AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Statistics

Wednesday 14 June 2023 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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ANSWER IN THE SPACES PROVIDED

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Answer **all** questions in the spaces provided.

1 The discrete random variable A takes only the values 0, 2 and 4, and has cumulative distribution function $\mathrm{F}(a) = \mathrm{P}(A \leq a)$

a	0	2	4
$\mathrm{F}(a)$	0.2	0.6	1

Find $\mathrm{P}(A = 2)$

Circle your answer.

[1 mark]

0 0.4 0.6 0.8

2 The time, T days, between rain showers in a city in autumn can be modelled by an exponential distribution with mean 1.25

Find the distribution of the number of rain showers per day in the city.

Tick (✓) **one** box.

[1 mark]

	Distribution	Mean
☐	Exponential	0.8
☐	Exponential	1.25
✓	Poisson	0.8
☐	Poisson	1.25

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3 The masses of tins of a particular brand of spaghetti are normally distributed with mean μ grams and standard deviation 4.1 grams.

A random sample of 11 tins of spaghetti has a mean mass of 401.8 grams.

Construct a 98% confidence interval for μ, giving your values to one decimal place.

[3 marks]

98% confidence ⇒ $z = 2.3263$

CI: $\bar{x} \pm z \times \sigma/\sqrt{n}$

$= 401.8 \pm 2.3263 \times 4.1/\sqrt{11}$

$= 401.8 \pm 2.8757...$

Lower limit: $401.8 - 2.8757... = 398.924...$

Upper limit: $401.8 + 2.8757... = 404.675...$

98% confidence interval for μ: $(398.9, 404.7)$

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4 The random variable X has a normal distribution with unknown mean μ and unknown variance σ^2

A random sample of 8 observations of X has mean $\bar{x} = 101.5$ and gives the unbiased estimate of the variance as $s^2 = 4.8$

The random sample is used to conduct a hypothesis test at the 10% level of significance with the hypotheses

$H_0 : \mu = 100$

$H_1 : \mu \neq 100$

Carry out the hypothesis test.

[5 marks]

σ^2 unknown and n small, so use a t-test with $\nu = 8 - 1 = 7$

Test statistic: $t = (\bar{x} - \mu) \div \sqrt{(s^2/n)}$

$= (101.5 - 100) \div \sqrt{(4.8/8)}$

$= 1.5/\sqrt{0.6} = 1.9364... \approx 1.94$

Two-tailed test at 10%, so 5% in each tail.

Critical values: $t_7 = \pm 1.895$

1.94 > 1.895, so the test statistic lies in the critical region.

$\therefore$ Reject H_0

There is sufficient evidence at the 10% level of significance

to suggest that the mean of X is not 100

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5 A school management team oversees 11 different schools.

The school management team allows each student in the schools to choose one enrichment activity from 11 possible activities.

The school management team count the number of students in each school choosing each of the possible activities. They then conduct a χ^2-test for association with the data they have gathered.

5 (a) Exactly one of the calculated expected frequencies for the χ^2-test is less than 5

Explain why the number of degrees of freedom for the test is 90

[2 marks]

One expected frequency is less than 5, so that cell must be combined:

two rows (schools) or two columns (activities) are merged.

This leaves an 11 × 10 table (one category fewer) instead of 11 × 11

ν = (rows − 1)(columns − 1)

= (11 − 1)(10 − 1) = 10 × 9 = 90

5 (b) The school management team claims that there is an association between the school a student attends and the activity they choose.

The test statistic is 124.8

Test the claim using the 1% level of significance.

[5 marks]

H_0: There is no association between the school a student attends and the activity they choose

H_1: There is an association between the school a student attends and the activity they choose

ν = 90, 1% level of significance:

Critical value $\chi^2_{90}(1\%) = 124.116$

Test statistic = 124.8

124.8 > 124.116, so it lies in the critical region

∴ Reject H_0

There is sufficient evidence at the 1% level to suggest

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that there is an association between the school a student attends and the activity they choose, so the claim is supported.

5 (c) During the hypothesis test, the value of $\frac{(O-E)^2}{E}$, where O is the observed frequency and E is the expected frequency, was calculated for each group of students.

The values for four groups of students are shown in the table below.

Group	$\frac{(O-E)^2}{E}$
Attends school 3 and chose activity 1	0.01
Attends school 8 and chose activity 3	18.5
Attends school 8 and chose activity 7	24.2
Attends school 11 and chose activity 7	49.0

State, with a reason, which of the four groups of students represents the strongest source of association.

[1 mark]

Attends school 11 and chose activity 7, because it has the highest value of $(O-E)^2/E$ (49.0), so it contributes the most.

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6 A game consists of two rounds.

The first round of the game uses a random number generator to output the score X, a real number between 0 and 10

6 (a) Find $P(X > 4)$

[1 mark]

$X \sim U(0, 10)$, a continuous rectangular distribution

$P(X > 4) = (10 - 4)/10 = 0.6$

6 (b) The second round of the game uses an unbiased dice, with faces numbered 1 to 6, to give the score Y

The variables X and Y are independent.

6 (b) (i) Find the mean total score of the game.

[3 marks]

$E(X) = (0 + 10)/2 = 5$

Y is discrete uniform on 1, 2, 3, 4, 5, 6:

$E(Y) = (1 + 2 + 3 + 4 + 5 + 6)/6 = 21/6 = 3.5$

$E(X + Y) = E(X) + E(Y)$

$= 5 + 3.5$

$= 8.5$

Mean total score = 8.5

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6 (b) (ii) Find the variance of the total score of the game.

[3 marks]

$\text{Var}(X) = (10 - 0)^2/12 = 100/12 = 25/3$

$\text{Var}(Y) = (6^2 - 1)/12 = 35/12$

X and Y independent, so

$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$

$= 100/12 + 35/12$

$= 135/12$

$= 11.25$

Variance of total score = 11.25

Turn over for the next question

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7 Company *A* uses a machine to produce toys.

The number of toys in a week that do not pass Company *A*'s quality checks is modelled by a Poisson distribution X with standard deviation 5

The machine producing the toys breaks down.

After it is repaired, 16 toys in the next week do not pass the quality checks.

7 (a) Investigate whether the average number of toys that do not pass the quality checks in a week has changed, using the 5% level of significance.

[7 marks]

Standard deviation 5 $\Rightarrow$ $\lambda = 5^2 = 25$

Let λ = mean number of toys per week not passing the checks

H_0: $\lambda = 25$

H_1: $\lambda \neq 25$

Under H_0, $X \sim Po(25)$

Two-tailed test at 5%, so 2.5% in each tail.

16 < 25, so consider the lower tail:

$P(X \leq 16) = 0.03774...$

$0.0377 > 0.025$

(equivalently $P(X \leq 15) = 0.0223$, so the lower critical region is $X \leq 15$, and 16 is not in it)

Not significant, so accept H_0

There is insufficient evidence at the 5% level to suggest that the average number of toys per week that do not pass the quality checks has changed.

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7 (b) For the test carried out in part **(a)**, state in context the meaning of a Type II error.

[1 mark]

Concluding that the average number of toys per week that do not pass the quality checks has not changed, when in fact it has changed.

7 (c) Company *B* uses a different machine to produce toys.

The number of toys in a week that do not pass Company *B*'s quality checks is modelled by a Poisson distribution Y with mean 18

The variables X and Y are independent.

Find the distribution of the total number of toys in a week produced by companies *A* and *B* that do not pass their quality checks.

[1 mark]

$X + Y \sim Po(25 + 18)$

$X + Y \sim Po(43)$

7 (d) State **two** reasons why a Poisson distribution may not be a valid model for the number of toys that do not pass the quality checks in a week.

[2 marks]

Reason 1 Toys failing the checks may not occur independently, e.g. one machine fault could cause several faulty toys in a row.

Reason 2 The rate of failing toys may not be constant over time, e.g. the number of toys produced each week may vary with demand.

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8 The continuous random variable X has probability density function

$$f(x) = \begin{cases} k \sin 2x & 0 \leq x \leq \frac{\pi}{6} \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

8 (a) Show that $k = 4$

[4 marks]

$\int_0^{\pi/6} k \sin 2x \, dx = 1$

$k \left[-\frac{1}{2} \cos 2x \right]_0^{\pi/6} = 1$

$k \left(-\frac{1}{2} \cos(\pi/3) + \frac{1}{2} \cos 0 \right) = 1$

$k \left(-\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times 1 \right) = 1$

$k \left(-\frac{1}{4} + \frac{1}{2} \right) = 1$

$k/4 = 1$

$\therefore \; k = 4$ as required

8 (b) Find the cumulative distribution function $F(x)$

[3 marks]

For $0 \leq x \leq \pi/6$:

$F(x) = \int_0^x 4 \sin 2t \, dt = \left[-2 \cos 2t \right]_0^x$

$= -2 \cos 2x + 2 = 2 - 2 \cos 2x$

$$F(x) = \begin{cases} 0 & x < 0 \\ 2 - 2 \cos 2x & 0 \leq x \leq \pi/6 \\ 1 & x > \pi/6 \end{cases}$$

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8 (c) Find the median of X, giving your answer to three significant figures.

[2 marks]

$F(m) = 0.5$: $2 - 2\cos 2m = 0.5$

$\cos 2m = 0.75$

$2m = \cos^{-1}(0.75) = 0.72273...$

$m = 0.36136...$

Median = 0.361 (3 s.f.)

8 (d) Find the mean of X giving your answer in the form $\frac{1}{a}(b\sqrt{3} - \pi)$ where a and b are integers.

[5 marks]

$E(X) = \int_0^{\pi/6} 4x \sin 2x \, dx$

By parts: $u = x$, $dv/dx = 4\sin 2x \Rightarrow du/dx = 1$, $v = -2\cos 2x$

$E(X) = [-2x\cos 2x]_0^{\pi/6} - \int_0^{\pi/6} -2\cos 2x \, dx$

$= [-2x\cos 2x + \sin 2x]_0^{\pi/6}$

$= (-2 \times \pi/6 \times \cos(\pi/3) + \sin(\pi/3)) - (0 + 0)$

$= -2 \times \pi/6 \times \frac{1}{2} + \sqrt{3}/2$

$= \sqrt{3}/2 - \pi/6$

$= \frac{1}{6}(3\sqrt{3} - \pi)$

$\therefore a = 6, \; b = 3$

END OF QUESTIONS

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