AQA

Please write clearly in block capitals.

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I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Mechanics

Friday 7 June 2024 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 A particle moves in a circular path so that at time t seconds its position vector, **r** metres, is given by

$$\mathbf{r} = 4\sin(2t)\mathbf{i} + 4\cos(2t)\mathbf{j}$$

Find the velocity of the particle, in m s^{-1}, when $t = 0$

Circle your answer.

[1 mark]

(8**i**) –8**j** 8**j** 8**i** – 8**j**

$v = dr/dt = 8\cos(2t)i - 8\sin(2t)j$; $t = 0$: $v = 8i$

2 As a particle moves along a straight horizontal line, it is subjected to a force F newtons that acts in the direction of motion of the particle.

At time t seconds, $F = \frac{t}{5}$

Calculate the magnitude of the impulse on the particle between $t = 0$ and $t = 3$

Circle your answer.

[1 mark]

0.3 N s 0.6 N s (0.9 N s) 1.8 N s

$I = \int_0^3 t/5\ dt = [t^2/10]_0^3 = 9/10 = 0.9$ N s

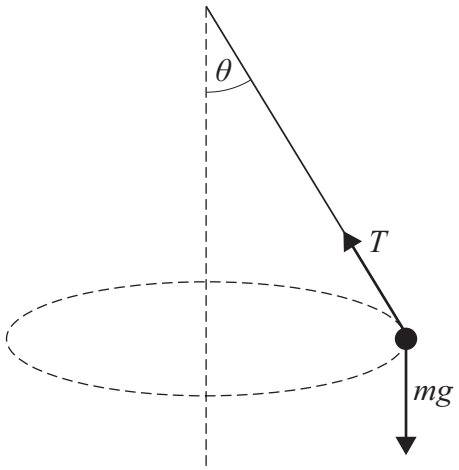
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3 A conical pendulum consists of a light string and a particle of mass m kg

The conical pendulum completes horizontal circles with radius r metres and angular speed ω radians per second. The string makes an angle θ with the downward vertical.

The tension in the string is T newtons.

The conical pendulum and the forces acting on the particle are shown in the diagram.



Which one of the following statements is correct?

Tick (✓) **one** box.

[1 mark]

$T\cos\theta = mr\omega^2$ ☐

$T\sin\theta = mr\omega^2$ ☑

$T\cos\theta = \frac{m\omega^2}{r}$ ☐

$T\sin\theta = \frac{m\omega^2}{r}$ ☐

Turn over ▶

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4 A particle of mass 3 kg is attached to one end of a light inextensible string.

The other end of the string is attached to a fixed point on a smooth horizontal surface.

The particle is set into motion so that it moves with a constant speed 4 m s^{-1} in a circular path with radius 0.8 metres on the horizontal surface.

4 (a) Find the acceleration of the particle.

[2 marks]

$a = v^2/r$

$= 4^2/0.8$

$= 20$ m s^{-2} (towards the centre)

4 (b) Find the tension in the string.

[1 mark]

$T = ma = 3 \times 20$

$T = 60$ N

4 (c) Show that the angular speed of the particle is 48 revolutions per minute correct to two significant figures.

[2 marks]

$\omega = v/r = 4/0.8 = 5$ rad s^{-1}

Revolutions per minute $= 5 \times 60/(2\pi)$

$= 47.746...$

$= 48$ rpm (2 s.f.) as required

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5 When a sphere of radius r metres is falling at v m s^{-1} it experiences an air resistance force F newtons.

The force is to be modelled as

$$F = kr^{\alpha}v^{\beta}$$

where k is a constant **with units** kg m^{-2}

5 (a) State the dimensions of F

[1 mark]

$[F] = MLT^{-2}$

5 (b) Use dimensional analysis to find the value of α and the value of β

[3 marks]

$[k] = ML^{-2}, \quad [r] = L, \quad [v] = LT^{-1}$

$[F] = [k][r]^{\alpha}[v]^{\beta}$

$MLT^{-2} = ML^{-2} \times L^{\alpha} \times (LT^{-1})^{\beta} = M\,L^{-2+\alpha+\beta}\,T^{-\beta}$

T: $-2 = -\beta \Rightarrow \beta = 2$

L: $1 = -2 + \alpha + \beta \Rightarrow \alpha + \beta = 3$

$\alpha = 3 - 2 \Rightarrow \alpha = 1$

M: $1 = 1$ (consistent)

$\therefore \alpha = 1, \quad \beta = 2$

Turn over ▶

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6 **In this question use $g = 9.8$ m s^{-2}**

A light elastic string has natural length 3 metres and modulus of elasticity 18 newtons.

One end of the elastic string is attached to a particle of mass 0.25 kg

The other end of the elastic string is attached to a fixed point O

The particle is released from rest at a point A, which is 4.5 metres vertically below O

6 (a) Calculate the elastic potential energy of the string when the particle is at A

[2 marks]

Extension at A = 4.5 − 3 = 1.5 m

EPE = $\lambda x^2/(2l)$ = $(18 \times 1.5^2)/(2 \times 3)$

EPE = 6.75 J

6 (b) The point B is 3 metres vertically below O

Calculate the gravitational potential energy gained by the particle as it moves from A to B

[2 marks]

AB = 4.5 − 3 = 1.5 m

GPE gained = mgh = 0.25 × 9.8 × 1.5

= 3.675 J

= 3.7 J (2 s.f.)

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6 (c) Find the speed of the particle at *B*

[3 marks]

At B the string is just taut, so EPE = 0

Energy: EPE lost = GPE gained + KE gained

$6.75 = 3.675 + \frac{1}{2} \times 0.25 \times v^2$

$0.125v^2 = 3.075$

$v^2 = 24.6$

$v = 4.959...$

$v = 5.0 \text{ m s}^{-1}$ (2 s.f.)

6 (d) The point *C* is 3.6 metres vertically below *O*

Explain, showing any calculations that you make, why the speed of the particle is increasing the first time that the particle is at *C*

[3 marks]

At C: extension = 3.6 − 3 = 0.6 m

Tension $T = \lambda x/l = 18 \times 0.6/3 = 3.6$ N

Weight = mg = 0.25 × 9.8 = 2.45 N

Resultant force = T − mg = 3.6 − 2.45 = 1.15 N upwards

(acceleration = 1.15/0.25 = 4.6 m s^{-2} upwards)

The first time at C, the particle is moving upwards (it was released from rest at A, below C).

The resultant force acts upwards, in the direction of motion, so the speed of the particle is increasing.

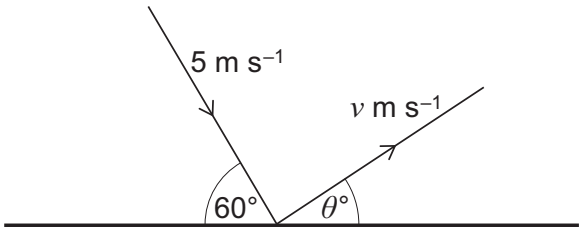
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7 A sphere, of mass 0.2 kg, moving on a smooth horizontal surface, collides with a fixed wall.

Before the collision the sphere moves with speed 5 m s^{-1} at an angle of 60° to the wall.

After the collision the sphere moves with speed v m s^{-1} at an angle of θ° to the wall.

The velocities are shown in the diagram below.



The coefficient of restitution between the wall and the sphere is 0.7

7 (a) Assume that the wall is smooth.

7 (a) (i) Find the value of v

Give your answer to two significant figures.

[4 marks]

Parallel to wall (smooth, no impulse): $v\cos\theta = 5\cos 60° = 2.5$

Perpendicular (restitution): $v\sin\theta = 0.7 \times 5\sin 60°$

$= 3.0310...$

$v^2 = (5\cos 60°)^2 + (3.5\sin 60°)^2$

$= 2.5^2 + 3.0310...^2 = 15.4375$

$v = \sqrt{15.4375} = 3.929...$

$v = 3.9$ (2 s.f.)

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7 (a) (ii) Find the value of θ

Give your answer to the nearest whole number.

[2 marks]

$\tan\theta = v\sin\theta / v\cos\theta = 3.5\sin60° / (5\cos60°)$

$= 3.0310... / 2.5 = 1.2124...$

$\theta = 50.48...°$

$\theta = 50$

7 (a) (iii) Find the magnitude of the impulse exerted on the sphere by the wall.

Give your answer to two significant figures.

[2 marks]

Taking away from the wall as positive (perpendicular to wall):

$I = 0.2 \times 3.929\sin50.48° - (-0.2 \times 5\sin60°)$

$= 0.2 \times (3.0310... + 4.3301...) = 1.4722...$

$I = 1.5$ N s (2 s.f.)

7 (b) In reality the wall is not smooth.

Explain how this would cause a change in the magnitude of the impulse calculated in part **(a)(iii)**.

[2 marks]

Friction acts parallel to the wall only, so it does not change the velocity components perpendicular to the wall; the component of the impulse perpendicular to the wall is unchanged.

Friction gives the impulse an extra component parallel to the wall, so the magnitude of the total impulse would increase.

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8 The finite region enclosed by the line $y = kx$, the x-axis and the line $x = 5$ is rotated through 360° around the x axis to form a solid cone.

8 (a) (i) Use integration to show that the position of the centre of mass of the cone is independent of k

[4 marks]

Let ρ be the density of the cone (uniform). Vertex at O, axis along x.

Mass: $\pi\rho \int_0^5 (kx)^2\,dx = \pi\rho k^2 [x^3/3]_0^5 = 125\pi\rho k^2/3$

Moments about O: $\pi\rho \int_0^5 x(kx)^2\,dx = \pi\rho k^2 [x^4/4]_0^5$

$= 625\pi\rho k^2/4$

So $(125\pi\rho k^2/3) \times \bar{x} = 625\pi\rho k^2/4$

$\bar{x} = (625\pi\rho k^2/4) \div (125\pi\rho k^2/3)$

$= (625 \times 3)/(4 \times 125)$

$= 15/4$

$\bar{x} = 15/4$ from the vertex, which does not contain k,

so the position of the centre of mass is independent of k.

8 (a) (ii) State the distance between the base of the cone and its centre of mass.

[1 mark]

Distance from base = $5 - 15/4 = 5/4$

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8 (b) State one assumption that you have made about the cone.

[1 mark]

The cone is a uniform solid (constant density).

8 (c) The plane face of the cone is placed on a rough inclined plane.

The coefficient of friction between the cone and the plane is 0.8

The angle between the plane and the horizontal is gradually increased from 0°

Find the range of values of k for which the cone slides before it topples.

[4 marks]

Let α be the angle of the plane to the horizontal.

On the point of sliding: $F = \mu R$, $mg\sin\alpha = 0.8\, mg\cos\alpha$

$\tan\alpha = 0.8 \quad (\alpha = 38.65...°)$

On the point of toppling: G is vertically above the lowest point of the base. Base radius = $5k$, G is $5/4$ above the base.

$\tan\alpha = 5k/(5/4) = 4k$

The cone slides before it topples if the sliding angle is smaller:

$0.8 < 4k$

$k > 1/5$

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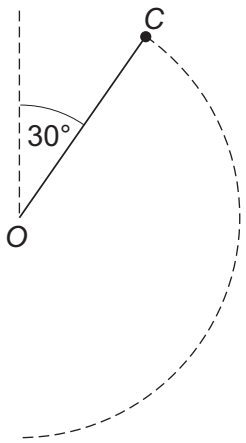
9 A small sphere, of mass m, is attached to one end of a light inextensible string of length a

The other end of the string is attached to a fixed point O

The sphere is at rest in equilibrium directly below O when it is struck, giving it a horizontal impulse of magnitude mU

After the impulse, the sphere follows a circular path in a vertical plane containing the point O until the string becomes slack at the point C

At C the string makes an angle of 30° with the upward vertical through O, as shown in the diagram below.



9 (a) Show that

$$U^2 = \frac{ag}{2}\left(4 + 3\sqrt{3}\right)$$

where g is the acceleration due to gravity.

[6 marks]

Impulse = change in momentum: $mU = mu \Rightarrow$ initial speed $u = U$

At C the string becomes slack, so $T = 0$. Resolving towards O at C:

$mg\cos30° = mv^2/a$

$v^2 = ag\cos30° = \frac{1}{2}ag\sqrt{3}$

Height of C above the lowest point $= a + a\cos30° = a(1 + \cos30°)$

Conservation of energy (lowest point to C):

$\frac{1}{2}mU^2 = \frac{1}{2}mv^2 + mga(1 + \cos30°)$

$U^2 = v^2 + 2ga(1 + \cos30°)$

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$U^2 = \frac{1}{2}ag\sqrt{3} + 2ga(1 + \frac{1}{2}\sqrt{3})$

$= \frac{1}{2}ag\sqrt{3} + 2ag + ag\sqrt{3}$

$= ag(2 + \frac{1}{2}\sqrt{3} + \sqrt{3})$

$= ag(2 + (3/2)\sqrt{3})$

$= (ag/2)(4 + 3\sqrt{3})$

$\therefore\ U^2 = (ag/2)(4 + 3\sqrt{3})$ as required

9 (b) With reference to any modelling assumptions that you have made, explain why giving your answer as an inequality would be more appropriate, and state this inequality.

[2 marks]

The model assumes no air resistance acts on the sphere.

In reality air resistance would act, so some energy would be lost

and a greater initial speed would be needed to reach C, so

$U^2 > (ag/2)(4 + 3\sqrt{3})$

END OF QUESTIONS

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