AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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I declare this is my own work.

A-level

FURTHER MATHEMATICS

Paper 3 Mechanics

Friday 13 June 2025 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

For Examiner's Use	
Question	Mark
1	
2	
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10	
11	
TOTAL	

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 A motorcycle has a maximum speed of 44 m s^{-1} on a straight, horizontal road.

The maximum power of the motorcycle is 55 kilowatts.

Find the magnitude of the total resistance force acting on the motorcycle at its maximum speed on this road.

Circle your answer.

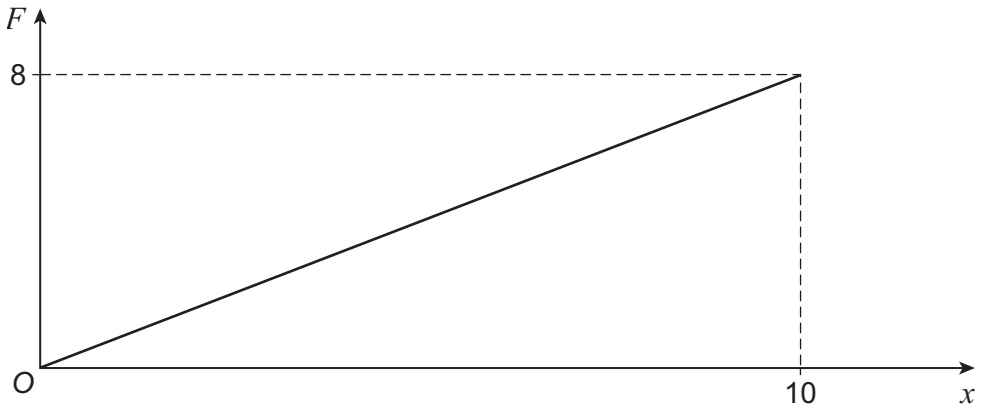
[1 mark]

1.25 N | 125 N | (1250 N) | 2420 N

$P = Fv \Rightarrow 55\,000 = F \times 44 \Rightarrow F = 1250$ N

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2 The graph shows how a force F newtons varies with displacement x metres.



Find the work done by the force as the displacement increases from $x = 5$ to $x = 10$

Circle your answer.

[1 mark]

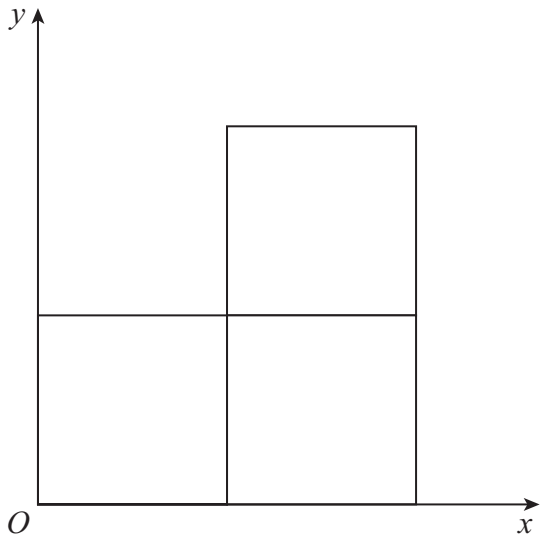
10 J | 30 J | 40 J | 80 J

Turn over for the next question

$F = 0.8x$: at $x = 5$, $F = 4$; at $x = 10$, $F = 8$
Work done = area = $\frac{1}{2}(4 + 8) \times 5 = 30$ J

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3 The diagram shows a uniform lamina made from three identical squares.



The coordinates of the centre of the mass of the lamina are $(\bar{x}, \bar{y})$

Which one of the following statements is correct?

Circle your answer.

[1 mark]

$\bar{x} = \bar{y}$ $\quad$ $\bar{x} > \bar{y}$ $\quad$ $\bar{x} < \bar{y}$ $\quad$ $\bar{x} = \frac{1}{3}\bar{y}$

$\bar{x} = (0.5 + 1.5 + 1.5)/3 = 7/6, \quad \bar{y} = (0.5 + 0.5 + 1.5)/3 = 5/6$
so $\bar{x} > \bar{y}$

Do not write outside the box

4 A ball is released from rest at a point A and falls vertically.

The ball is falling at a speed of 8 m s^{-1} when it hits a horizontal surface.

The ball rebounds vertically at a speed of 5 m s^{-1}

4 (a) Find the coefficient of restitution between the ball and the surface.

[1 mark]

e = speed of separation / speed of approach = $5/8 = 0.625$

4 (b) The ball is then released from rest at a different point B and falls onto the same surface.

The ball rebounds vertically at a speed of 7 m s^{-1}

Find the speed at which the ball was falling when it hit the surface in this case.

[2 marks]

$e = 7/u \Rightarrow 5/8 = 7/u$

$u = 7 \times 8/5 = 56/5$

Speed = 11.2 m s^{-1}

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5 **In this question use g = 9.81 m s^{-2}**

A ball of mass 0.5 kg is projected with an initial speed of 16 m s^{-1} from a point on a horizontal surface.

5 (a) Calculate the initial kinetic energy of the ball.

[1 mark]

$KE = \frac{1}{2} \times 0.5 \times 16^2 = 64$ J

5 (b) During the subsequent motion, the ball reaches a maximum height of 6 metres above the horizontal surface.

5 (b) (i) Find the maximum gravitational potential energy of the ball, relative to the horizontal surface.

[1 mark]

$GPE = mgh = 0.5 \times 9.81 \times 6 = 29.43$ J

5 (b) (ii) Find the minimum speed of the ball during its motion.

[3 marks]

Minimum speed at the maximum height.

Energy: $64 = 29.43 + \frac{1}{2} \times 0.5 \times v^2$

$v^2 = (64 - 29.43)/0.25 = 138.28$

$v = 11.759...$

Minimum speed = 11.8 m s^{-1} (3 s.f.)

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5 (b) (iii) State one assumption that you have made about the forces acting on the ball.

[1 mark]

There is no air resistance acting on the ball.

Turn over for the next question

Do not write outside the box

6 A bead of mass m kg moves on a smooth horizontal wire through a fluid which has density ρ kg m^{-3}

A possible model for the acceleration, a m s^{-2}, of the bead when its speed is v m s^{-1} is

$$a = -km^{\alpha}\rho^{\beta}v^{\gamma}$$

where k is a positive dimensionless constant and α, β and γ are constants.

Use dimensional analysis to find the value of α, the value of β and the value of γ

[3 marks]

$[a] = LT^{-2}$, $[m] = M$, $[\rho] = ML^{-3}$, $[v] = LT^{-1}$

$LT^{-2} = M^{\alpha}(ML^{-3})^{\beta}(LT^{-1})^{\gamma} = M^{\alpha+\beta}L^{-3\beta+\gamma}T^{-\gamma}$

T: $-\gamma = -2 \Rightarrow \gamma = 2$

L: $-3\beta + \gamma = 1 \Rightarrow -3\beta + 2 = 1 \Rightarrow \beta = 1/3$

M: $\alpha + \beta = 0 \Rightarrow \alpha = -1/3$

$\alpha = -1/3$, $\beta = 1/3$, $\gamma = 2$

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7 The force $\mathbf{F} = \begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix}$ acts at the point with coordinates (3, 5, –1)

Show that the magnitude of the moment of this force about the origin is $k\sqrt{6}$ where k is a positive integer to be found.

[3 marks]

Moment $M = r \times F = (3i + 5j - k) \times (2i - 4j + 3k)$

$= (5\times3 - (-1)(-4))\, i - (3\times3 - (-1)(2))\, j + (3(-4) - 5\times2)\, k$

$= (15 - 4)\, i - (9 + 2)\, j + (-12 - 10)\, k = 11i - 11j - 22k$

$|M| = \sqrt{(11^2 + (-11)^2 + (-22)^2)} = \sqrt{(121 + 121 + 484)}$

$= \sqrt{726} = \sqrt{(121 \times 6)} = 11\sqrt{6}$

$\therefore$ magnitude is $k\sqrt{6}$ with $k = 11$

Turn over for the next question

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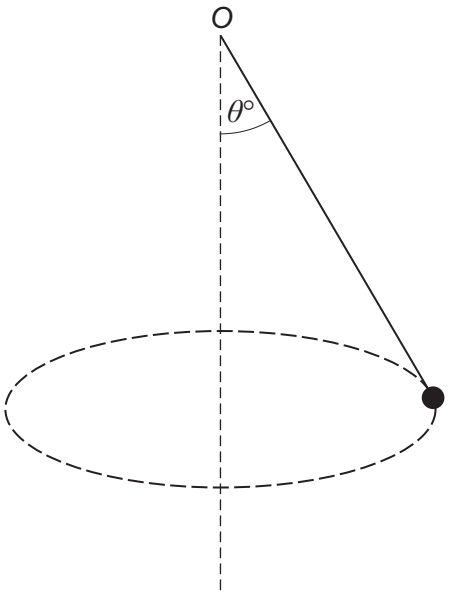
8 **In this question use $g = 9.8$ m s^{-2}**

A conical pendulum consists of a light inextensible string of length 1.2 metres and a small metal sphere of mass 0.2 kg

One end of the string is attached to a fixed point O and the other end of the string is attached to the sphere.

The sphere is set into motion, so that it follows a horizontal circular path of radius 0.6 metres.

The string makes an angle of $\theta°$ to the vertical through O as shown in the diagram.



8 (a) Show that the tension in the string is 2.3 newtons correct to two significant figures.

[3 marks]

sin θ = 0.6/1.2 = 0.5 ⇒ θ = 30

Resolve vertically: T cos 30° = 0.2 × 9.8 = 1.96

T = 1.96 / cos 30° = 2.263...

T = 2.3 N (2 s.f.) as required

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8 (b) Find the speed of the sphere.

[3 marks]

Resolve horizontally (towards centre):

$T \sin 30° = 0.2v^2/0.6$

$v^2 = 3 \times 2.263... \times 0.5 = 3.394...$

$v = 1.842...$

Speed = $1.8\ m\ s^{-1}$ (2 s.f.)

Turn over for the next question

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9 Two discs, *A* and *B*, are moving on a smooth horizontal surface.

The two discs collide.

Disc *A* has mass 0.8 kg and before the collision had velocity $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ m s^{-1}

Disc *B* has mass 1.2 kg and before the collision had velocity $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$ m s^{-1}

After the collision, disc *A* has velocity $\begin{bmatrix} 1.1 \\ 1.5 \end{bmatrix}$ m s^{-1}

9 (a) Find the velocity of disc *B* after the collision.

[3 marks]

Conservation of momentum:

$0.8(5, 3) + 1.2(-2, -3) = 0.8(1.1, 1.5) + 1.2\ v_B$ [as column vectors]

$(4 - 2.4, 2.4 - 3.6) = (0.88, 1.2) + 1.2\ v_B$

$(1.6, -1.2) - (0.88, 1.2) = 1.2\ v_B$

$1.2\ v_B = (0.72, -2.4)$

$v_B = (0.6, -2)$ i.e. $0.6i - 2j$ m s^{-1}

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9 (b) Find the magnitude of the impulse exerted on disc *A* during the collision.

Give your answer to two significant figures.

[3 marks]

$I = m(v - u) = 0.8[(1.1, 1.5) - (5, 3)]$

$= 0.8(-3.9, -1.5) = (-3.12, -1.2)$ N s

$|I| = \sqrt{(3.12^2 + 1.2^2)} = \sqrt{11.1744} = 3.342...$

Magnitude of impulse = 3.3 N s (2 s.f.)

9 (c) The discs are in contact for 0.1 seconds during the collision.

Assume that while the two discs are in contact, the force exerted on disc *B* by disc *A* is constant.

Find the force exerted on disc *B* by disc *A* during the collision.

Give your answer as a vector.

[2 marks]

Impulse on B = −(impulse on A) = (3.12, 1.2) N s

$F \times 0.1 = (3.12, 1.2) \Rightarrow F = (31.2, 12)$

$F = 31.2i + 12j$ N

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10 A uniform semicircular lamina has radius a

10 (a) Show that the centre of mass is a distance $\frac{4a}{3\pi}$ from the straight edge of the lamina.

[5 marks]

Place the straight edge along the x-axis, centre O at the origin.

Curved edge: $x^2 + y^2 = a^2$, so the half-width at height y is $\sqrt{(a^2 - y^2)}$.

Let the mass per unit area be σ. Take a strip parallel to the straight edge at height y, thickness δy: mass $\approx \sigma \times 2\sqrt{(a^2 - y^2)}\,\delta y$

Total mass $M = \sigma \times \frac{1}{2}\pi a^2$

Moments about the straight edge:

$M\bar{y} = \int_0^a \sigma y \times 2\sqrt{(a^2 - y^2)}\,dy$

$= \sigma \int_0^a 2y(a^2 - y^2)^{\frac{1}{2}}\,dy$

$= \sigma \left[-\frac{2}{3}(a^2 - y^2)^{3/2} \right]_0^a$

$= \sigma \left(0 + \frac{2}{3}(a^2)^{3/2} \right) = \frac{2}{3}\sigma a^3$

So $\frac{1}{2}\pi a^2 \sigma \bar{y} = \frac{2}{3}\sigma a^3$

$\bar{y} = (2a^3/3) \div (\pi a^2/2) = 4a^3/(3\pi a^2)$

$\bar{y} = 4a/(3\pi)$

$\therefore$ the centre of mass is $4a/(3\pi)$ from the straight edge, as required.

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10 (b) The lamina is suspended in equilibrium from one end of the straight edge.

Find the acute angle between the straight edge and the vertical.

Give your answer to the nearest degree.

[3 marks]

Suspended from end A of the straight edge, so AG is vertical.

G is a from A along the edge (to the centre) and $4a/(3\pi)$ perpendicular to it.

Angle θ between the straight edge and the vertical:

$\tan\theta = (4a/(3\pi)) / a = 4/(3\pi) = 0.4244...$

$\theta = 22.997...°$

$\theta = 23°$ (nearest degree)

Turn over for the next question

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11 **In this question use $g = 9.8\ \text{m s}^{-2}$**

A rough slope is inclined at an angle of 30° to the horizontal.

A light elastic string has natural length 0.8 metres and modulus of elasticity 60 newtons.

One end of the elastic string is attached to a point *P* on the slope.

The other end of the elastic string is attached to a block of mass 2 kg

The block is released from rest at a point down the slope from *P* where the length of the elastic string is 2.4 metres.

The coefficient of friction between the block and the slope is 0.3

Assume that there is no air resistance.

11 (a) Calculate the elastic potential energy of the elastic string when the block is released.

[2 marks]

Extension = 2.4 − 0.8 = 1.6 m

EPE = $\lambda x^2/(2l)$ = 60 × $1.6^2/(2 \times 0.8)$ = 96 J

11 (b) Find the maximum speed of the block as it moves.

[6 marks]

Block moves up the slope; maximum speed when acceleration is zero.

R = 2 × 9.8 cos 30°, friction F = 0.3 × 2 × 9.8 cos 30° = 5.092... N

At max speed: T = 2 × 9.8 sin 30° + 0.3 × 2 × 9.8 cos 30°

60e/0.8 = 9.8 + 5.092... ⇒ 75e = 14.892...

e = 0.1986 m

Distance moved d = 1.6 − 0.1986 = 1.4014 m

Energy: initial EPE = final EPE + GPE gained + work against friction + KE

EPE at max speed = $60 \times 0.1986^2/(2 \times 0.8)$ = 1.478... J

GPE gained = 2 × 9.8 × 1.4014 sin 30° = 13.73... J

Work against friction = 5.092... × 1.4014 = 7.136... J

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Q11(b) continued

$96 = 1.478... + 13.73... + 7.136... + \frac{1}{2} \times 2 \times v^2$

$v^2 = 73.65...$

$v = 8.582...$

Maximum speed = $8.6\ m\ s^{-1}$ (2 s.f.)

11 (c) The model is refined to include air resistance.

Explain how the distance that the block travels before reaching its maximum speed would change.

[2 marks]

Air resistance acts down the slope, opposing the motion, so the maximum speed (zero acceleration) is reached when the tension is greater, i.e. when the extension is larger.

So the distance travelled before reaching maximum speed would be less.

END OF QUESTIONS

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