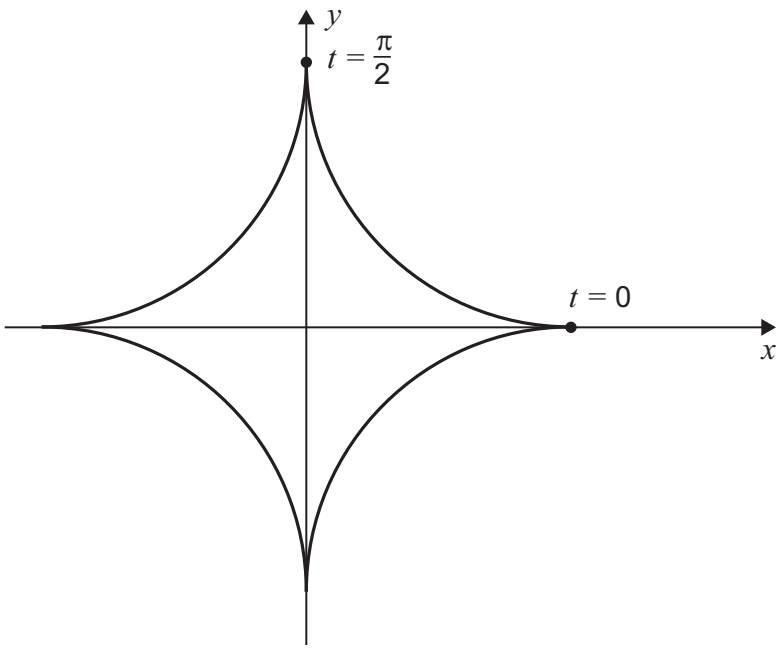
$$x + y = c$$

where c is a constant.

[2 marks]

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7



The diagram shows a curve known as an astroid.

The curve has parametric equations

$$x = 4\cos^3 t$$

$$y = 4\sin^3 t$$

$$(0 \leq t < 2\pi)$$

The section of the curve from $t = 0$ to $t = \frac{\pi}{2}$ is rotated through 2π radians about the x-axis.

Show that the curved surface area of the shape formed is equal to $\frac{b\pi}{c}$, where b and c are integers.

[7 marks]

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8 The complex number z satisfies the equations

$$|z^* - 1 - 2\mathrm{i}| = |z - 3|$$

and

$$|z - a| = 3$$

where a is real.

Show that a must lie in the interval $\left[1 - s\sqrt{t},\ 1 + s\sqrt{t}\right]$, where s and t are prime numbers.

[6 marks]

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9 (a) The line L has polar equation

$$r = \frac{7}{4}\sec\theta \qquad \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$$

Show that L is perpendicular to the initial line.

[2 marks]

9 (b) The curve C has polar equation

$$r = 3 + \cos\theta \qquad (-\pi < \theta \leq \pi)$$

Find the polar coordinates of the points of intersection of L and C

Fully justify your answer.

[5 marks]

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9 (c) The region R is the set of points such that

$$r > \frac{7}{4}\sec\theta$$

and

$$r < 3 + \cos\theta$$

Find the exact area of R

[7 marks]

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10 In a colony of seabirds, there are y birds at time t years.

10 (a) The rate of reduction in the number of birds due to birds dying or leaving the colony is proportional to the number of birds.

In one year the reduction in the number of birds due to birds dying or leaving the colony is equal to 16% of the number of birds at the start of the year.

If no birds are born or join the colony, find the constant k such that

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -ky$$

Give your answer to three significant figures.

[4 marks]

10 (b) A wildlife protection group takes measures to support the colony.

The rate of reduction in the number of birds due to birds dying or leaving the colony is the same as in part **(a)**, but in addition:

- The rate of increase in the number of birds due to births is $20t$ per year.
- The wildlife protection group brings 45 birds into the colony each year.

Write down a first-order differential equation for y and t

[2 marks]

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10 (c) The initial number of birds is 340

Solve your differential equation from part **(b)** to find y in terms of t

[5 marks]

10 (d) Describe two limitations of the model you have used.

[2 marks]

Limitation 1 _____

Limitation 2 _____

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11 The Cartesian equation of the line L_1 is

$$\frac{x+1}{3} = \frac{-y+5}{2} = \frac{2z+5}{3}$$

The Cartesian equation of the line L_2 is

$$\frac{2x-1}{2} = \frac{y-14}{m} = \frac{z+12}{p}$$

The non-singular matrix $\mathbf{N} = \begin{bmatrix} -0.5 & 1 & 2 \\ 1 & b & 4 \\ -3 & -2 & c \end{bmatrix}$ maps the line L_1 onto the line L_2

Calculate the values of the constants b, c, m and p

Fully justify your answers.

[9 marks]

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12 The integral S_n is defined by

$$S_n = \int_0^a x^n \sinh x \, \mathrm{d}x \qquad (n \geq 0)$$

12 (a) Show that for $n \geq 2$

$$S_n = n(n-1)S_{n-2} + a^n \cosh a - na^{n-1} \sinh a$$

[7 marks]

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12 (b) Hence show that

$$\int_0^1 x^4 \sinh x \, \mathrm{d}x = \frac{9}{2}\mathrm{e} + \frac{65}{2}\mathrm{e}^{-1} - 24$$

[5 marks]

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13 (a) Two of the solutions to the equation $\cos 6\theta = 0$ are $\theta = \frac{\pi}{4}$ and $\theta = \frac{3\pi}{4}$

Find the other solutions to the equation $\cos 6\theta = 0$ for $0 \leq \theta \leq \pi$

[2 marks]

13 (b) Use de Moivre's theorem to show that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

[5 marks]

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13 (c) Use the fact that $\theta = \frac{\pi}{4}$ and $\theta = \frac{3\pi}{4}$ are solutions to the equation $\cos 6\theta = 0$ to find a factor of $32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$ in the form $(a\cos^2\theta + b)$, where a and b are integers.

[4 marks]

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13 (d) Hence show that

$$\cos\left(\frac{11\pi}{12}\right) = -\sqrt{\frac{2+\sqrt{3}}{4}}$$

[5 marks]

END OF QUESTIONS

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