AQA

Please write clearly in block capitals.

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I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Mechanics

Wednesday 14 June 2023 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 State the dimensions of power.

Circle your answer.

[1 mark]

(circled) ML^2T^{-3} ML^3T^{-3} ML^3T^{-2} ML^2T^{-2}

$[P] = [F][v] = MLT^{-2} \times LT^{-1} = ML^2T^{-3}$

2 The force $(3\mathbf{i} + 4\mathbf{j})$ N acts at the point with coordinates (0, 2)

The unit vectors **i** and **j** are directed along the x-axis and the y-axis respectively.

Calculate the magnitude of the moment of this force about the origin.

Circle your answer.

[1 mark]

(circled) 6 N m 8 N m 10 N m 14 N m

Moment $= x F_y - y F_x = 0 \times 4 - 2 \times 3 = -6$

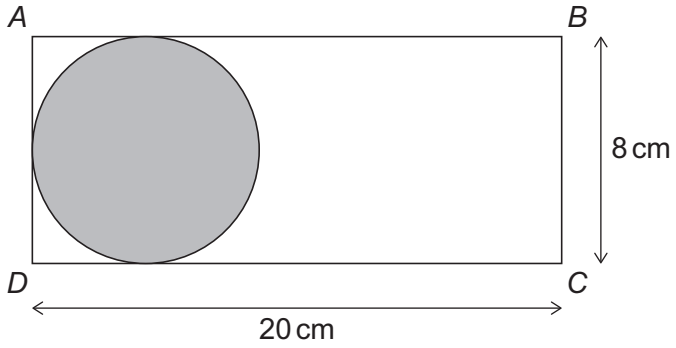
Magnitude $= 6$ N m

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3 A uniform disc has mass 6 kg and diameter 8 cm

A uniform rectangular lamina, *ABCD*, has mass 4 kg, width 8 cm and length 20 cm

The disc is fixed to the lamina to form a composite body as shown in the diagram below.



The sides *AB*, *AD* and *CD* are tangents to the disc.

Calculate the distance of the centre of mass of the composite body from *AD*

Circle your answer.

[1 mark]

4 cm 5.6 cm 6.4 cm 8.8 cm

4 A car of mass 1400 kg drives around a horizontal circular bend of radius 60 metres.

The car has a constant speed of 12 m s^{-1} on the bend.

Calculate the magnitude of the resultant force acting on the car.

[2 marks]

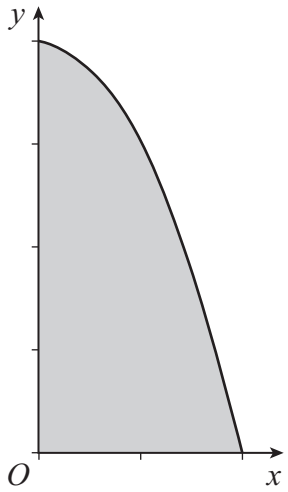
Acceleration towards centre: $a = v^2/r = 12^2/60 = 2.4$ m s^{-2}

Resultant force: $F = ma = 1400 \times 2.4$

$F = 3360$ N

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5 A region bounded by the curve with equation $y = 4 - x^2$, the x-axis and the y-axis is shown below.



The region is rotated through 360° around the x-axis to create a uniform solid.

5 (a) Show that the distance of the centre of mass of the solid from the circular face is $\frac{5}{8}$

[5 marks]

Curve meets x-axis when $4 - x^2 = 0 \Rightarrow x = 2$, so limits 0 to 2.

Volume $V = \pi\int_0^2 y^2\,dx = \pi\int_0^2 (4 - x^2)^2\,dx$

$= \pi\int_0^2 (16 - 8x^2 + x^4)\,dx = \pi[16x - 8x^3/3 + x^5/5]_0^2$

$= \pi(32 - 64/3 + 32/5) = 256\pi/15$

Let density be ρ, so mass $= 256\pi\rho/15$

Taking moments about the circular face ($x = 0$):

$(256\pi\rho/15)\,\bar{x} = \rho\pi\int_0^2 x\,y^2\,dx = \rho\pi\int_0^2 x(4 - x^2)^2\,dx$

$\int_0^2 x(4 - x^2)^2\,dx = \int_0^2 (16x - 8x^3 + x^5)\,dx$

$= [8x^2 - 2x^4 + x^6/6]_0^2 = 32 - 32 + 64/6 = 32/3$

So $(256\pi\rho/15)\,\bar{x} = 32\pi\rho/3$

$\bar{x} = (32/3) \div (256/15) = (32 \times 15)/(3 \times 256) = 480/768$

$\bar{x} = 5/8$

$\therefore$ the centre of mass is 5/8 from the circular face, as required.

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5 (b) The solid is suspended in equilibrium from a point on the edge of the circular face.

Find the angle between the circular face and the horizontal, giving your answer to the nearest degree.

[2 marks]

G is 5/8 from the face; the face has radius 4.

Suspended from A on the rim, AG is vertical.

Angle between face and vertical: $\tan\alpha = (5/8)/4 \Rightarrow \alpha = 8.88°$

Angle with horizontal θ: $\tan\theta = 4/(5/8) = 32/5$

$\theta = 81.1...° = 81°$

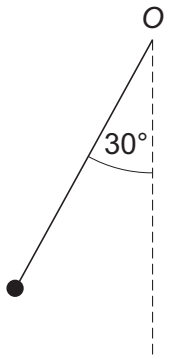
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6 **In this question use $g = 10\,\text{m s}^{-2}$**

A sphere of mass 0.8 kg is attached to one end of a string of length 2 metres.

The other end of the string is attached to a fixed point O

The sphere is released from rest with the string taut and at an angle of 30° to the vertical, as shown in the diagram below.



6 (a) Find the speed of the sphere when it is directly below O

[3 marks]

Height fallen to lowest point: $h = 2 - 2\cos 30° = 2(1 - \cos 30°)$

Energy: KE gained = GPE lost

$\frac{1}{2} \times 0.8 \times v^2 = 0.8 \times 10 \times 2(1 - \cos 30°)$

$v^2 = 40(1 - \cos 30°) = 5.358...$

$v = 2.314...$

Speed = 2 m s^{-1} (1 s.f., as $g = 10$)

6 (b) State one assumption that you made about the string.

[1 mark]

The string is light and inextensible.

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6 (c) As the sphere moves, the string makes an angle θ with the downward vertical.

By finding an expression for the tension in the string in terms of θ, show that the tension is a maximum when the sphere is directly below *O*

[6 marks]

Energy from release to angle θ (drop = 2cosθ − 2cos30°):

½ m v² = mg × 2(cosθ − cos30°) ⇒ v² = 4g(cosθ − cos30°)

v² = 40cosθ − 40cos30° = 40cosθ − 20√3

Resolving radially (towards O): T − mg cosθ = m v²/r

T − 0.8 × 10 cosθ = 0.8(40cosθ − 20√3)/2

T − 8cosθ = 16cosθ − 8√3

T = 24cosθ − 8√3

T is greatest when cosθ is greatest, i.e. cosθ = 1

cosθ = 1 ⇒ θ = 0, so the string is vertical

∴ the tension is a maximum when the sphere is directly below O.

6 (d) A physics student conducts an experiment and uses a device to measure the tension in the string when the sphere is directly below *O*

They find that the tension is 9.5 newtons.

Explain why this result is reasonable, showing any calculations that you make.

[2 marks]

Predicted maximum: T = 24 − 8√3 = 10.14 N

Measured 9.5 N is slightly less than 10.14 N. This is reasonable because air resistance would reduce the speed of the sphere, so the tension below O would be a little less than the prediction.

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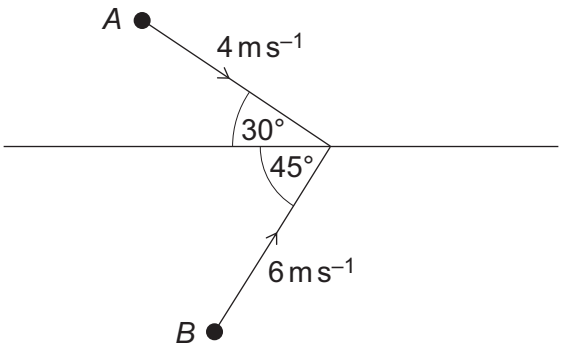
7 Two particles, A and B, are moving on a smooth horizontal surface.

A straight line has been marked on the surface and the particles are on opposite sides of the line.

Particle A has mass 2 kg and moves with velocity $4\,\text{m s}^{-1}$ at an angle of 30° to the line.

Particle B has mass 3 kg and moves with velocity $6\,\text{m s}^{-1}$ at an angle of 45° to the line.

The particles and their velocities are shown in the diagram below.



The particles collide when they reach the line and then move together as a single combined particle.

7 (a) Show that the magnitude of the impulse on particle A during the collision is 7.55 N s correct to three significant figures.

[8 marks]

Take i along the line (to the right) and j perpendicular, upwards.

$u_A = (4\cos30°\,i - 4\sin30°\,j) = 3.464\,i - 2\,j$

$u_B = (6\cos45°\,i + 6\sin45°\,j) = 4.243\,i + 4.243\,j$

Conservation of momentum (combined mass 5 kg):

$2u_A + 3u_B = 5v$

i: $2(4\cos30°) + 3(6\cos45°) = 5v_x \Rightarrow 8\cos30° + 18\cos45° = 5v_x$

j: $2(-4\sin30°) + 3(6\sin45°) = 5v_y \Rightarrow -8\sin30° + 18\sin45° = 5v_y$

$v_x = (6.928... + 12.727...)/5 = 3.931...$

$v_y = (-4 + 12.727...)/5 = 1.745...$

$v = 3.931\,i + 1.745\,j$

Impulse on A: $I_A = m_A(v - u_A)$

$I_A = 2[(3.931 - 3.464)\,i + (1.745 - (-2))\,j]$

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$I_A = 2(0.4671\,i + 3.7456\,j) = 0.934\,i + 7.491\,j$

$|I_A| = \sqrt{(0.934^2 + 7.491^2)}$

$|I_A| = \sqrt{56.99\ldots} = 7.549\ldots$

$|I_A| = 7.55$ N s (3 s.f.), as required.

7 (b) State the magnitude of the impulse on *B* during the collision, giving a reason for your answer.

[2 marks]

7.55 N s, because by Newton's third law the impulse on B is equal in magnitude but opposite in direction to the impulse on A.

7 (c) Find the size of the angle between the straight line and the **impulse** acting on *B*, giving your answer to the nearest degree.

[2 marks]

$I_B = -0.934\,i - 7.491\,j$, $\tan\alpha = 7.491/0.934$

$\alpha = 82.9\ldots^\circ = 83^\circ$

7 (d) During the collision, one particle crosses the straight line.

State which particle crosses the line, giving a reason for your answer.

[1 mark]

B crosses the line: its velocity component perpendicular to the line stays the same sign (4.24 to 1.75), while A's reverses.

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8 **In this question use $g = 9.8\,\text{m}\,\text{s}^{-2}$**

A block has mass 10 kg and is at rest 1 metre from a fixed point O on a horizontal surface.

One end of an elastic string is attached to the block and the other end of the elastic string is attached to the point O

The elastic string has modulus of elasticity 40 newtons and natural length 2 metres.

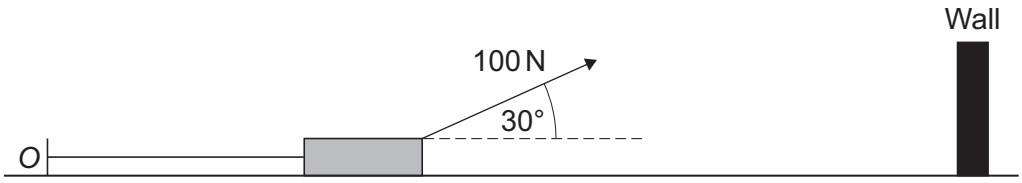
The coefficient of friction between the block and the surface is 0.6

A force is applied to the block so that it starts to move towards a vertical wall.

The block moves on a line that is perpendicular to the wall.

The force has magnitude 100 newtons and acts at an angle of 30° to the horizontal.

The situation is shown in the diagram below.



8 (a) Show that the distance that the block has moved, when the forces acting on it are in equilibrium, is 3.9 metres correct to two significant figures.

[4 marks]

Resolving vertically: $R + 100\sin30° = 10 \times 9.8$

$R = 98 - 50 = 48$ N, friction $F = \mu R = 0.6 \times 48 = 28.8$ N

Let d be the distance moved. Block starts 1 m from O and the

string is 2 m long, so extension $= (1 + d) - 2 = d - 1$

Tension (Hooke): $T = \lambda x/l = 40(d - 1)/2 = 20(d - 1)$

Equilibrium, resolving horizontally:

$100\cos30° = 0.6 \times 48 + 20(d - 1)$

$86.60... = 28.8 + 20(d - 1)$

$d - 1 = 57.80.../20 = 2.890...$

$d = 3.890...$

$d = 3.9$ m (2 s.f.), as required.

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8 (b) State one limitation of the model that you have used.

[1 mark]

Air resistance on the block has been ignored.

8 (c) Find the maximum speed of the block.

[4 marks]

Maximum speed when the forces are in equilibrium (acceleration 0),

i.e. after moving d = 3.890 m, with extension 2.890 m.

Work-energy: work by force − work against friction − EPE = KE

EPE = $\lambda x^2/(2l) = 40 \times 2.890^2/(2 \times 2) = 83.53$ J

$100\cos 30° \times 3.890 - 28.8 \times 3.890 - 83.53 = \frac{1}{2} \times 10\ v^2$

$336.90 - 112.04 - 83.53 = 5v^2$

$5v^2 = 141.33...$

$v^2 = 28.27...$

$v = 5.317...$

Maximum speed = 5.32 m s^{-1} (3 s.f.)

Turn over ▶

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8 (d) The vertical wall is 8.7 metres from *O*

Determine whether the block reaches the wall, showing any calculations that you make.

[4 marks]

Let x be the distance moved when the block comes to rest (KE = 0).

$100\cos 30° \, x - 28.8x - \frac{1}{2} \times (40/2) \times (x - 1)^2 = 0$

$86.60x - 28.8x - 10(x^2 - 2x + 1) = 0$

$-10x^2 + (100\cos 30° - 28.8 + 20)x - 10 = 0$

$10x^2 - 77.80x + 10 = 0$

$x = [77.80 + \sqrt{(77.80^2 - 400)}]/20 = 7.649...$

Distance from O = 1 + 7.649 = 8.65 m (3 s.f.)

8.65 m < 8.7 m, so modelling the block as a particle it stops

0.05 m (5 cm) short of the wall and does not reach it.

However, if the length of the block in front of the string attachment

is more than 5 cm, the block itself would reach the wall.

END OF QUESTIONS

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