# A-level
# FURTHER MATHEMATICS
# 7367/3S

Paper 3 Statistics

**Mark scheme**

June 2025

Version: 1.0 Final

256A7367/3S/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

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# Mark scheme instructions to examiners

## General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

**Key to mark types**

| | |
|---|---|
| M | mark is for method |
| R | mark is for reasoning |
| A | mark is dependent on M marks and is for accuracy |
| B | mark is independent of M marks and is for method and accuracy |
| E | mark is for explanation |
| F | follow through from previous incorrect result |

**Key to mark scheme abbreviations**

| | |
|---|---|
| CAO | correct answer only |
| CSO | correct solution only |
| ft | follow through from previous incorrect result |
| 'their' | indicates that credit can be given from previous incorrect result |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| NMS | no method shown |
| PI | possibly implied |
| sf | significant figure(s) |
| dp | decimal place(s) |
| ISW | Ignore Subsequent Workings |

Examiners should consistently apply the following general marking principles:

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**Diagrams**

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

**Work erased or crossed out**

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

**Choice**

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

## AS/A-level Maths/Further Maths assessment objectives

| AO | | Description |
|---|---|---|
| **AO1** | AO1.1a | Select routine procedures |
| | AO1.1b | Correctly carry out routine procedures |
| | AO1.2 | Accurately recall facts, terminology and definitions |
| **AO2** | AO2.1 | Construct rigorous mathematical arguments (including proofs) |
| | AO2.2a | Make deductions |
| | AO2.2b | Make inferences |
| | AO2.3 | Assess the validity of mathematical arguments |
| | AO2.4 | Explain their reasoning |
| | AO2.5 | Use mathematical language and notation correctly |
| **AO3** | AO3.1a | Translate problems in mathematical contexts into mathematical processes |
| | AO3.1b | Translate problems in non-mathematical contexts into mathematical processes |
| | AO3.2a | Interpret solutions to problems in their original context |
| | AO3.2b | Where appropriate, evaluate the accuracy and limitations of solutions to problems |
| | AO3.3 | Translate situations in context into mathematical models |
| | AO3.4 | Use mathematical models |
| | AO3.5a | Evaluate the outcomes of modelling in context |
| | AO3.5b | Recognise the limitations of models |
| | AO3.5c | Where appropriate, explain how to refine models |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **1** | Circles the 4th answer. | 1.1b | B1 | 6400 |
| | **Question total** | | **1** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **2** | Circles the 1st answer. | 2.2a | B1 | $-6$ |
| | **Question total** | | **1** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **3** | Ticks the 1st box. | 1.2 | B1 | He could have made a Type I error but not a Type II error. |
| | **Question total** | | **1** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **4(a)** | Obtains<br>$\sin(3 \times 0.3) - \sin(3 \times 0.1)$<br>**PI**<br>Condone missing brackets or poor bracketing<br>eg $\sin 3(0.3) - \sin 3(0.1)$ | 1.1a | M1 | $F(0.3) - F(0.1)$<br>$= \sin(3 \times 0.3) - \sin(3 \times 0.1)$<br>$= 0.488$ |
| | Obtains **AWRT** 0.488 | 1.1b | A1 | |
| | **Subtotal** | | **2** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **4(b)** | Obtains $\sin(3 \times \text{median}) = 0.5$ | 1.1a | M1 | $\sin(3m) = 0.5$ |
| | Completes reasoned argument to show median = $\frac{\pi}{18}$<br>Must see $3 \times \text{median} = \frac{\pi}{6}$<br>before final answer | 2.1 | R1 | $3m = \frac{\pi}{6}$<br>$m = \frac{\pi}{18}$ |
| | **Subtotal** | | **2** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **4(c)** | Obtains $3\cos(3x)$ | 1.1a | M1 | $f(x) = \begin{cases} 3\cos(3x) & 0 \le x \le \frac{\pi}{6} \\ 0 & \text{otherwise} \end{cases}$ |
| | Obtains correct fully defined function for $f(x)$ including $f(x) =$ | 1.1b | A1 | |
| | **Subtotal** | | **2** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **6** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **5(a)** | Obtains $\frac{n^2-1}{12}$ or $\frac{(n+1-n)^2}{12}$ **OE** | 1.1a | M1 | $\text{Var}(D) = \frac{n^2-1}{12}$<br>$\text{Var}(R) = \frac{(n+1-n)^2}{12} = \frac{1}{12}$<br>$\text{Var}(D+R) = \frac{n^2-1}{12} + \frac{1}{12}$<br>$\text{Var}(D+R) = \frac{n^2}{12}$ |
| | Obtains $\frac{n^2-1}{12}$ and $\frac{(n+1-n)^2}{12}$ **OE** | 1.1b | A1 | |
| | Completes reasoned argument to obtain $\text{Var}(D+R) = \frac{n^2}{12}$<br>$\text{Var}(D)$ and $\text{Var}(R)$ do not need to be explicitly identified but the addition needs to be seen before the simplified final answer<br><br>An unsimplified $\text{Var}(R)$ in terms of $n$ needs to be seen | 2.1 | R1 | |
| | **Subtotal** | | **3** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **5(b)** | Obtains<br>$\text{E}(D+R) = \frac{n+1}{2} + \frac{n+(n+1)}{2}$ **OE**<br>**PI** | 1.1b | B1 | $\text{E}(D+R) = \frac{n+1}{2} + \frac{n+(n+1)}{2}$<br>$\text{E}(D+R) = \frac{3n}{2} + 1$<br>$\frac{3n}{2} + 1 = \frac{n^2+44}{12}$<br>$\frac{n^2}{12} - \frac{3n}{2} + \frac{8}{3} = 0$<br>$2n^2 - 36n + 64 = 0$<br>$n = 2, 16$<br>$n = 16$ |
| | Forms an equation using<br>their $\text{E}(D+R) = \frac{n^2+44}{12}$<br>Their $\text{E}(D+R)$ must be labelled or correct | 1.1a | M1 | |
| | Completes reasoned argument by solving a correct equation to obtain $n = 16$<br><br>$n = 2$ must be seen and rejected<br>Allow any or missing reason for rejection | 2.1 | R1 | |
| | **Subtotal** | | **3** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **6** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **6(a)** | Obtains $\bar{x} = \frac{1003.2}{11}$ **OE**<br>**PI** | 1.1b | B1 | $\bar{x} = \frac{1003.2}{11} = 91.2$<br>$s^2 = \frac{1}{10}\left(91747.84 - \frac{1003.2^2}{11}\right)$<br>$s^2 = 25.6$<br>$t_{10}(95\%) = 1.812$<br>$91.2 \pm 1.812 \times \sqrt{\frac{25.6}{11}}$<br>(88.44, 93.96) |
| | Obtains $s^2 = 25.6$ **OE** or<br>$s =$ **AWRT** 5.06<br>**PI** | 1.1b | B1 | |
| | Obtains $t_{10}(95\%) =$ **AWRT** 1.81<br>**PI** | 2.2a | B1 | |
| | Evaluates<br>their $\bar{x}$ + their $t \times \frac{\text{their } s}{\sqrt{11}}$ or<br>$\bar{x}$ – their $t \times \frac{\text{their } s}{\sqrt{11}}$<br>where $t$ is a value between 0 and 4<br>Condone $z$ value<br>Condone $\sigma$ for $s$<br>**PI** | 1.1a | M1 | |
| | Obtains correct confidence interval.<br>**AWRT** 88.44 and 93.96<br>Allow **AWRT** 88.43 and 93.97 only if their $t$ is stated as 1.8125 | 1.1b | A1 | |
| | **Subtotal** | | **5** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **6(b)** | States both hypotheses using correct language | 2.5 | B1 | $H_0 : \mu = 90$<br>$H_1 : \mu \neq 90$ |
| | Evaluates the *t* model by correctly identifying whether 90 lies within their confidence interval and makes a correct corresponding conclusion whether to reject $H_0$<br>or<br>Correctly calculates *t* test statistic **AWRT** 0.8, compares it to their *t* value and makes a correct corresponding conclusion whether to reject $H_0$<br>**OE**<br>Condone Accept $H_0$/Reject $H_1$ | 3.5a | M1 | 90 is in the confidence interval<br>Do not reject $H_0$<br><br>Insufficient evidence to suggest that the mean satisfaction score for the university is not 90 |
| | Concludes, from a fully correct comparison, in context by referring to mean score<br>(The conclusion must not be definite. eg use of suggest/support)<br><br>Note an award of B0 M1 R1 is possible | 2.2b | R1 | |
| | **Subtotal** | | **3** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **8** | |

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td rowspan="7">7(a)</td><td>Obtains correct probability distribution table or function for $X$ or $X^n$<br>PI</td><td>3.3</td><td>M1</td><td rowspan="7"><table><tr><td>$x$</td><td>0</td><td>10</td><td>100</td></tr><tr><td>$x^n$</td><td>0</td><td>$10^n$</td><td>$100^n$</td></tr><tr><td>P($X = x$)</td><td>0.59</td><td>0.37</td><td>0.04</td></tr></table><br>E($X$) = 0 × 0.59 + 10 × 0.37 + 100 × 0.04<br>E($X$) = 7.7<br>E($X^n$) = $10^n$ × 0.37 + $100^n$ × 0.04<br>0.37($10^n$) + 0.04($100^n$) = 50 × 7.7<br>0.04($10^n$)$^2$ + 0.37($10^n$) – 385 = 0<br>$10^n$ = 93.59<br>$n$ = $\log_{10}$ 93.59<br>$n$ = 1.97<br>New scores = 0, 94, 8759</td></tr>
<tr><td>Uses their discrete random variable model to obtain E($X$) or E($X^n$)</td><td>3.4</td><td>M1</td></tr>
<tr><td>Obtains<br>E($X$) = 10 × 0.37 + 100 × 0.04<br>and<br>E($X^n$) = $10^n$ × 0.37 + $100^n$ × 0.04<br>OE</td><td>1.1b</td><td>A1</td></tr>
<tr><td>Forms equation<br>their E($X^n$) = 50 × their E($X$)</td><td>3.1b</td><td>M1</td></tr>
<tr><td>Obtains correct quadratic equation for their 7.7 in terms of $10^n$<br>0.04($10^n$)$^2$ + 0.37($10^n$) = 385<br>OE<br>PI by sight of AWRT 94 or AWRT 2</td><td>1.1a</td><td>M1</td></tr>
<tr><td>Obtains $10^n$ = AWRT 94<br>PI</td><td>1.1b</td><td>A1</td></tr>
<tr><td>Obtains the scores for the new edition of the game, 0, 94, 8759</td><td>3.2a</td><td>A1</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>7</td><td></td></tr>
</table>

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **7(b)** | Refines, or describes how to refine, the correct scores from part **(a)** to give scores with a mean value at least 385 and all scores are multiples of 10<br><br>eg round all the scores **up** to the nearest 10 or round $n$ to 2<br><br>Note for 0, 90, 8760 award E0 but for e.g. 10, 90, 8760 award E1 | 3.5c | E1 | New scores = 0, 100, 8760 |
| | **Subtotal** | | **1** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **8** | |

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td rowspan="6">8</td><td>States both hypotheses using correct language OE. Variables need to be stated in at least the null hypothesis.</td><td>2.5</td><td>B1</td><td rowspan="6">$H_0$: There is no association between favourite coffee shop and favourite restaurant.<br>$H_1$: There is an association between favourite coffee shop and favourite restaurant.<br><br><table><tr><td>$O$</td><td>$C$</td><td>$D$</td></tr><tr><td>$A$</td><td>52</td><td>78</td></tr><tr><td>$B$</td><td>88</td><td>82</td></tr></table><br><table><tr><td>$E$</td><td>$C$</td><td>$D$</td></tr><tr><td>$A$</td><td>$60\frac{2}{3}$</td><td>$69\frac{1}{3}$</td></tr><tr><td>$B$</td><td>$79\frac{1}{3}$</td><td>$90\frac{2}{3}$</td></tr></table><br>$\sum\frac{(\lvert O-E\rvert-0.5)^2}{E}=\frac{\left(\left\lvert 52-60\frac{2}{3}\right\rvert-0.5\right)^2}{60\frac{2}{3}}+\frac{\left(\left\lvert 78-69\frac{1}{3}\right\rvert-0.5\right)^2}{69\frac{1}{3}}+\frac{\left(\left\lvert 88-79\frac{1}{3}\right\rvert-0.5\right)^2}{79\frac{1}{3}}+\frac{\left(\left\lvert 82-90\frac{2}{3}\right\rvert-0.5\right)^2}{90\frac{2}{3}}$<br>= 3.638<br><br>$\chi^2$ cv for 1 df = 3.841<br><br>3.638 < 3.841<br><br>Do not reject $H_0$<br><br>Insufficient evidence to suggest that there is an association between favourite coffee shop and favourite restaurant.</td></tr>
<tr><td>Obtains correct observed or expected frequencies<br>Allow expected frequencies to be rounded to 3 s.f.<br>PI</td><td>3.3</td><td>M1</td></tr>
<tr><td>Obtains correct observed and expected frequencies<br>Allow expected frequencies to be rounded to 3 s.f.<br>PI</td><td>1.1b</td><td>A1</td></tr>
<tr><td>Calculates test statistic<br>$\sum\frac{(\lvert O-E\rvert-0.5)^2}{E}$ or $\sum\frac{(O-E)^2}{E}$<br>Condone missing modulus signs<br>OE<br>PI</td><td>3.4</td><td>M1</td></tr>
<tr><td>Obtains correct value of<br>$\sum\frac{(\lvert O-E\rvert-0.5)^2}{E}$ = AWRT 3.6</td><td>1.1b</td><td>A1</td></tr>
<tr><td>Obtains correct critical value<br>AWRT 3.8<br>or<br>obtains $p$ = AWRT 0.056</td><td>2.2a</td><td>B1</td></tr>
<tr><td>Evaluates $\chi^2$-test statistic by comparing their test statistic with AWRT 3.8 or 5.0 or their $p$ (a test statistic must have been calculated) with 0.05 and makes a correct corresponding conclusion whether to reject $H_0$<br>Condone Accept $H_0$ or reject $H_1$</td><td>3.5a</td><td>M1</td></tr>
</table>

| | | | | |
|---|---|---|---|---|
| | Concludes, from a fully correct comparison, in context by referring to association between coffee shop and restaurant **OE**<br>(The conclusion must not be definite. eg use of suggest/support) | 2.2b | R1 | |
| | | | | |
| | **Question total** | | **8** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **9(a)** | States or uses $X \sim$ Exp(60), Exp(120) or Exp(2)<br>**PI** by use of correct pdf | 3.3 | M1 | $X \sim$ Exp(60)<br>$\mathrm{E}(X) = \lim_{n\to\infty}\left(\int_0^n 60x\mathrm{e}^{-60x}\ \mathrm{d}x\right)$ |
| | Obtains $\int \lambda x\mathrm{e}^{-\lambda x}\ (\mathrm{d}x)$,or<br>$\int \frac{1}{\lambda}x\mathrm{e}^{-\frac{1}{\lambda}x}\ (\mathrm{d}x)$ for their $\lambda$ | 3.4 | M1 | $= \lim_{n\to\infty}\left(\left[-x\mathrm{e}^{-60x}\right]_0^n - \int_0^n -\mathrm{e}^{-60x}\ \mathrm{d}x\right)$<br>$= \lim_{n\to\infty}\left(\left[-x\mathrm{e}^{-60x} - \frac{1}{60}\mathrm{e}^{-60x}\right]_0^n\right)$ |
| | Obtains $-x\mathrm{e}^{-\lambda x} - \int -\mathrm{e}^{-\lambda x}\ (\mathrm{d}x)$ or<br>$-x\mathrm{e}^{-\frac{1}{\lambda}x} - \int -\mathrm{e}^{-\frac{1}{\lambda}x}\ (\mathrm{d}x)$ for their $\lambda$<br>**PI**<br>The two terms may be seen on different lines | 1.1a | M1 | $= \left(-n\times 0 - \frac{1}{60}\times 0\right)$<br>$-\left(-0\times\mathrm{e}^{-60\times 0} - \frac{1}{60}\times\mathrm{e}^{-60\times 0}\right)$<br>$= \frac{1}{60}$ of 30 minutes |
| | Obtains $-x\mathrm{e}^{-\lambda x} - \frac{1}{\lambda}\mathrm{e}^{-\lambda x}$ for their $\lambda$<br>The two terms may be seen on different lines | 1.1b | A1F | Mean time = $\frac{1}{60}\times 30$<br>= 0.5 minutes |
| | Applies correct limits to correctly obtain correct value of the mean $\frac{1}{\lambda}$<br>where $\lambda$ = 60, 120 or 2<br>Condone lack of limiting process. | 1.1b | A1 | |
| | Completes reasoned argument to find that the mean time between text messages being sent is 0.5<br>Do not condone the lack of a limiting process nor missing dx<br>For $\lambda$ = 60, need to see $\frac{1}{60}\times 30$<br>For $\lambda$ = 120, need to see $\frac{1}{120}\times 60$<br>For $\lambda$ = 2, need to see $\lambda = \frac{60}{30}$ or 60 per 30 min cancelled to 2 per min | 2.1 | R1 | |
| | **Subtotal** | | **6** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **9(b)** | Use Poisson model $Y \sim$ Po(120) to calculate one of<br>P($Y > 130$) = **AWRT** 0.1685<br>P($Y \leq 130$) = **AWRT** 0.83<br>P($Y \geq 130$) = **AWRT** 0.19<br>P($Y < 130$) = **AWRT** 0.81<br>Condone incorrect labelling | 3.4 | M1 | $Y \sim$ Po(120)<br>P($Y > 130$) = 1 – P($Y \leq 130$)<br>P($Y > 130$) = 1 – 0.8315<br>P($Y > 130$) = 0.1685<br>P($Y > 130$) = 0.169 |
| | Completes reasoned argument to find that the probability that the company sends more than 130 texts in an hour is 0.169, correct to three decimal places.<br><br>Must see either a more accurate value for<br>P($Y > 130$) = **AWRT** 0.1685 or<br>P($Y \leq 130$) = **AWRT** 0.8315, condone truncated 0.8314 provided incorrect 0.1686 **not** also seen,<br>before the final answer with correct labelling **OE**<br><br>The Poisson model used, Po(120), must be stated. | 2.1 | R1 | |
| | **Subtotal** | | **2** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **9(c)** | States or uses $A \sim$ B(7, 0.169)<br>Allow the use of a more accurate value for 0.169 | 3.3 | M1 | $A \sim$ B(7, 0.169)<br>P($A = 2$) = |
| | Calculates<br>$\binom{7}{2} \times 0.169^2 \times (1-0.169)^{7-2}$<br>**PI** | 3.4 | M1 | $\binom{7}{2} \times 0.169^2 \times (1-0.169)^{7-2}$<br>P($A = 2$) = 0.24 |
| | Obtains P($A = 2$) = **AWRT** 0.24<br>Do not ignore subsequent working | 1.1b | A1 | |
| | **Subtotal** | | **3** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **11** | |

| | | | | |
|---|---|---|---|---|
| | **Paper total** | | **50** | |