AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Discrete

Thursday 11 June 2020 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

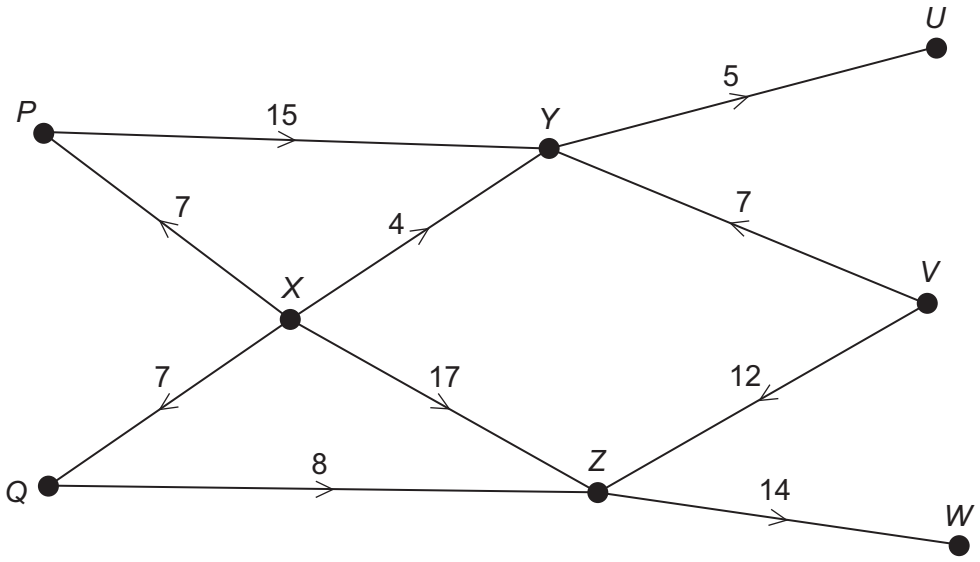
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The diagram below shows a network of pipes with their capacities.



A supersource and a supersink will be added to the network.

To which nodes should the supersource and supersink be connected?

Tick (✓) **one** box.

[1 mark]

Supersource	Supersink	
P, *Q*	*U*, *V*, *W*	☐
U, *V*, *W*	*P*, *Q*	☐
V, *X*	*U*, *W*	☐
U, *W*	*V*, *X*	☐

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2 Which of the following statements is true about the operation of matrix multiplication on the set of all 2×2 real matrices?

Tick (✓) **one** box.

[1 mark]

Matrix multiplication is associative and commutative. ☐

Matrix multiplication is associative but not commutative. ☐

Matrix multiplication is commutative but not associative. ☐

Matrix multiplication is not commutative and not associative. ☐

Turn over for the next question

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3 A company is installing an internal telephone network between the offices in a council building. Each office is required to be connected with telephone cables, either directly or indirectly, to every other office in the building.

The lengths of cable, in metres, needed to connect the offices are shown in the table below.

	Education	**Housing**	**Refuse Collection**	**Payroll**	**Social Care**	**Transport**
Education	–	27	13	35	16	24
Housing	27	–	29	30	22	24
Refuse Collection	13	29	–	26	23	17
Payroll	35	30	26	–	20	40
Social Care	16	22	23	20	–	21
Transport	24	24	17	40	21	–

The council wants the total length of cable that is used to be as small as possible.

The cost to the council to install one metre of cable is £8

3 (a) (i) Find the minimum total cost to the council to install the cable required for the internal telephone network.

[4 marks]

Do not write outside the box

3 (a) (ii) Suggest a reason why the total cost to the council for installing the internal telephone network is likely to be different from your answer to part **(a)(i)**.

[1 mark]

_____

_____

_____

3 (b) Before the company starts installing the cable, it is told that the Education office cannot be connected directly to the Transport office due to issues with the building.

Explain the possible impact of this on your answer to part **(a)(i)**.

[2 marks]

_____

_____

_____

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Turn over for the next question

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4 Joe, a courier, is required to deliver parcels to six different locations, *A*, *B*, *C*, *D*, *E* and *F*.

Joe needs to start and finish his journey at the depot.

The distances, in miles, between the depot and the six different locations are shown in the table below.

	Depot	***A***	***B***	***C***	***D***	***E***	***F***
Depot	–	18	17	15	16	19	30
A	18	–	29	20	25	35	21
B	17	29	–	26	30	16	14
C	15	20	26	–	28	31	27
D	16	25	30	28	–	34	24
E	19	35	16	31	34	–	28
F	30	21	14	27	24	28	–

The minimum total distance that Joe can travel in order to make all six deliveries, starting and finishing at the depot, is L miles.

4 (a) Using the nearest neighbour algorithm starting from the depot, find an upper bound for L.

[2 marks]

Do not write outside the box

4 (b) By deleting the depot, find a lower bound for L.

[3 marks]

4 (c) Joe starts from the depot, delivers parcels to all six different locations and arrives back at the depot, covering 134 miles in the process.

Joe claims that this is the minimum total distance that is possible for the journey.

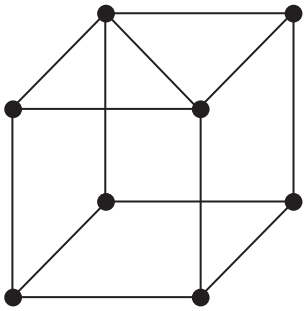
Comment on Joe's claim.

[1 mark]

Turn over for the next question

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5 The planar graph P is shown below.



5 (a) Determine the number of faces of P.

[2 marks]

5 (b) Akwasi claims that P is semi-Eulerian as it is connected and it has exactly two vertices with even degree.

Comment on the validity of Akwasi's claim.

[2 marks]

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6 The group $(G, \blacktriangle)$ has the elements e, r, r^2, q, qr and qr^2, where $r^2 = r \blacktriangle r$, $qr = q \blacktriangle r$, $qr^2 = q \blacktriangle r^2$ and e is the identity element of G.

The elements q and r have the following properties:

$$r \blacktriangle r \blacktriangle r = e$$

$$q \blacktriangle q = e$$

$$r^2 \blacktriangle q = q \blacktriangle r$$

6 (a) (i) State the order of G.

[1 mark]

6 (a) (ii) Prove that the inverse of qr is qr.

[3 marks]

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6 (b) Complete the Cayley table for elements of G.

[3 marks]

▲	e	r	r^2	q	qr	qr^2
e	e	r	r^2	q	qr	qr^2
r	r	r^2	e			
r^2	r^2	e	r			
q	q	qr	qr^2	e		
qr	qr	qr^2	q	r^2		
qr^2	qr^2	q	qr	r	r^2	e

6 (c) State the name of a group which is isomorphic to G.

[1 mark]

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7 An engineering company makes brake kits and clutch kits to sell to motorsport teams.

The table below summarises the time taken and costs involved in making the two different types of kit.

Type of kit	Time taken to make a kit (hours)	Cost to engineering company per kit (£)	Profit to engineering company per kit (£)
Brake kit	5	500	2000
Clutch kit	3	200	1000

The workers at the engineering company have a combined 2500 hours available to make the kits every month.

The engineering company has £200 000 available to cover the costs of making the kits every month.

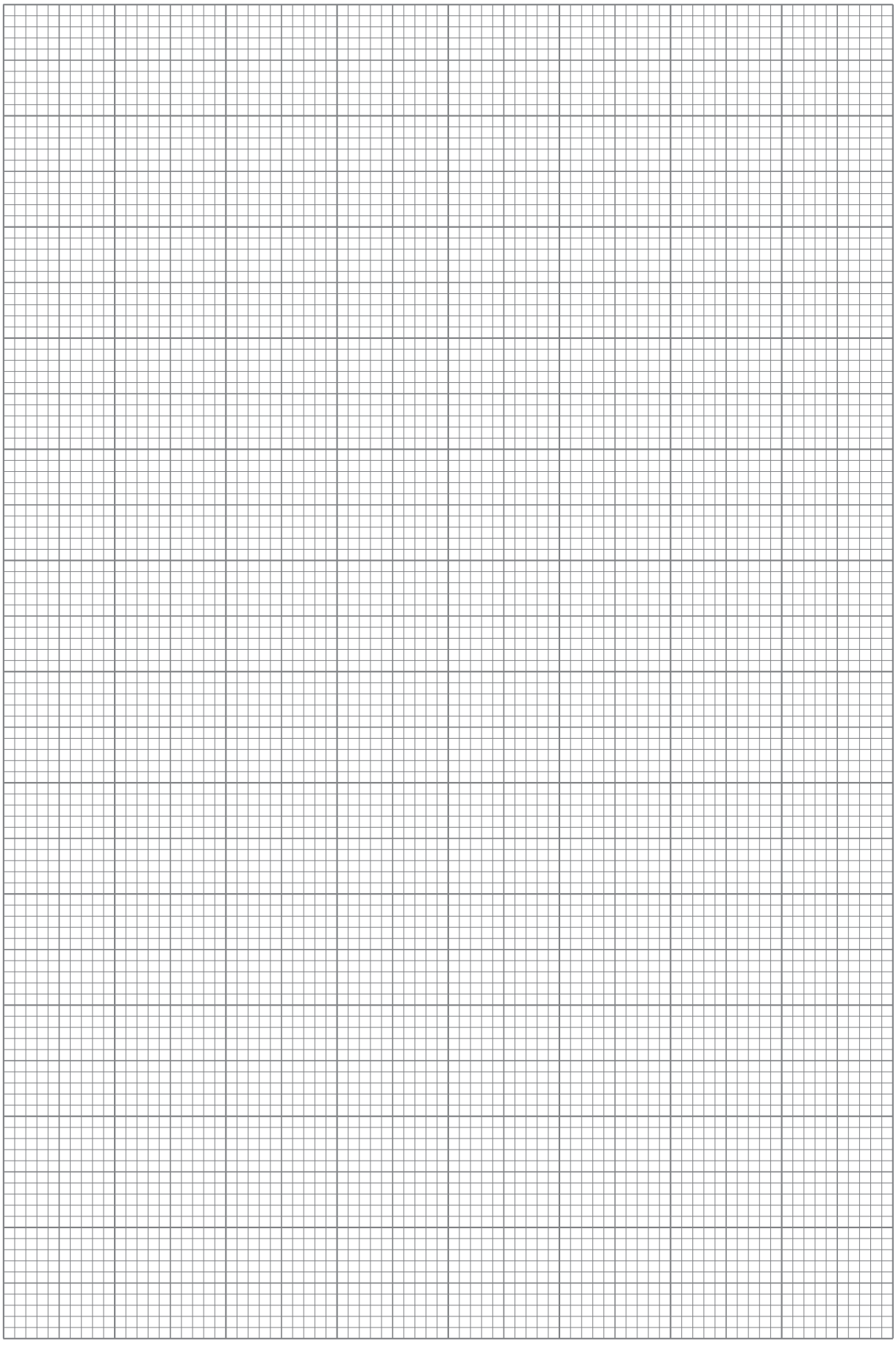
To meet the minimum demands of the motorsport teams, the engineering company must make at least 100 of each type of kit every month.

7 (a) Using a graphical method on the grid opposite, find the number of each type of kit that the engineering company should make every month, in order to maximise its total monthly profit.

Show clearly how you obtain your answer.

[8 marks]

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7 (b) Give a reason why the engineering company may not be able to make the number of each kit that you found in part **(a)**.

[1 mark]

7 (c) During one particular month the engineering company removes the need to make at least 100 of each type of kit.

Explain whether or not this has an effect on your answer to part **(a)**.

[2 marks]

Turn over for the next question

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8 Daryl and Clare play a zero-sum game.

The game is represented by the following pay-off matrix for Daryl.

		Clare		
	Strategy	**X**	**Y**	**Z**
Daryl	**A**	-1	3	4
	B	3	2	1

8 (a) (i) Show that the game does not have a stable solution.

[3 marks]

8 (a) (ii) The value of the game for Daryl is V.

Using your answer to part **(a)(i)**, state the interval in which V lies.

[1 mark]

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8 (b) Daryl plays strategy **A** with probability $\sin^2\theta$ and strategy **B** with probability $\cos^2\theta$, where $0° \leq \theta \leq 90°$

8 (b) (i) Show that the expected gain for Daryl when Clare plays strategy **X** is

$$3 - 4\sin^2\theta$$

[2 marks]

8 (b) (ii) Find an expression in terms of $\sin^2\theta$ for the expected gain for Daryl when Clare plays strategy **Y**.

[1 mark]

8 (b) (iii) Find an expression in terms of $\sin^2\theta$ for the expected gain for Daryl when Clare plays strategy **Z**.

[1 mark]

Question 8 continues on the next page

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8 (c) Daryl plays her optimal mixed strategy.

Using your answers to part **(b)**, find the value of θ, giving your answer to three significant figures.

[4 marks]

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