AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

# A-level
# FURTHER MATHEMATICS

## Paper 3 Discrete

Friday 13 June 2025 Afternoon Time allowed: 2 hours

**Materials**
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics or Statistics). You will have 2 hours to complete **both** papers.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

| For Examiner's Use | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| **TOTAL** | |

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

**1** The group $A$ is of order 9

The group $B$ is a proper, non-trivial subgroup of $A$

Find the order of $B$

Circle your answer.

**[1 mark]**

1 3 9 18

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**2** The simplex algorithm is used to solve a linear programming problem, which results in the following correct simplex tableau.

| $P$ | $x$ | $y$ | $r$ | $s$ | **value** |
|---|---|---|---|---|---|
| 1 | $\frac{2}{3}$ | 0 | $\frac{5}{3}$ | 0 | 25 |
| 0 | $\frac{5}{3}$ | 0 | $-\frac{1}{3}$ | 1 | 7 |
| 0 | $\frac{1}{3}$ | 1 | $\frac{1}{3}$ | 0 | 5 |

Which one of the following gives the correct optimal values of $P$, $x$ and $y$?

Tick (✓) **one** box.

**[1 mark]**

$P = 1,\ x = \frac{2}{3},\ y = 0$ ☐

$P = 1,\ x = \frac{5}{3},\ y = 1$ ☐

$P = 25,\ x = 0,\ y = 5$ ☐

$P = 25,\ x = 7,\ y = 5$ ☐

**Turn over for the next question**

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**3** The group $(X, *)$ is an abelian group of order 4

The elements of $X$ are

$$e,\ a,\ b \text{ and } c$$

where $e$ is the identity element of $(X, *)$

The following is a partially completed Cayley table for $(X, *)$

| $*$ | $e$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e$ | $e$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $e$ | $c$ | $b$ |
| $b$ | $b$ | | $e$ | $a$ |
| $c$ | $c$ | | | $e$ |

Which one of the following fully completed Cayley tables for $(X, *)$ is correct?

Tick (✓) **one** box.

**[1 mark]**

| $*$ | $e$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e$ | $e$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $e$ | $c$ | $b$ |
| $b$ | $b$ | $a$ | $e$ | $a$ |
| $c$ | $c$ | $b$ | $c$ | $e$ |

☐

| $*$ | $e$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e$ | $e$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $e$ | $c$ | $b$ |
| $b$ | $b$ | $a$ | $e$ | $a$ |
| $c$ | $c$ | $c$ | $b$ | $e$ |

☐

| $*$ | $e$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e$ | $e$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $e$ | $c$ | $b$ |
| $b$ | $b$ | $c$ | $e$ | $a$ |
| $c$ | $c$ | $a$ | $b$ | $e$ |

☐

| $*$ | $e$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e$ | $e$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $e$ | $c$ | $b$ |
| $b$ | $b$ | $c$ | $e$ | $a$ |
| $c$ | $c$ | $b$ | $a$ | $e$ |

☐

*Do not write outside the box*

**4** A water company is planning to install a new sewer system between 5 districts of a city.

The lengths, in miles, of possible sewer pipes between districts is shown in the table.

| | Blacon | Hoole | Lache | Newton | Upton |
|---|---|---|---|---|---|
| Blacon | – | 3.3 | 4.0 | 2.9 | 2.7 |
| Hoole | 3.3 | – | 3.6 | 1.3 | 1.7 |
| Lache | 4.0 | 3.6 | – | 3.4 | 4.3 |
| Newton | 2.9 | 1.3 | 3.4 | – | 1.4 |
| Upton | 2.7 | 1.7 | 4.3 | 1.4 | – |

The water company wants to create a network connecting, directly or indirectly, each of the 5 districts using the minimum possible total length of sewer pipes.

**4 (a)** Find the minimum possible total length of sewer pipes required.

**[3 marks]**

**4 (b)** As part of their planning, the water company carries out a geological survey of the city.

As a result, the water company cannot connect Hoole directly to Upton in the plans for the new sewer system.

Explain the effect, if any, this has on your answer to part **(a)**.

**[2 marks]**

**Turn over ▶**

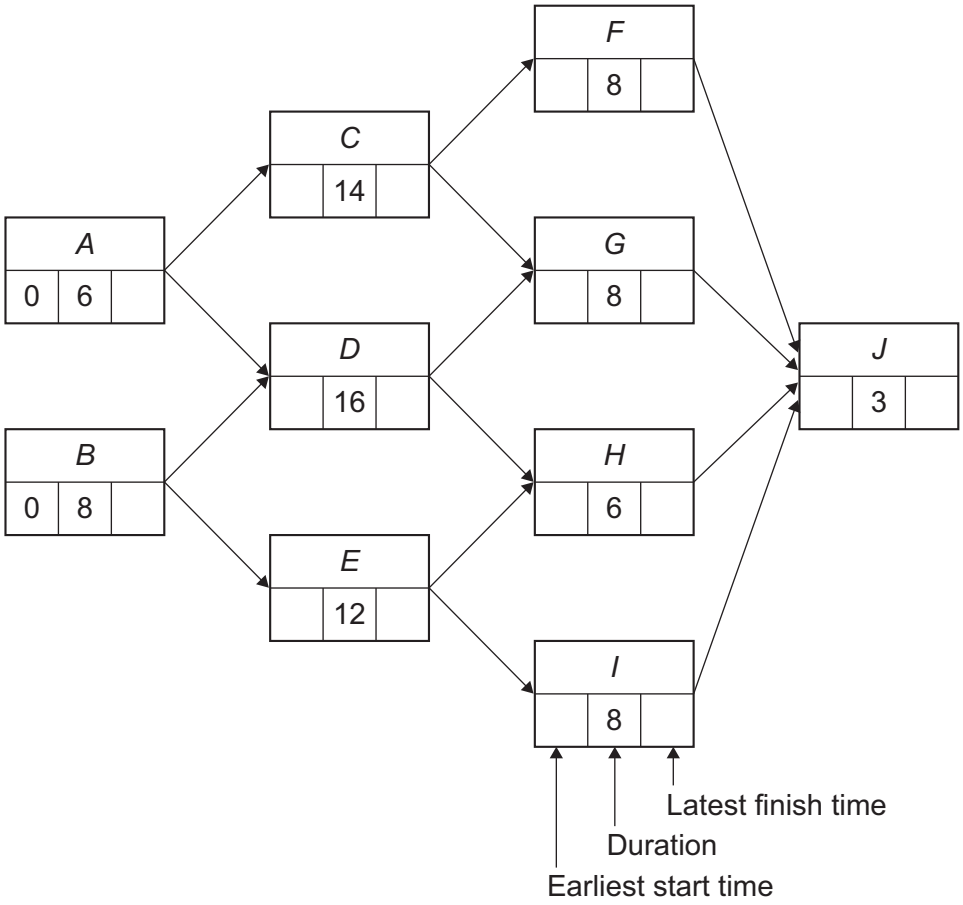
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**5** A project is being planned by Lepton Electricity Installations.

The project is broken down into 10 separate activities *A*, *B*, …, *J* and the activity network for the project is shown in **Figure 1**

The duration of each activity is given in days.

**Figure 1**




**5 (a) (i)** Find the earliest start time and the latest finish time for each activity and write these values on the activity network in **Figure 1**

**[2 marks]**

**5 (a) (ii)** Determine the float of activity *F*

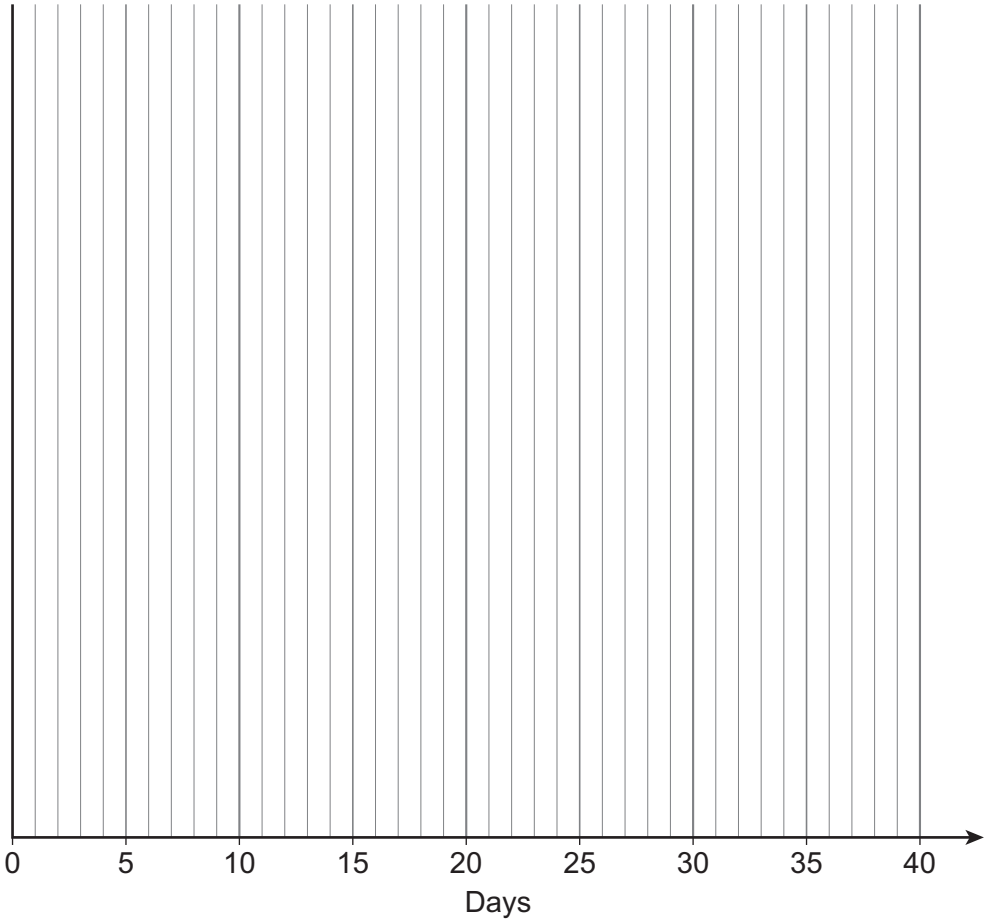
**[1 mark]**

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**5 (b)** On **Figure 2** draw a cascade diagram (Gantt chart) for the project, assuming that each activity starts as early as possible.

**[3 marks]**

**Figure 2**

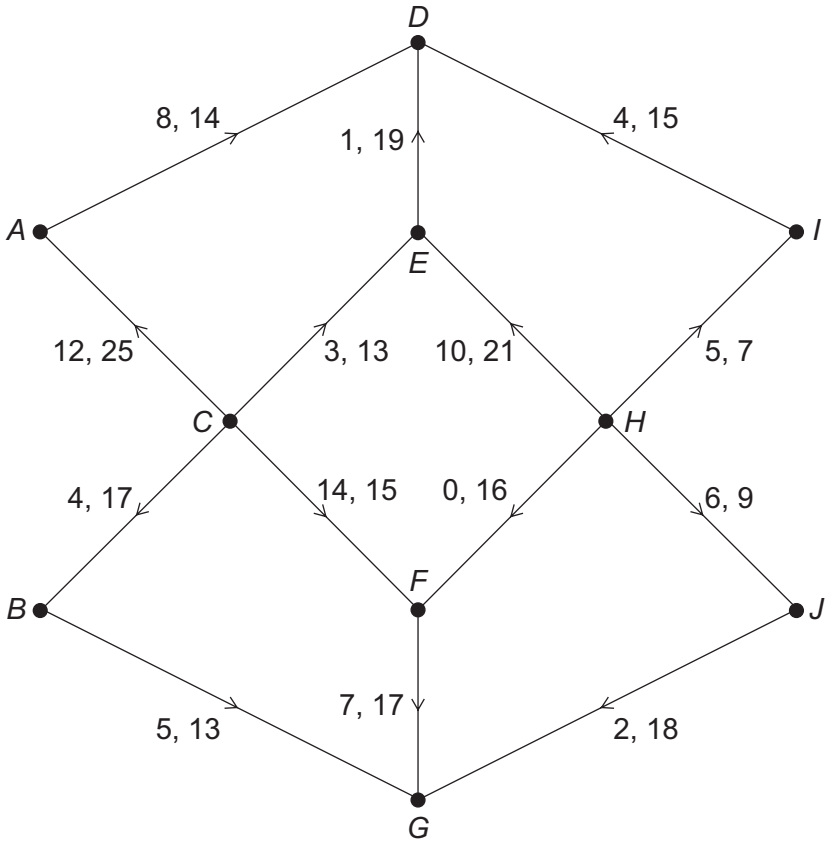



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**6** The network below represents a system of pipes that transport chemicals at a factory.

The numbers on each arc represent the lower and upper capacity of each pipe in litres per second.




**6 (a)** The management team at the factory plan to connect a supersource and a supersink to the network.

Determine which node(s) should connect to the supersource and which node(s) should connect to the supersink.

**[2 marks]**

Node(s) connected to supersource _____

Node(s) connected to supersink _____

*Do not write outside the box*

**6 (b)** Find the value of the cut $\{A, B, C, E, F, H\}\ \{D, G, I, J\}$

**[1 mark]**

**6 (c) (i)** Andrew, a manager at the factory, finds a flow of 79 litres per second through the network. He claims this is the maximum possible flow through the network.

Explain whether this claim is correct.

Fully justify your answer.

**[2 marks]**

**6 (c) (ii)** Andrew also claims that introducing a supersource and a supersink to the network will increase the maximum flow through the network.

Explain why this claim is incorrect.

**[1 mark]**

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**7** A large bakery makes and sells two different types of loaf: malt and rye.

Each day, the bakery makes $x$ malt loaves and $y$ rye loaves.

The bakery wants to maximise the total profit, in £s, it generates each day from these two types of loaf.

The bakery formulates their situation as a linear programming problem by considering:

the total profit, £$P$, generated on all the malt loaves and rye loaves made each day

the total time available to make the loaves each day

the ingredients available to make the loaves each day

a minimum number of each type of loaf made each day.

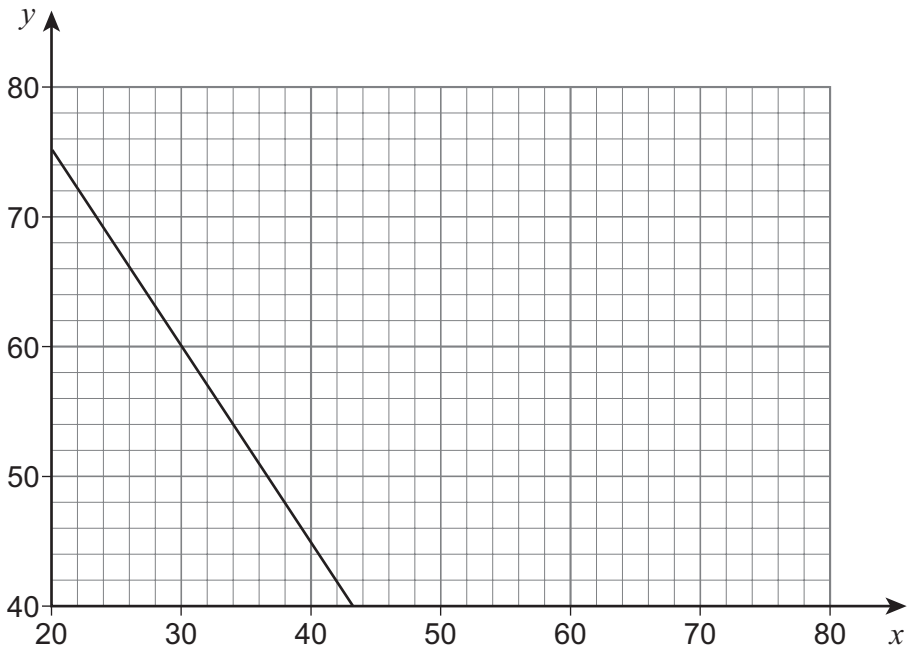
The bakery's linear programming problem is shown below.

Maximise $P = 2x + 3y$

subject to
$$x + 2y \leq 150$$
$$3x + 2y \leq 210$$
$$x \geq 20$$
$$y \geq 40$$

and $x$, $y$ are integers

**7 (a)** The graph shows the line $3x + 2y = 210$



*Do not write outside the box*

**7 (a) (i)** On the graph, draw the line $x + 2y = 150$

**[1 mark]**

**7 (a) (ii)** On the graph, identify the feasible region of the linear programming problem.

**[1 mark]**

**7 (b) (i)** Hence, find the number of malt loaves and the number of rye loaves the bakery should make each day.

**[2 marks]**

Number of malt loaves _____

Number of rye loaves _____

**7 (b) (ii)** Find the maximum total profit, in £s, the bakery will generate each day.

**[1 mark]**

**7 (b) (iii)** Give a reason why your answer to part **(b)(ii)** may not be accurate.

**[1 mark]**

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**8** The connected planar graph $G$ has $n^2$ vertices, $2n$ edges and $3n - 4$ faces, where $n$ is an integer.

**8 (a) (i)** Show that $n = 2$

**[3 marks]**

**8 (a) (ii)** Hence, write down the number of vertices, edges and faces for $G$

**[1 mark]**

Number of vertices _____

Number of edges _____

Number of faces _____

*Do not write outside the box*

**8 (b) (i)** In the case when $G$ is Eulerian, draw $G$

**[1 mark]**

**8 (b) (ii)** Alexie claims that in the case when $G$ is Eulerian, $G$ is also a bipartite graph.

Comment on the validity of Alexie's claim.

**[2 marks]**

**8 (c) (i)** In the case when $G$ is semi-Eulerian, draw $G$

**[1 mark]**

**8 (c) (ii)** Zackary claims that in the case when $G$ is semi-Eulerian, $G$ is also a tree.

Comment on the validity of Zackary's claim.

**[2 marks]**

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**9 (a)** John and Elizabeth play a zero-sum game that does **not** have a stable solution.

This game is represented by the following pay-off matrix for John.

| | | Elizabeth | | |
|---|---|---|---|---|
| | **Strategy** | **X** | **Y** | **Z** |
| John | **A** | 3 | −3 | −1 |
| | **B** | −1 | 4 | 3 |
| | **C** | −2 | 1 | 1 |

**9 (a) (i)** State, with a reason, which strategy John should never play.

**[1 mark]**

**9 (a) (ii)** Find the optimal mixed strategy for John.

Fully justify your answer.

**[6 marks]**

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**Question 9 continues on the next page**

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**9 (b)** John and Elizabeth play a different zero-sum game that does **not** have a stable solution.

This game is represented by the following pay-off matrix for Elizabeth.

| | | John | | |
|---|---|---|---|---|
| | **Strategy** | **$J_1$** | **$J_2$** | **$J_3$** |
| Elizabeth | **$E_1$** | 6 | 2 | 4 |
| | **$E_2$** | 3 | 4 | 3 |
| | **$E_3$** | 7 | 1 | 8 |

To determine her optimal mixed strategy for this game, Elizabeth begins by defining the following variables.

$v$ = the value of the game for Elizabeth

$p_1$ = the probability of Elizabeth playing strategy **$E_1$**

$p_2$ = the probability of Elizabeth playing strategy **$E_2$**

$p_3$ = the probability of Elizabeth playing strategy **$E_3$**

Elizabeth then **starts** to formulate her situation as a linear programming problem by writing

Maximise $P = v$

subject to $p_1 + p_2 + p_3 \leq 1$

$p_1, p_2, p_3 \geq 0$

Write down the **three** remaining inequalities needed for the linear programming problem for Elizabeth's situation to be **fully** formulated.

**[2 marks]**

Inequality 1 _____

Inequality 2 _____

Inequality 3 _____

*Do not write outside the box*

**Turn over for the next question**

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**ANSWER IN THE SPACES PROVIDED**

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**10** The set $S$ contains the elements $(a, b)$ where $a, b \in \mathbb{R}$ **except** $(0, 0)$

The group $G$ is formed by $S$ under the binary operation $\lozenge$ where

$$(a, b) \lozenge (c, d) = (a \times d + b \times c, b \times d - a \times c)$$

**10 (a)** Prove that $(0, 1)$ is the identity element of $G$

**[2 marks]**

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**10 (b)** It is given that $(a, b) \in G$

Find, in terms of $a$ and $b$, the inverse of $(a, b)$ in $G$

**[3 marks]**

**END OF QUESTIONS**

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| Question number | **Additional page, if required.**<br>**Write the question numbers in the left-hand margin.** |
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