AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Mechanics

Thursday 11 June 2020 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 A rigid rod, *AB*, has mass 2 kg and length 4 metres.

Two particles of masses 5 kg and 3 kg are fixed to *A* and *B* respectively to create a composite body, as shown in the diagram.



Find the distance of the centre of mass of the composite body from *B*.

Circle your answer.

[1 mark]

1.5 metres | 1.6 metres | (2.4 metres) | 2.5 metres

Moments about B: $10\bar{x} = 5 \times 4 + 2 \times 2 + 3 \times 0 = 24$

$\bar{x} = 24/10 = 2.4$ m from B

2 The tension, T newtons, in a spring is given by $T = 20e$, where e metres is the extension of the spring.

Calculate the work done when the extension is increased from 0.2 metres to 0.4 metres.

Circle your answer.

[1 mark]

0.4 J | 0.9 J | (1.2 J) | 1.6 J

Work done $= \int_{0.2}^{0.4} 20e \, de = [10e^2]_{0.2}^{0.4}$

$= 10(0.16 - 0.04) = 1.2$ J

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3 The speed, v, of a particle moving in a horizontal circle is given by the formula $v = r\omega$ where:

$v =$ speed

$r =$ radius

$\omega =$ angular speed.

Show that the dimensions of angular speed are T^{-1}

[2 marks]

$v = r\omega \Rightarrow [v] = [r][\omega]$

$LT^{-1} = L \times [\omega]$

$[\omega] = LT^{-1} \div L$

$[\omega] = T^{-1}$

$\therefore$ the dimensions of angular speed are T^{-1}

Turn over for the next question

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4 A car has mass 1000 kg and travels on a straight horizontal road.

The maximum speed of the car on this road is $48\,m\,s^{-1}$

In a simple model, it is assumed that the car experiences a resistance force that is proportional to its speed.

When the car travels at $20\,m\,s^{-1}$, the magnitude of the resistance force is 600 newtons.

4 (a) Show that the maximum power of the car is 69 120 W

[2 marks]

Resistance = kv, where k is a constant

At 20 m s^{-1}: 600 = 20k ⇒ k = 30, so resistance = 30v

At maximum speed the acceleration is zero, so

driving force = resistance = 30 × 48 = 1440 N

Maximum power P = Fv = 30 × 48 × 48

P = 69120 W, as required.

4 (b) Find the maximum acceleration of the car when it is travelling at $25\,m\,s^{-1}$

[3 marks]

At 25 m s^{-1}: maximum driving force = P/v = 69120/25 = 2764.8 N

Resistance = 30 × 25 = 750 N

Newton's second law: F − 30v = 1000a

69120/25 − 30 × 25 = 1000a

2764.8 − 750 = 1000a

a = 2014.8/1000 = 2.0148

Maximum acceleration = 2.01 m s^{-2} (3 s.f.)

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4 (c) Find the maximum acceleration of the car when it is travelling at $3\,\text{m}\,\text{s}^{-1}$

[1 mark]

$69120/3 - 30 \times 3 = 1000a \Rightarrow 23040 - 90 = 1000a$

$a = 22950/1000 = 22.95$

Maximum acceleration = $23.0\ \text{m}\ \text{s}^{-2}$ (3 s.f.)

4 (d) Comment on the validity of the model in the context of your answers to parts **(b)** and **(c)**.

[2 marks]

The acceleration of $23.0\ \text{m}\ \text{s}^{-2}$ at $3\ \text{m}\ \text{s}^{-1}$ is very large (over twice g), which is unrealistic for a car, whereas $2.01\ \text{m}\ \text{s}^{-2}$ at $25\ \text{m}\ \text{s}^{-1}$ is reasonable. So the model seems valid at higher speeds but is not valid at low speeds.

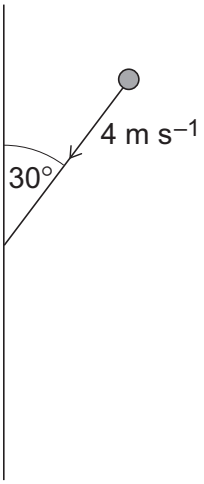
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5 A ball, of mass 0.3 kg, is moving on a smooth horizontal surface.

The ball collides with a smooth fixed vertical wall and rebounds.

Before the ball hits the wall, the ball is moving at $4\,\text{m}\,\text{s}^{-1}$ at an angle of 30° to the wall as shown in the diagram.



The magnitude of the force, F newtons, exerted on the ball by the wall at time t seconds is modelled by

$$F = kt^2(0.1 - t)^2 \quad \text{for} \quad 0 \leq t \leq 0.1$$

where k is a constant.

The ball is in contact with the wall for 0.1 seconds.

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5 (a) Show that the impulse exerted on the ball by the wall while they are in contact has magnitude $\frac{k}{3\,000\,000}$

Fully justify your answer.

[4 marks]

Impulse = $\int F\,dt$ over the time of contact

$I = \int_0^{0.1} kt^2(0.1 - t)^2\,dt$

$= k\int_0^{0.1} t^2(0.01 - 0.2t + t^2)\,dt$

$= k\int_0^{0.1} (t^2/100 - t^3/5 + t^4)\,dt$

$= k[t^3/300 - t^4/20 + t^5/5]_0^{0.1}$

$= k(0.001/300 - 0.0001/20 + 0.00001/5 - 0)$

$= k(1/300000 - 1/200000 + 1/500000)$

$= k(10 - 15 + 6)/3000000$

$= k/3000000$

$F \geq 0$ throughout $0 \leq t \leq 0.1$ and always acts perpendicular to the wall (the wall is smooth), so the impulse is perpendicular to the wall with magnitude $k/3000000$ N s, as required.

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5 (b) Explain why $1\,800\,000 < k \leq 3\,600\,000$

Fully justify your answer.

[5 marks]

Component of velocity perpendicular to the wall before impact:

4 sin 30° = 2 m s^{-1} (parallel component 4 cos 30° unchanged,

as the wall is smooth). Let the rebound perpendicular speed be v.

Maximum: the ball cannot gain kinetic energy ($e \leq 1$), so at most

it rebounds with the same perpendicular speed, $v \leq 2$.

$I_{max} = 0.3 \times 2 - (-0.3 \times 2) = 2(0.3 \times 4 \sin 30°) = 1.2$ N s

Minimum: the ball rebounds, so the impulse must be greater than

the impulse needed to reduce the perpendicular velocity to zero

($e > 0$, $v > 0$).

$I_{min} > 0.3 \times 4 \sin 30° = 0.6$ N s

So $0.6 < I \leq 1.2$

$0.6 < k/3000000 \leq 1.2$

$0.6 \times 3000000 < k \leq 1.2 \times 3000000$

$\therefore 1800000 < k \leq 3600000$

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5 (c) Given that $k = 2\,400\,000$

Find the speed of the ball after the collision with the wall.

[4 marks]

Parallel component of velocity unchanged (smooth wall):

$4 \cos 30° = 2\sqrt{3} = 3.464 \text{ m s}^{-1}$

$I = 2400000/3000000 = 0.8 \text{ N s}$

Perpendicular to the wall (away from wall positive):

$0.3v - (-0.3 \times 4 \sin 30°) = 0.8$

$0.3v + 0.6 = 0.8 \Rightarrow v = 2/3 \text{ m s}^{-1}$

Speed $= \sqrt{((2/3)^2 + (4 \cos 30°)^2)}$

$= \sqrt{(4/9 + 12)} = \sqrt{(112/9)} = 3.5276...$

Speed $= 3.53 \text{ m s}^{-1}$ (3 s.f.)

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6 A particle moves with constant speed on a circular path of radius 2 metres.

The centre of the circle has position vector 2**j** metres.

At time $t = 0$, the particle is at the origin and is moving in the positive **i** direction.

The particle returns to the origin every 4 seconds.

The unit vectors **i** and **j** are perpendicular.

6 (a) Calculate the angular speed of the particle.

[2 marks]

One revolution takes 4 seconds: $4\omega = 2\pi$

$\omega = \pi/2$ rad s^{-1}

6 (b) Write down an expression for the position vector of the particle at time t seconds.

[2 marks]

Centre (0, 2), radius 2, starting at origin moving in the i direction:

$r = 2\sin(\omega t)\,i + (2 - 2\cos(\omega t))\,j$

$r = 2\sin(\pi t/2)\,i + (2 - 2\cos(\pi t/2))\,j$ metres

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6 (c) Find an expression for the acceleration of the particle at time t seconds.

[3 marks]

$v = dr/dt = \pi \cos(\pi t/2)\, i + \pi \sin(\pi t/2)\, j$

$a = dv/dt = -(\pi^2/2) \sin(\pi t/2)\, i + (\pi^2/2) \cos(\pi t/2)\, j$ m s^{-2}

6 (d) State the magnitude of the acceleration of the particle.

[1 mark]

$|a| = \sqrt{((\pi^2/2)^2(\sin^2 + \cos^2))} = \pi^2/2$ m s^{-2} $(= 4.93$ m s$^{-2})$

6 (e) State the time when the acceleration is first directed towards the origin.

[1 mark]

a is directed towards the centre $(0, 2)$, so it points at the origin

when the particle is at $(0, 4)$, half a revolution later: $t = 2$ seconds

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7 **In this question use $g = 9.8\,\text{m s}^{-2}$**

A box, of mass 8 kg, is on a rough horizontal surface.

A string attached to the box is used to pull it along the surface.

The string is inclined at an angle of 40° above the horizontal.

The tension in the string is 50 newtons.

As the box moves a distance of x metres, its speed increases from $2\,\text{m s}^{-1}$ to $5\,\text{m s}^{-1}$

The coefficient of friction between the box and the surface is 0.4

7 (a) By using an energy method, find x.

[6 marks]

Resolving vertically: $R + 50 \sin 40° = 8 \times 9.8$

$R = 78.4 - 50 \sin 40° = 78.4 - 32.139... = 46.260...$ N

Friction (moving, so limiting): $F = \mu R = 0.4(78.4 - 50 \sin 40°)$

$F = 18.504...$ N

Work done by tension = $50 \cos 40° \times x = 38.302... x$

Work done against friction = $0.4(78.4 - 50 \sin 40°)\, x = 18.504... x$

Change in kinetic energy = $\frac{1}{2} \times 8 \times 5^2 - \frac{1}{2} \times 8 \times 2^2$

$= 100 - 16 = 84$ J

Work-energy principle:

work done by tension − work done against friction = gain in KE

$50 \cos 40° x - 0.4(78.4 - 50 \sin 40°)\, x = \frac{1}{2} \times 8 \times 5^2 - \frac{1}{2} \times 8 \times 2^2$

$38.302... x - 18.504... x = 84$

$19.797... x = 84$

$x = 84 \div 19.797... = 4.2429...$

$x = 4.24$ m (3 s.f.)

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7 (b) Describe how the model could be refined to obtain a more realistic value of x and use an energy argument to explain whether this would increase or decrease the value of x.

[2 marks]

Include air resistance acting on the box. This would increase the value of x, because more work would need to be done (against air resistance as well as friction) to give the same gain in kinetic energy, and the net work done per metre would be smaller.

Turn over ▶

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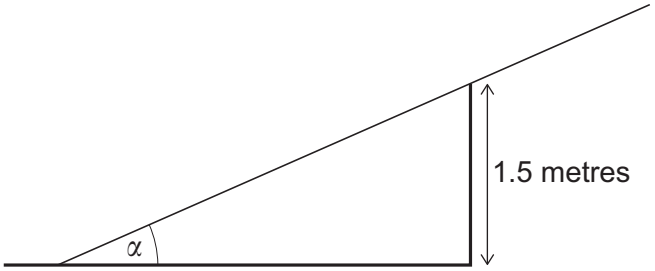
8 A ladder has length 4 metres and mass 20 kg

The ladder rests in equilibrium with one end on a horizontal surface and the ladder resting on the top of a vertical wall.

In this position the ladder is on the point of slipping.

The top of the wall is 1.5 metres above the horizontal surface.

The angle between the ladder and the horizontal surface is α, as shown in the diagram.



The coefficient of friction between the ladder and the wall is 0.5

The coefficient of friction between the ladder and the ground is also 0.5

Show that

$$\cos\alpha \sin^2\alpha = \frac{3}{10}$$

stating clearly any assumptions you make.

[8 marks]

Assume the ladder is uniform (weight 20g acts at its midpoint, 2 m from the base) and both contacts are on the point of slipping (limiting).

Forces: at the ground, R up and friction F_1 towards the wall; at the wall top, S perpendicular to the ladder and friction F_2 up the ladder.

$F_1 = 0.5R$, $F_2 = 0.5S$. Distance from base to wall top = $1.5/\sin\alpha$

Resolving horizontally: $F_2\cos\alpha + F_1 = S\sin\alpha$

Resolving vertically: $R + S\cos\alpha + F_2\sin\alpha = 20g$

Moments about the base: $20g \times 2\cos\alpha = S \times 1.5/\sin\alpha$

Horizontal: $\frac{1}{2}S\cos\alpha + \frac{1}{2}R = S\sin\alpha \Rightarrow R = 2S\sin\alpha - S\cos\alpha$

Vertical: $(2S\sin\alpha - S\cos\alpha) + S\cos\alpha + \frac{1}{2}S\sin\alpha = 20g$

$(5/2)\,S\sin\alpha = 20g \Rightarrow S = 8g/\sin\alpha$

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Q8 continued

Substitute into the moments equation:

$40g \cos \alpha = (8g/\sin \alpha) \times (1.5/\sin \alpha)$

$40g \cos \alpha = 12g/\sin^2 \alpha$

$\cos \alpha \sin^2 \alpha = 12/40$

$\therefore \cos \alpha \sin^2 \alpha = 3/10$, as required.

END OF QUESTIONS

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