AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Statistics

Friday 13 June 2025 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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9	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The continuous random variable X has standard deviation 16

Find $\text{Var}(5X - 12)$

Circle your answer.

[1 mark]

80 400 6388 (6400)

2 The discrete random variable Y has probability distribution

y	-6	2	3	18
$\text{P}(Y = y)$	0.56	0.02	0.21	0.21

Find the mode of Y

Circle your answer.

[1 mark]

(-6) 0.21 0.56 18

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3 Cosmo is carrying out a hypothesis test.

He rejects the null hypothesis.

State which of the following statements is correct.

Tick (✓) **one** box.

[1 mark]

He could have made a Type I error but not a Type II error. ✓

He could have made a Type II error but not a Type I error.

He could have made both a Type I error and a Type II error.

He has made neither a Type I nor a Type II error.

Turn over for the next question

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4 The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ \sin 3x & 0 \le x \le \frac{\pi}{6} \\ 1 & x > \frac{\pi}{6} \end{cases}$$

4 (a) Find $P(0.1 < X < 0.3)$

Give your answer to three decimal places.

[2 marks]

$P(0.1 < X < 0.3) = F(0.3) - F(0.1)$

$= \sin(3 \times 0.3) - \sin(3 \times 0.1)$

$= \sin 0.9 - \sin 0.3$

$= 0.78333... - 0.29552... = 0.48781...$

$= 0.488$ (3 d.p.)

4 (b) Show that the median of X is equal to $\frac{\pi}{18}$

[2 marks]

Median m satisfies $F(m) = 0.5$

$\sin(3m) = 0.5$

$3m = \frac{\pi}{6}$ (since $0 \le 3m \le \frac{\pi}{2}$)

$m = \frac{\pi}{18}$ as required

Do not write outside the box

4 (c) Find $f(x)$, the probability density function of X

[2 marks]

$f(x) = F'(x) = \frac{d}{dx}(\sin 3x) = 3\cos 3x$

$$f(x) = \begin{cases} 3\cos 3x & 0 \leq x \leq \frac{\pi}{6} \\ 0 & \text{otherwise} \end{cases}$$

Turn over for the next question

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5 The discrete random variable D has a discrete uniform distribution taking values 1, 2, 3, …, n

The continuous random variable R has a rectangular distribution taking values between n and $n + 1$

The random variables D and R are independent.

5 (a) Show that $\text{Var}(D + R) = \frac{n^2}{12}$

[3 marks]

$\text{Var}(D) = \frac{n^2 - 1}{12}$

$\text{Var}(R) = \frac{((n + 1) - n)^2}{12} = \frac{1}{12}$

D and R are independent, so $\text{Var}(D + R) = \text{Var}(D) + \text{Var}(R)$

$\text{Var}(D + R) = \frac{n^2 - 1}{12} + \frac{1}{12}$

$= \frac{n^2}{12}$ as required

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5 (b) It is given that $\mathrm{E}(D + R) = \dfrac{n^2 + 44}{12}$

Show that $n = 16$

[3 marks]

$E(D) = \frac{n+1}{2}$, $E(R) = \frac{n+(n+1)}{2} = \frac{2n+1}{2}$

$E(D+R) = \frac{n+1}{2} + \frac{2n+1}{2} = \frac{3n+2}{2}$

So $\frac{3n+2}{2} = \frac{n^2+44}{12}$

$6(3n+2) = n^2 + 44$

$18n + 12 = n^2 + 44$

$n^2 - 18n + 32 = 0$

$(n-2)(n-16) = 0$

$n = 2$ or $n = 16$

Reject $n = 2$, so $n = 16$ as required

Turn over for the next question

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6 Students are given a questionnaire asking them about their experiences while studying at their university.

Each student's answers are used to calculate that student's satisfaction score for their university.

The satisfaction scores, X, for a particular university are known to have a normal distribution.

A random sample of 11 students from the university is taken and their satisfaction scores are recorded. The summarised results are

$$\sum x = 1003.2 \quad \text{and} \quad \sum x^2 = 91747.84$$

6 (a) Construct a 90% confidence interval for the population mean satisfaction score for the university.

Give your values to two decimal places.

[5 marks]

$\bar{x} = \frac{1003.2}{11} = 91.2$

$s^2 = \frac{1}{10}\left(91747.84 - \frac{1003.2^2}{11}\right) = \frac{256}{10} = 25.6$

$s = \sqrt{25.6} = 5.0596...$

$\nu = 11 - 1 = 10$; 90% confidence $\Rightarrow t_{10} = 1.812$

CI: $\bar{x} \pm t\frac{s}{\sqrt{n}} = 91.2 \pm 1.812 \times \frac{\sqrt{25.6}}{\sqrt{11}}$

$= 91.2 \pm 2.7642...$

90% confidence interval = (88.44, 93.96)

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6 (b) The university claims that the mean satisfaction score is 90

Test the university's claim, using the 10% level of significance.

[3 marks]

H_0: $\mu = 90$ H_1: $\mu \neq 90$ (μ = mean satisfaction score)

A two-tailed test at the 10% level matches the 90% CI

90 lies within the confidence interval (88.44, 93.96)

($t = \frac{91.2 - 90}{\sqrt{(25.6 \div 11)}} = 0.787$, $-1.812 < 0.787 < 1.812$)

Do not reject H_0

There is insufficient evidence at the 10% level to suggest that the mean satisfaction score for the university is not 90, so there is no reason to doubt the university's claim.

Turn over for the next question

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7 A company sells a particular type of game.

In the game, a player can score 0, 10 or 100 points.

The probability of scoring 10 points is 0.37

The probability of scoring 100 points is 0.04

For a new edition of the game, the company is considering replacing the points scored per game X with X^n, where n is a positive constant, while leaving the probabilities unchanged.

The company wants the mean score of the new edition to be equal to 50 times the mean score of the original game.

7 (a) Find the possible points a player can score in the new edition.

Give your values to the nearest whole number.

[7 marks]

X = points scored in the original game

$P(X = 0) = 1 - 0.37 - 0.04 = 0.59$

x	0	10	100
x^n	0	10^n	100^n
$P(X = x)$	0.59	0.37	0.04

$E(X) = 0 \times 0.59 + 10 \times 0.37 + 100 \times 0.04 = 3.7 + 4 = 7.7$

$E(X^n) = 0 \times 0.59 + 10^n \times 0.37 + 100^n \times 0.04$

Require $E(X^n) = 50 \times E(X) = 50 \times 7.7 = 385$

$0.37(10^n) + 0.04(100^n) = 385$

Since $100^n = (10^n)^2$, let $u = 10^n$:

$0.04u^2 + 0.37u - 385 = 0$

$u = \dfrac{-0.37 \pm \sqrt{(0.37^2 + 4 \times 0.04 \times 385)}}{2 \times 0.04}$

$u = 93.591...$ ($u = 10^n > 0$, so reject the negative root)

$n = \log_{10} 93.591... = 1.9712...$

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Scores in the new edition:

$0^n = 0$

$10^n = 93.591... \approx 94$

$100^n = (10^n)^2 = 8759.3... \approx 8759$

Possible points: 0, 94 and 8759

7 (b) A prototype of the new edition is produced for the company to assess.

For the new edition, the company now decides that it wants

- the points scored changed so that they are all multiples of 10
- the mean score to be at least 50 times the mean score of the original game.

State how your answer to part **(a)** can be refined.

[1 mark]

Round each score up to the nearest multiple of 10, so the scores become 0, 100 and 8760 (mean is then at least 385).

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8 In a village there are two coffee shops *A* and *B*, and two restaurants *C* and *D*

A random sample of 300 villagers is asked to state their favourite coffee shop and their favourite restaurant.

130 villagers said that their favourite coffee shop is *A*

160 villagers said that their favourite restaurant is *D*

88 villagers said that their favourite coffee shop is *B* and their favourite restaurant is *C*

Elaine claims that there is an association between favourite coffee shop and favourite restaurant.

Test Elaine's claim, using the 5% level of significance.

[8 marks]

H_0: there is no association between favourite coffee shop and favourite restaurant

H_1: there is an association between favourite coffee shop and favourite restaurant

O	C	D	Total
A	52	78	130
B	88	82	170
Total	140	160	300

E	C	D
A	60.667	69.333
B	79.333	90.667

E = row total × column total ÷ 300, e.g. 130 × 140 ÷ 300 = 60.667

All E ≥ 5; 2 × 2 table, so use Yates' correction

$$\chi^2 = \Sigma \frac{(|O - E| - 0.5)^2}{E}$$

$$= \frac{(|52 - 60.667| - 0.5)^2}{60.667} + \frac{(|78 - 69.333| - 0.5)^2}{69.333}$$

$$+ \frac{(|88 - 79.333| - 0.5)^2}{79.333} + \frac{(|82 - 90.667| - 0.5)^2}{90.667}$$

$$= 1.0994 + 0.9619 + 0.8407 + 0.7356$$

$$= 3.6376... = 3.638$$

$\nu = (2 - 1)(2 - 1) = 1$; 5% critical value $\chi^2_1 = 3.841$

3.638 < 3.841, so not in the critical region

Do not reject H_0

There is insufficient evidence at the 5% level to suggest that there

Do not write outside the box

is an association between favourite coffee shop and favourite restaurant, so Elaine's claim is not supported.

Turn over for the next question

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9 A company sends out text messages to customers at random times during working hours.

The mean number of text messages sent per 30 minutes is 60

One text message being sent is independent of any other text message being sent.

The company sends text messages at a constant mean rate and sends them individually.

9 (a) Using integration, show that the mean time between text messages being sent is 0.5 minutes.

[6 marks]

X = time (minutes) between consecutive text messages

Rate $\lambda = \frac{60}{30} = 2$ text messages per minute

$X \sim Exp(2)$, $f(x) = 2e^{-2x}$ for $x \geq 0$

$E(X) = \int_0^\infty 2x\, e^{-2x}\, dx = \lim_{n\to\infty} \int_0^n 2x\, e^{-2x}\, dx$

By parts: $u = x$, $\frac{dv}{dx} = 2e^{-2x} \Rightarrow \frac{du}{dx} = 1$, $v = -e^{-2x}$

$\int_0^n 2x\, e^{-2x}\, dx = [\, -x\, e^{-2x}\,]_0^n - \int_0^n -e^{-2x}\, dx$

$= [\, -x\, e^{-2x} - \frac{1}{2} e^{-2x}\,]_0^n$

$= (\, -n\, e^{-2n} - \frac{1}{2} e^{-2n}\,) - (\, -0 \times e^0 - \frac{1}{2} e^0\,)$

$= \frac{1}{2} - n\, e^{-2n} - \frac{1}{2} e^{-2n}$

As $n \to \infty$, $n\, e^{-2n} \to 0$ and $e^{-2n} \to 0$

$E(X) = \lim_{n\to\infty} (\, \frac{1}{2} - n\, e^{-2n} - \frac{1}{2} e^{-2n}\,) = \frac{1}{2}$

Mean time between text messages = 0.5 minutes, as required

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9 (b) Show that the probability that the company sends more than 130 text messages in an hour is 0.169, correct to three decimal places.

[2 marks]

Y = number of text messages sent in one hour

$Y \sim Po(2 \times 60)$, i.e. $Y \sim Po(120)$

$P(Y > 130) = 1 - P(Y \leq 130) = 1 - 0.8315$

$= 0.1685 = 0.169$ (3 d.p.) as required

9 (c) The company sends out text messages during a 7-hour interval on working days.

Find the probability that the company sends more than 130 text messages in an hour in exactly 2 of the 7 hours.

Give your answer to two decimal places.

[3 marks]

A = number of the 7 hours with more than 130 text messages

$A \sim B(7, 0.169)$

$P(A = 2) = {}^{7}C_{2} \times 0.169^{2} \times (1 - 0.169)^{5}$

$= 21 \times 0.028561 \times 0.39628...$

$= 0.23768... = 0.24$ (2 d.p.)

END OF QUESTIONS

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